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BY

ANDREW JAMIESON, M.Inst.C.E.,

FORMERLY PROFESSOR OF ENGINEERING IN THE GLASGOW AND WEST OF SCOTLAND TECHNICAL COLLEGE; MEMBER OF THE INSTITUTION OF ELECTRICAL ENGINEERS; FELLOW OF THE ROYAL SOCIETY, EDINBURGH; AUTHOR OF TEXT-BOOKS ON STEAM AND STEAM ENGINES, APPLIED MECHANICS AND MECHANICAL ENGINEERING, MAGNETISM AND ELECTRICITY, ELECTRICAL RULES AND TABLES, ETC.

VOLUME V.

THEORY OF MACHINES AND PRACTICAL MECHANISMS.

SEVENTH EDITION, REVISED AND ENLARGED.

With Numerous Diagrams, Special Plates, and Examination Questions.

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PREFACE

TO THE SEVENTH EDITION.

It has been found necessary to still further subdivide this wide and all-important subject of Advanced Applied Mechanics and Mechanical Engineering.

In order to do so with the least departure and derangement of the previous volumes and editions, it has been advisable and convenient to follow the recent subdivision of this subject as stated in the "Rules and Syllabus of Examinations applying to the Election of Associate Members of The Institution of Civil Engineers."

Moreover, this particular method of subdivision is practised by several Universities and Technical Colleges. It is also being advocated by Teachers in connection with the Boards of Education, and, to a certain extent, by those connected with the City and Guilds of London Examinations in Mechanical Engineering.

Consequently, Volume I. will deal with "Applied Mechanics" proper, Volume II. will discuss and give practical illustrations of "Strength and Elasticity of Materials," Volume III. will be confined to "The Theory of Structures," Volume IV. to "Hydraulics, Hydraulic and Refrigerating Machinery," whilst Volume V. will be greatly enlarged, and treat upon "The Theory of Machines."

Separate Contents and Index have been carefully arranged for each Volume. These enable students to find the details and pages where the different subjects are treated. The Author's system of Engineering Symbols, Abbreviations, and Index Letters have been printed at the beginning of each volume.

It is thus hoped, that the size and cost of each volume will suit the requirements of every Student of Engineering.

The Author has again to thank his chief assistant, Mr. John Ramsay, A.M.Inst.C.E., for his help, and his Publishers for the special care with which they have prepared these five volumes. The Author is specially indebted to Professor Alexander MacLay, B.Sc., C.E., and the Technical Publishing Company, Ltd., Manchester, for permission to abstract certain parts and figures from his excellent book on "*Loci* in Mechanical Drawing, including Point Paths in Mechanisms." These additions appear in Vol. V. upon the "Theory of Machines," along with the new Lectures dealing with the "Kinematics of Machinery."

The Author will feel much obliged to Engineers, Teachers, his B.Sc. and C.E. "Correspondence Students," also to any other Students of Engineering, for hints which may tend to further enhance the attractiveness and usefulness of these works.

ANDREW JAMIESON.

Consulting Engineer and Electrician,
16 Rosslyn Terrace, Kelvinside,
Glasgow, *July*, 1910.

PREFATORY NOTE TO VOLUME V.

In drawing up the details of the different Lectures of this volume, I have tried to follow the Syllabus of the final B.Sc. Examination in Engineering for External Students, as printed in the Regulations of the University of London :—

Kinematic chains ; mechanisms ; virtual centres ; relative velocities in mechanisms ; belt, rope, and chain gearing ; wheel trains and teeth ; bevel, epicyclic, worm, and helical gearing ; cams ; velocity and acceleration diagrams ; forces in a machine ; crank effort diagrams ; inertia of reciprocating parts ; governors ; flywheels ; friction and efficiency ; lubrication ; brakes and dynamometers ; valve gears and valve diagrams ; principles of balancing, with application to simple cases.

This Syllabus covers that of the Institution of Civil Engineers for their Associate Membership Examination in the subject "Theory of Machines," as well as that of several other Engineering Courses.

I could not very well adhere to the *precise* order of the London University Syllabus, since I tried to put the different sections in what appeared to me to be a somewhat more natural sequence for teaching purposes. This order will at once be gathered from an examination of the Contents as printed on pages ix to xvi.

Those who desire to still further study any part of this interesting and instructive subject should consult at least one or more of the following or other large treatises :—

(1) *Kinematics of Machines*, by J. R. Dudley, B.Sc. (John Wiley & Sons, New York ; or Chapman & Hall, Limited, London.)

(2) *Kinematics*, by C. W. MacCord, M.A., Sc.D. (John Wiley & Sons, New York.)

(3) *Kinematics of Machinery*, by Professor Reuleaux. Translated by Professor Kennedy. (M'Millan & Co., London.)

ANDREW JAMIESON.

16 Rosslyn Terrace, Kelvinside,
Glasgow, *July*, 1910.

INSTRUCTIONS FOR ANSWERING HOME EXERCISES.

(As used in my Engineering and Electrical Science Correspondence System.)

---

1. Use ordinary foolscap paper, and write on the left side *only*, leaving the facing page blank for my corrections and remarks.

2. Put the date of the Exercises at the left-hand top corner; your Name and Address in full, the name of the Subject or Section, as well as number of Lecture or Exercise, in the centre of the first page. The number of each page should be put in the right-hand top corner.

3. Leave a margin 1¼ inches wide on the left-hand side of each page, and in this margin place the *number* of the question *and nothing more.* Also, leave a clear space of *at least* 2 inches deep between your answers.

4. Be sure you understand *exactly* what the question requires you to answer, then give *all* it requires, but *no more.* If unable to fully answer any question, write down your own best attempt and state your difficulties.

5. Make your answers concise, clear, and exact, and always accompany them, if possible, by an *illustrative sketch.* Try to give (1) Side View, (2) Plan, (3) End View. Where asked, or advisable, give Sections, or Half Outside Views and Half-Sections for (1), (2), and (3).

6. Make all sketches large, open, and in the centre of the page. Do not crowd any writing about them. Simply print sizes and index letters (or names of parts), with a bold Sub-heading of what each figure or set of figures represent.

7. The character of the sketches will be carefully considered in awarding marks to the several answers. Neat sketches and "*index letters*," having the first letter of the name of the part, will always receive more marks than a bare written description.

8. All students are strongly recommended to *first* thoroughly study the Lecture, *second* to work out each answer in scroll, and *third* to compare it with the question and the text-book in order to see that each item has been answered. The final copy should be done without *any aid or reference* to the scroll copy or to *any text-book.* This acts as the best preparation for sitting at Examinations.

9. Reasonable and easily intelligible contractions (*e.g.*, mathematical, mechanical or electrical, and chemical symbols) are permitted.

10. Each corrected answer which has the symbols R.W. marked thereupon must be carefully re-worked as *one* of the set of answers for the following week.

ANDREW JAMIESON.

16 Rosslyn Terrace, Kelvinside, Glasgow.

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VOL. III.—Theory of Structures and Graphic Statics, with Applications to Roofs, Cranes, Beams, Girders, and Bridges.

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VOL. V.—Theory of Machines.—Tooth, Friction, Belt, Rope, Chain and Miscellaneous Gearing, with their Applications to Machines.—Shapes and Strengths of Teeth.—Automatic Tooth-Cutting Machines.— Velocity-Ratio and Power Transmitted by Gearing. —Motion and Energy.—Practical Applications to Governors, Flywheels, and Centrifugal Machines.

MECHANICAL ENGINEERING SYMBOLS, ABBREVIATIONS, AND INDEX LETTERS

USED IN VOLUMES I. TO V.

Of Professor Jamieson's "Applied Mechanics."

Prefatory Note.—It is very tantalising, as well as a great inconvenience to Students and Engineers, to find so many different symbol letters and terms used for denoting one and the same thing by various writers on mechanics. It is a pity, that British Civil and Mechanical Engineers have not as yet *standardised* their symbols and nomenclature as Chemists and Electrical Engineers have done. The Committee on Notation of the Chamber of Delegates to the International Electrical Congress, which met at Chicago in 1893, recommended a set of "Symbols for Physical Quantities and Abbreviations for Units," which have ever since been (almost) universally adopted throughout the world by Electricians.* This at once enables the results of certain new or corroborative investigations and formulæ, which may have been made and printed anywhere, to be clearly understood anywhere else, without having to specially interpret the precise meaning of each symbol letter.

In the following list of symbols, abbreviations and index letters, the *first* letter of the chief noun or most important word has been used to indicate the same. Where it appeared necessary, the *first* letter or letters of the adjectival substantive or qualifying words have been added, either as a following or as a subscript or suffix letter or letters. For certain specific quantities, ratios, coefficients and angles, small Greek letters have been used, and I have added to this list the complete Greek alphabet, since it may be refreshing to the memory of some to again see and read the names of these letters, which were no doubt quite familiar to them when at school.

* These "Symbols for Physical Quantities and Abbreviations for Units" will be found printed *in full* in the form of a table at the commencement of Munro and Jamieson's *Pocket-Book of Electrical Rules and Tables.* If a similar recommendation were authorised by a committee composed of delegates from the chief Engineering Institutions, it would be gladly adopted by "The Profession" in the same way that the present work of "The Engineering Standards Committee" is being accepted.

TABLE OF MECHANICAL ENGINEERING QUANTITIES, SYMBOLS, UNITS AND THEIR ABBREVIATIONS.

(*As used in Vols. I. to V. of Prof. Jamieson's "Applied Mechanics."*)

Quantities.	Symbols.	Defining Equations.	Practical Units.	Abbreviations of the Practical Units.
FUNDAMENTAL.				
Length,	L, l	...	Yard,	yd.
			Foot,	ft.
			Inch,	in.
Mass,	M, m	...	Pound,	lb.
Time,	T, t	...	Second,	s.
			Minute,	m.
			Hour,	h.
GEOMETRIC.				
Surface,	S, s	$S = L^2$	Square foot,	sq. ft.
			Square inch,	sq. in.
Volume,	V	$V = L^3$	Cubic foot,	cb. ft.
			Cubic inch,	cb. in.
Angle, $\angle$	$\left\{\begin{matrix}\alpha, \beta \\ \theta, \phi\end{matrix}\right\}$	$\alpha = \frac{\text{arc}}{\text{radius}}$	Degree,	1°
			Minute,	$1'$
			Second,	$1''$
			Radian $= \frac{180^\circ}{\pi}$	rn.
MECHANICAL.				
Velocity,	v	$v = \frac{L}{T}$	Foot per second,	$\frac{\text{ft.}}{\text{s.}}$
Angular velocity,	ω	$\omega = \frac{v}{L} = \frac{\theta}{t}$	Revs. per second,	r.p.s.
			Revs. per minute,	r.p.m.
			Radians per second,	ω
Acceleration,	a, g	$a = \frac{v}{T}$	Foot per sec. per sec.	$\frac{\text{ft.}}{\text{s}^2}$
Force,	F, f	...	Pound weight (gravitational unit),	lb. wt. (or lb.)
	W, w	$F = M a$	Poundal (absolute unit),	pdl.
Pressure (per unit area),	p	$p = \frac{F}{s}$	Pound per sq. inch,	lb. $\square''$
Work,	$(W\ h)$	$W\ h = F\ L$	Foot-pound,	ft.-lb.
Potential energy,	E_P	$E_P = W\ h$	Foot-pound,	ft.-lb.
Kinetic energy,	E_K	$E_K = \frac{W\ v^2}{2\ g}$	Foot-pound,	ft.-lb.
Power or activity,	HP	H.P. $= \frac{W\ h}{T}$	Horse power,	H.P.
			Ft.-lb. per min.,	ft.-lb./m.
			Ft.-lb. per sec.,	ft.-lb./s.
Moment of inertia,	I	$I = M\ k^2$	... ...	lb.-ft.^{2}
Density,	ρ	$\rho = \frac{M}{V}$	Pound per cb. ft.,	$\frac{\text{lb.}}{\text{ft.}^3}$
			Pound per cb. in.,	$\frac{\text{lb.}}{\text{in.}^3}$

OTHER SYMBOLS AND ABBREVIATIONS IN VOLS. I. TO V.

A for Areas.
B, b ,, Breadths.
C, c, k ,, Constants, ratios.
c.g. ,, Centre of gravity.
D, d ,, Diameters depths, deflections.
D_1, D_2, D_3 ,, Drivers in gearing.
E ,, Modulus of elasticity.
e ,, Velocity ratio in wheel gearing.
F_1, F_2, F_3 ,, Followers in gearing.
f_s, f_t ,, Forces of shear and tension.
H, h ,, Heights, heads.
H.P., h.p. ,, Horse-power.
B.H.P. ,, Brake horse-power.
E.H.P. ,, Effective ,,
I.H.P. ,, Indicated ,,
k ,, { Radius of gyration, or, Coef. of discharge in hydraulics.
N, n ,, Numbers—*e.g.*, number of revs. per min., number of teeth, &c.
P, Q ,, Push or pull forces.
R_1 R_2 ,, Reactions, resultants, radii, resistances.
s ,, { Seconds, space, surface. Displacement, distance.
S F ,, Shearing force.
T M ,, Torsional moment.
T R ,, Torsional resistance.
B M ,, Bending moment.
M R ,, Moment of resistance.
R M ,, Resisting moment.
T_s, T_t ,, Tensions on driving and slack sides of belts or ropes, &c.
W_L, W_T, W_U ,, Lost, total, and useful work.

x, y, z for Unknown quantities.
Z ,, Modulus of section.
Z_t ,, ,, tension.
Z_c ,, ,, compression.

Δ, δ, d for Differential signs which are prefixed to another letter; then the two together represent a very small quantity.
ϵ, e ,, Represents base of Naperian Logs = 2·7182; for example, $\log_e 3 = 1{\cdot}1$.
η ,, Efficiency.
λ ,, Length ratio of ship to model.
μ ,, Coefficient of friction.
π ,, Circumference of a circle ÷ its diameter.
ρ ,, Radius of curvature, radian.

Σ for Symbol for sum total of a number of quantities.
$\int_0^x$,, Sign of integration or summation between limits 0 and x.
$\sim$,, Sign for the difference between two quantities.
□ ,, Sign for square—*e.g.*, 10 □″ = 10 square inches.
— ,, Sign over two letters, $\overline{PQ}$, for a force acting from P to → Q, means that they represent a vector quantity, which has (1) magnitude, (2) direction, (3) sense.
$\geqq$,, Sign for equal to or greater than.
$\leqq$,, Sign for equal to or less than.

GREEK ALPHABET.

A	α	Alpha.	I	ι	Iota.	P	ρ	Rho.
B	β	Beta.	K	κ	Kappa.	Σ	σ or ς	Sigma.
Γ	γ	Gamma.	Λ	λ	Lambda	T	τ	Tau.
Δ	δ	Delta.	M	μ	Mu	Υ	υ	Upsilon.
E	ϵ	Epsilon.	N	ν	Nu	Φ	ϕ	Phi.
Z	ζ	Zeta.	Ξ	ξ	Xi.	X	χ	Chi.
H	η	Eta.	O	o	Omicron.	Ψ	ψ	Psi.
Θ	ϑ	Theta.	Π	π	Pi.	Ω	ω	Omĕga.

VOLUME V.

ON

THEORY OF MACHINES.*

LECTURE I.

CONTENTS.—Loci—Oblique, Rectangular, and Polar Co-ordinates—Plotting the Path of a Moving Point—On Drawing Curves of Experimental Results — Graphic Representation of Natural Laws — Geometrical Loci in Kinematic, Geometric, or Algebraic Form—Example I.—The Conicograph—Explanation of Terms—Definition of the Locus of a Point—Exercises on Loci—Exercises I. and II.—Questions.

Loci.—To make the position of any point in space perfectly definite, we require to know its distance from three fixed planes of reference. On paper we require two fixed lines of reference. For, if we consider the means of fixing the position of a point on paper, we shall find that we cannot do with less than two measurements, and that these two must be made in a particular way.

Suppose that we begin with one fixed point of reference. The distance of any other point from that may be given. But the one measurement would not fix the position of that other point, as it might occupy any position in the circumference of a circle, having the given distance as its radius, and the fixed point of reference as its centre. Next, the distances of any point from two fixed points of reference may be given. This would be ambiguous, as the point required might then occupy a position on either side of the line, joining the two fixed points. Similarly we might use three fixed points of reference, but that involves three measurements.

If now we take a point and a line for reference, we have the same ambiguity as when using two points. With a given

* As mentioned in the Preface, I am specially indebted to Prof. M'Lay and the Technical Publishing Company, Ltd., Manchester, for permission to abstract certain parts and figures from his book on "Loci," &c. My publishers, Charles Griffin & Co., Ltd., London, have, however, kindly redrawn all the said figures, and had the blocks made, for clearness, on a larger scale, whilst the said abstracts and sub-headings to the figures have been so edited as to conform with the other four previous volumes of this series on *Applied Mechanics and Mechanical Engineering*.

distance as radius, draw a circle round the fixed point and a parallel to the line at a distance equal to the radius. The circle cuts the parallel in two places, except in one particular case—

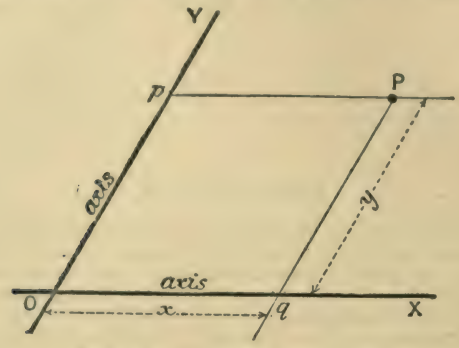

FIG. 1.—OBLIQUE CO-ORDINATES.

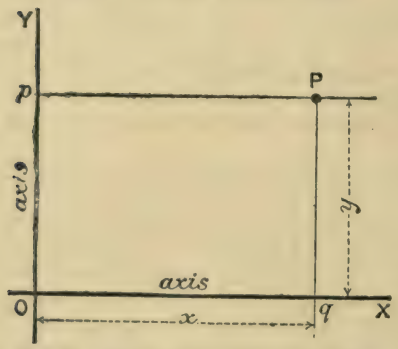

FIG. 2.—RECTANGULAR CO-ORDINATES.

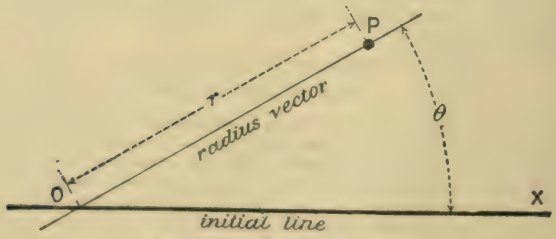

FIG. 3.—POLAR CO-ORDINATES.

viz., when the distance given is exactly half the perpendicular from the fixed point to the fixed line.

If we next try two fixed lines of reference, we shall find that they will not answer the purpose when they are drawn parallel to each other, but that they will do so perfectly when they cross one another. They may cross at any angle, but are generally drawn at right angles as being more convenient. The two parallels drawn from any point to these lines of reference or "axes," give its "co-ordinate distances" from them.

Oblique and Rectangular Co-ordinates.

In Fig. 1, O X and O Y are the axes set obliquely, and in Fig. 2 they are shown set at right angles. Pp and Pq are the co-ordinate distances of a point P, more often referred to in geometry simply as x and y respectively.

Polar Co-ordinates.

Another equally simple way of fixing the position of a point is that by two measurements, one linear and one angular, shown in Fig 3. O is any fixed point, called the "pole," and O X a line of reference drawn through it, as "initial line." The line joining any point P, with the pole O, is its "radius vector," and the angle between it and the initial line is the "vectorial angle." The two measurements required are the length of the radius vector and the magnitude of the vectorial angle.

Plotting the Path of a Moving Point.

Having thus found a means of fixing the position of any point P, when two measurements are given, let us now consider the case when P is represented as a point moving in a certain path on the paper. This is a part of mechanical drawing of very great importance, as it forms the basis of all diagrammatic representations, when the product of two variable but mutually dependent quantities has to be represented.

When a sufficient number of the co-ordinate distances are either given or obtainable, we can then find as many points, representing successive positions of the moving point. These, when plotted on the paper and a fair curve drawn through them, give a more or less accurate representation of the path, according to the number of points taken.

The line or curve so found may represent :—

(1) The results of some experiment ;
(2) Some physical law ;
(3) Some geometrical curve or figure.

In (1) the data are supplied in the form of tabulated results. For example, in experiments on the strength of materials in testing machines, the two quantities of which we wish to know the relation throughout are the stress and the strain. In other words, the resistance of the test piece and the corresponding deformation which it suffers during the experiment. Observations are made as to the amount of the one, and measurements taken as to the extent of the other, which when represented to suitable scales, give us the series of co-ordinate distances necessary for drawing the curve, which will show the behaviour of the material under a constantly increasing load (see Vol. II., Lect. VII., &c., for Stress-Strain Diagrams).

In (2) the data may be presented in the form of an equation, which is the algebraic expression of the law, and the curve drawn is termed the "*locus*" of the equation, in which x and y stand for the co-ordinate distances as shown in figs. 1 and 2, or r and θ as in Fig. 3. By assuming a series of values of x, for example, and finding the corresponding values of y, we can obtain as many co-ordinate distances as are necessary to draw the curve required (see Vol. I., Lect. X., for Plotting Curves of Varying Velocities).

In (3) the data is usually supplied in the ordinary way for constructions in geometrical exercises; but the connection between these geometric curves or figures may be made more apparent to the mind of the student by considering them also as "loci," or paths of points moving on the paper according to certain conditions. The conditions may be given in the equational form, with reference to the axes before referred to, but they may be expressed more simply as follows, beginning with those known as the conic sections :—

(*a*) *A Straight Line.*—The path of a point which moves so that its distances from two fixed lines or axes are either always equal or in some fixed proportion to one another.

(*b*) *A Circle.*—The path of a point which moves so as to be always at the same distance from some fixed point (its centre).

(*c*) *A Parabola*
(*d*) *An Ellipse*
(*e*) *An Hyperbola*

is the name given to the curve which shows the path of a point, according as its distance from a fixed point is

equal to—
less or greater (in some fixed proportion) than

its distance from a fixed straight line.

(*f*) *A Cycloid*
(*g*) *An Epicycloid*
(*h*) *An Hypocycloid*

is the name given to the curve which shows the path described by a point on the circumference of a circle, according as the circle rolls

on a straight line, upon a circle or under a circle.

(*i* to *n*) Each of the figures *f*, *g*, and *h* may be prolate or curtate, according as the describing point is within or without the rolling circle, and then the curves are called "trochoids."

(*o*) *An Involute.*—The curve described by a point at the end of a wire while it is being unwound from the surface of a cylinder (in a plane perpendicular to its axis).

(*p*) *A Spiral.*—The path of a point which moves round a fixed point as centre, and at the same time moves from it.

On Drawing Curves of Experimental Results.—The two variable but mutually dependent elements, previously referred to, may be correlative quantities of any kind. Among those most frequently represented by diagram are such as stress and strain; pressure and volume; motion and resistance; composition and strength; capacity and production.

Graphic Representation of Natural Laws.—We now come to the second class of curves (according to our previous division) which are of interest to engineers.

In curves drawn to experimental data some kind of proportionality may or may not exist between the pair of co-ordinate distances for any one point and those of any other; but in those now to be considered the co-ordinate distances are in some fixed proportion throughout, the relation being expressed in the form of an equation. While therefore in the one case a particular statement is necessary for fixing each individual point in the curve; in the other, one general statement—the equation—includes every point. The equation is reduced to figures, and from these the curve is drawn precisely in the same manner as has been indicated in the last section.

Geometrical Loci.—The next class of curves (according to our division) is that representing geometrical conditions.

The conditions may be given in (1) *kinematic*, (2) *geometric*, or (3) *algebraic* form.

(1) As those for determining the path of a material particle moving in a plane, when the only thing considered is the motion, as change of position, not the force producing or opposing it.

(2) As those for the construction in whole or in part of some curve or figure in practical plane geometry.

(3) As those which may be put in the form of an equation in plane co-ordinate geometry.

The curve may thus represent (1) the plane path of a moving point in some mechanism; hence of use as an introduction to the kinematics (or geometry) of machinery. (2) The outline or figure of some object or detail in constructive engineering; therefore useful in mechanical drawing generally. (3) The

locus of an equation, useful in putting a mathematical result in graphic form.

The following simple examples will illustrate the distinction:—

The straight line O Q, in Fig. 4, may represent—

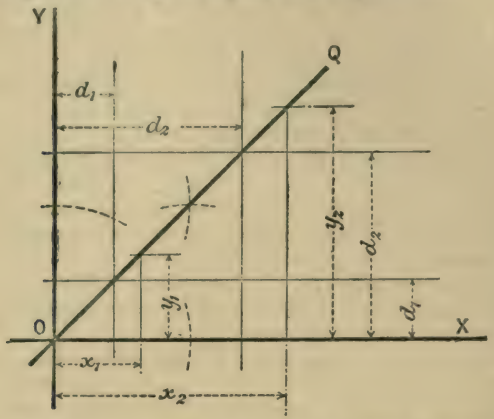

FIG. 4.—ILLUSTRATING CURVE OF PLANE MOTION.

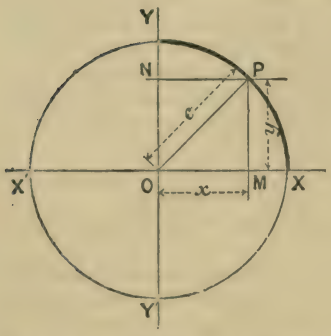

FIG. 5.—ILLUSTRATING OUTLINE CURVE.

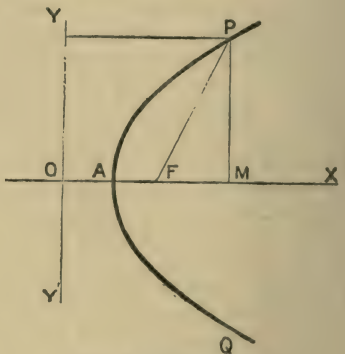

FIG. 6.—LOCUS OF AN EQUATION.

1. The path of a point which moves equidistantly between the two lines O X, O Y, as found by drawing two pairs of parallels at distances d_1 and d_2 to O X and O Y, and a line O Q through their intersections.

2. The locus of the equation $x = y$, as found by measuring off two values of x and the same of y, as shown by x_1, y_1 and x_2, y_2.

The curve X P Y, in Fig. 5, may represent—

1. The path of a point which moves round the fixed point O at a constant distance O P from it.

2. A circle drawn from O as centre, with radius O P.

3. The locus of the equation $x^2 + y^2 = c^2$, in which $x = O M$; $y = P M$, and $c = O P$.

Or again, the curve Q A P, in Fig. 6, may represent—

1. The path of a point which moves and keeps at an equal distance from the fixed point F and the straight line Y O Y'.

2. A parabola with F as focus, described by any of the geometrical methods, to be afterwards shown.

3. The locus of the equation $y^2 = 4a(x - a)$, in which $O M = x$; $P M = y$; $O A = A F = a$.

EXAMPLE I.—Show the path of a point on a line, when the extremities of the line are constrained to move in different paths.

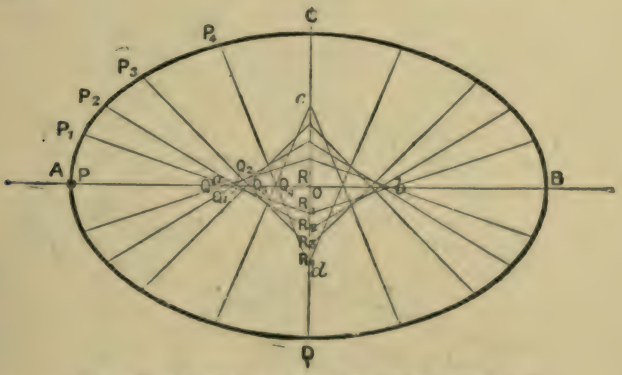

FIG. 7.—DIAGRAM OF AN ELLIPTIC POINT PATH.

Case (*a*).—When both extremities of a line are moving in straight lines, the one in a direction at right angles to the other.

ANSWER.—In Fig. 7, Q R is the line, and P is a point on the line produced. While Q moves along AB and R along CD, P describes the elliptic path ACBD.

Construction.—Draw two lines, A B and C D, at right angles to one another, meeting at O. Along OA measure $Oa = RQ$, say 1 in., and $aA = QP$, say $1\frac{1}{2}$ in. A is the first position of P. Let R move down OD to position R_1, Q will come into position Q_1 on AO. Draw a line through R_1Q_1 and produce it towards Q_1, making $Q_1P_1 = QP$. Similarly, find the positions P_2, P_3, P_4, making $Q_4P_4 = Q_3P_3 = Q_2P_2 = Q_1P_1 = QP$. When R is at *d*, Q is at O, and P is at C: thus $CO = PQ$. Q now moves along OB, and R returns along DO, while P traces out CB, the second quadrant of

the ellipse. When P reaches B, Q is at b, and R has returned to O. R now begins to move along OC, while Q returns to O, and P traces out the arc BD. Lastly, while R comes back along CO to O, and Q returns along OA to its original position at a, P traces out the remainder of the ellipse and completes the figure ACBD. It will be observed that PQ + QR = OA = OB, the semi-major axes of the ellipse, and that PQ = OC = OD,

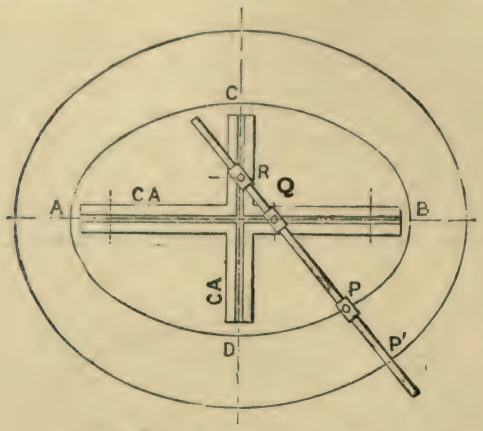

FIG. 8.—ELLIPTIC TRAMMEL.

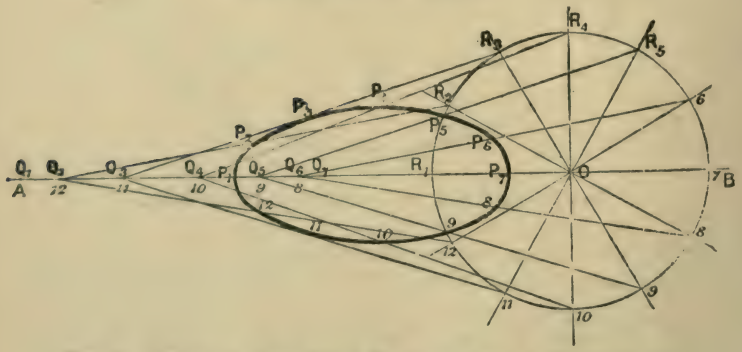

FIG. 9.—DIAGRAM OF A POINT PATH, SUCH AS A POINT IN THE CONNECTING-ROD OF A RECIPROCATING ENGINE.

the semi-minor axes. When these are given, then this is one of the methods used for drawing an ellipse in the form of the elliptic trammel, as shown by Fig. 8. It is often thus employed.

Elliptic Trammel.—In Fig. 8 there are two grooved cross arms CA of hard wood or of metal which give the necessary constrained

axial motion to pins Q and R on the cylindrical bar PQR. The relative positions of the sliding pieces P, Q, and R can be altered at pleasure to give different sizes or shapes of figures. When the adjustable sliding piece at P is shifted out to P′, whilst Q and R remain as set for the inner curve, the outer ellipse is described. It will be observed that this is not a parallel ellipse, but differs both in shape and size.

Case (*b*).—When the one extremity of a line moves in a straight line, and the other end in a circle.

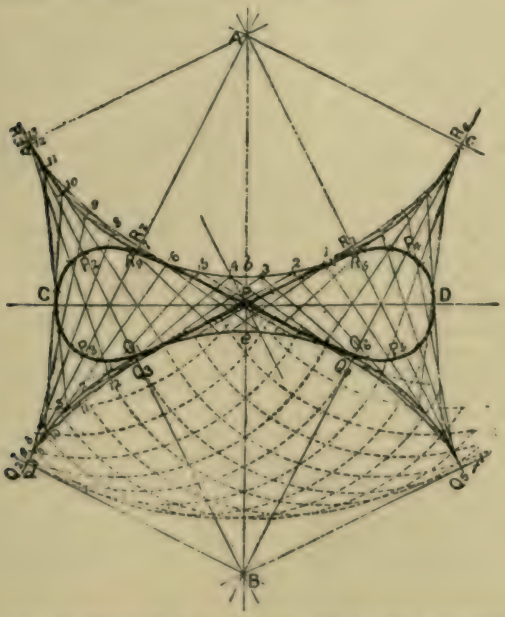

FIG. 10.—DIAGRAM OF POINT PATH, WHEN THE EXTREMITIES MOVE IN CIRCULAR ARCS.

ANSWER.—In Fig. 9, Q has a reciprocating straight line motion and R has a motion in a circle. Let the line be 4½ feet long and the stroke or range of motion be say 3 feet. Draw to a scale of $\frac{1}{12}$ the path of a point P situated midway between Q and R.

Construction.—Draw a line A B, and at a point O on it as centre, and radius, 1 foot 6 inches, describe a circle and divide its circumference into twelve equal parts. Beginning with point R_1 on the circle as centre, and with radius 4½ ft., cut the line AB at Q_1. Then from R_2 on the circle with the same radius cut the line at point Q_2. In the same way we find the remaining points 3 to 12 on the straight line, and join R_1 Q_1, R_2 Q_2, etc., and bisect each line so drawn to obtain twelve points on the elliptic path of P, as shown by the curve P_1, P_2, P_3, etc., in the figure.

Case (*c*).—When the extremities are constrained to move in circular arcs.

ANSWER.—In Fig. 10 the paths of the extremities of the line are arcs of equal radius, placed at a short distance apart, and while the extremities of the given line move along these, its middle point describes a double-looped curve, resembling the figure 8. Let A and B, two points 6 inches apart, be the centres of two arcs of circles *a b c*, and *d e f* of 2½ inches radius. Further, let the line QR, whose extremities are to move along these arcs, be equal in length to their common tangent, and let P, the point whose path is to be traced, be as in the last case the middle point of the line QR. The central position of P will, of course, be situated on the line of centres AB and midway between A and B—viz., at the point P_1. Also, the central positions of the line QR will be the tangents Q_1R_1, Q_4R_4, through P_1, to the arcs *a b c* and *d e f*, and the extreme positions of the line QR will coincide with the tangents to the arcs through their centres A and B, viz., Q_3R_3, Q_2R_2, Q_6R_6, and Q_5R_5, the corresponding positions of P being at P_2, P_3, P_4, and P_5. The extreme positions of P are at the points C and D on the line CD, the perpendicular to AB through its middle point P_1.

Construction.—Draw a straight line A B, and on it mark two points, A and B, 6 inches apart. With these as centres, radius 2½ inches, draw the arcs *a b c* and *d e f*. Bisect A B at P_1. Through P_1 draw C D, a perpendicular to A B, and tangents Q_1R_1, Q_4R_4 to the arcs. From centre A draw A*d*, A*f*, tangents to the arc *d e f*, and from centre B draw B*a*, B*c* to the arc *a b c*. By joining the points Q_1R_1, Q_4R_4, we show the middle positions, and by joining the points Q_3R_3, Q_2R_2, Q_6R_6, and Q_5R_5, the extreme positions of Q R in its range of motion. By bisecting each of these lines, the corresponding position of P is determined.

For the intermediate positions of the line, take a number of points on the one arc, and with these as centres, radius Q R, cut the other arc. Thus, on arc *a b c* take a number of points 1, 2, 3, &c., as successive positions of R, and with these as centres, radius Q R, cut the arc *d e f* at 1, 2, 3, &c., for the corresponding positions of Q. Thus, beginning with R at position R_1, Q is then at Q_1 and P at P_1, the intersection of A B and C D. Supposing the line to begin to move from that towards the left hand, R moves along the arc from R_1 through R_2 to R_3, its extreme position, while Q moves from Q_1 to Q_2 at *d*, and back again to Q_3. When Q reaches Q_4, then R is at R_4, and P has returned to its middle position, as at first, having traced out the left-hand loop of the curve. During the second half of the motion, Q moves from Q_4 to Q_5, its extreme position, and back to Q_1, while R has passed from R_4 through R_5 to R_6, and back to R_1, and P has traced out the right-hand loop and thus completed the figure.

By making the construction follow these movements, the student will find no difficulty in tracing the curve of P's motion. (Notice that the middle portions of the path are straight lines—a fact which was ingeniously taken advantage of by Watt in his Parallel Motion.)

The Conicograph.—As an example which will bring the idea of a point path before the mind of the student in a practical form, I may here refer to an ingeniously simple instrument, called the conicograph; and as it illustrates the principle which runs through a number of the preceding exercises, it may not be out of place to describe it at some length.

The conicograph is the invention of Mr. A. C. Campbell, of

Bridgeport, Conn., U.S.A.,* and its object is to enable the draughtsman to describe those curves known as the "conic sections," particularly the ellipse, parabola, and hyperbola. The student will have no difficulty in recognising at once the principle upon which the conicograph works—viz., that of describing the path of a point moving equidistantly between *a fixed point and a line or a circle.*

Instruments have been devised for describing each of those curves; but the conicograph is perhaps unique in being able to describe all three. It consists of six links, four of which are connected together by pins and eyes to form the jointed paral-

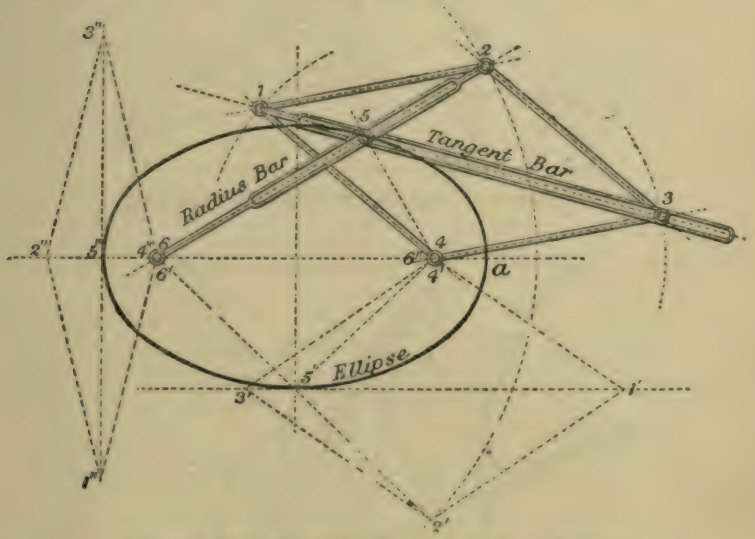

FIG. 11.—THE CONICOGRAPH DESCRIBING AN ELLIPSE.

lelogram 1, 2, 3, 4, in Figs. 11, 12, and 13. The fifth link 1, 3 is a diagonal of the parallelogram, and a tangent to the curve; hence referred to as the tangent bar. It is jointed by an eye to the pin at 1, and connected by a sliding piece to the joint at 3. The sixth link 2, 5, 6 forms the radius bar of the apparatus. It is connected with the tangent bar by a compound sliding piece at 5, which carries the describing point. It is further connected to the parallelogram at the pin joint 2, which describes the *line or circle* referred to above; and lastly, it either turns round or

* *Trans. Amer. Soc. Mech. Eng., VIII.*, 145.

slides along its other extremity at the joint 6. If we consider the straight line 2, 2′ of Fig. 12 as an arc of a circle of infinite

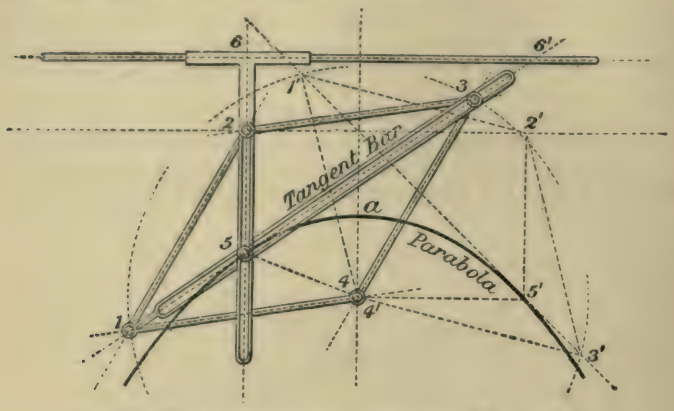

FIG. 12.—THE CONICOGRAPH DESCRIBING A PARABOLA.

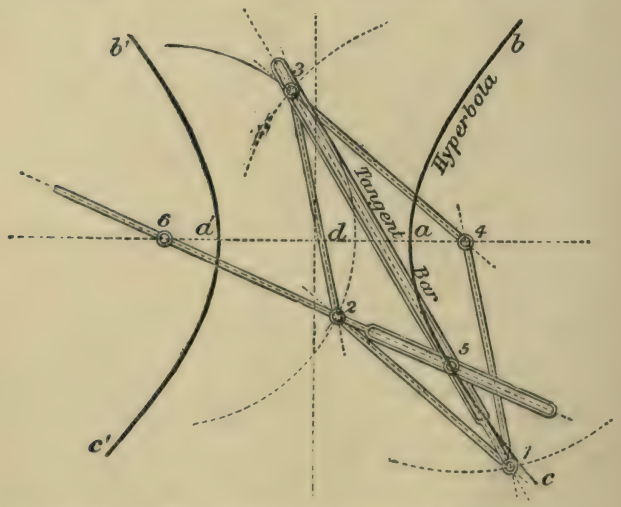

FIG. 13.—THE CONICOGRAPH DESCRIBING A HYPERBOLA.

radius, then we may say 2, 5, 6 is a radius bar in all three examples, turning round about the joint 6 at its further extremity.

Since from the nature of the mechanism as so constructed the describing point at joint 5 can only move in the parallelogram along the diagonal 1, 3, it is in all positions equidistant from the joints 2 and 4—the extremities of the other diagonal of the parallelogram. Thus it follows, that if one of these joints is centred at a *fixed point*, while the other is made to move in a *straight line or in a circle*, that we have the conditions necessary for describing the required curves.

(*a*) By causing joint 2 to move in the straight line 2, 2′, Fig. 12, and fixing the opposite joint of the parallelogram to turn round at 4, then the describing point at 5 will move equidistantly between the line 2, 2′ and the fixed point 4, and thus trace out a parabola.

(*b*) In Fig. 11 the joint 2 is caused to move in the circle 2, 2′, while the opposite joint is fixed to turn at 4, a point *within* the circle 2, 2′. The result is that the describing point at 5, moving equidistantly as it does between the circle 2, 2′ and the fixed point 4 *within* it, traces out part of an ellipse.

(*c*) In Fig. 13 the joint 2 again moves in a circle, viz., the arc 2, 3, while the opposite joint is fixed to turn at 4, a point *without* the circle 2, 3. The describing point at 5 now moves equidistantly between the circle 2, 3 and the fixed point 4 *without* it, and thus describes a hyperbola.

Figs. 11, 12, and 13 are diagrammatic representations of the construction of the instrument. For the ellipse and the hyperbola the construction is the same, but the order of the joints on the radius bar is different, as it depends upon the relative position of the fixed point and the circle. For the ellipse 2 falls beyond 5, whereas for the hyperbola it falls within 5, the order in the one case being 6, 5, 2, and in the other 6, 2, 5.

For the parabola the radius bar is fitted with a T-head for sliding along the edge of a T-square or a straight-edge; for 2, instead of describing a circle round 6, now moves parallel to it in the straight line 2, 2^1.

In Fig. 11 one position of the instrument is shown at 1.....6, and a subsequent position at 1′.....6′, when the describing point has passed from 5 to 5′. For the description of the left-hand side of the ellipse, joints 4 and 6 have to exchange places. The dotted Fig. 1″.....6″ to the left shows the position of the instrument when the describing point is at 5″. The same exchange of position has to take place between the joints 4 and 6 in drawing the left-hand curve in Fig. 13.

Explanation of Terms: Exercises on Loci.—The student should now be able of himself to find his way through a number of problems on loci, with a little assistance from notes added to them.

Those which follow are classified, not as to the manner in which they are derived, but according to result. In this way, while they form good exercises in drawing, in that they require neat and skilful working in order to obtain correct results, at

the same time many of them are useful as methods of constructing certain curves and figures, and to this attention is also directed. More important, perhaps than either of these considerations, is the training in giving a graphical representation of geometrical conditions.

A word or two may be helpful here as to the meaning of the terms used in the following heading. Previously, a locus was referred to in one place as the curve corresponding to an equation; in another as the path of a point moving on the paper according to certain conditions. The student will now be in a position to understand the general definition of the term.

Definition of the Locus of a Point.*

When a point is restricted by conditions of any kind to occupy any of a particular series of positions, that series of positions is called the locus of the point.

The term "point path" may be used to distinguish those loci which may be conceived of as being the result of mechanical movements of any kind from those which are derived from purely mathematical or geometrical considerations. For example, to an ellipse, as a curve traced through points which all lie in the same relation as to distance from a given fixed point and straight line; or, as the graphic representation of the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we would apply the name *geometrical locus*. But to an ellipse as described by means of a string tied to the foci; by the conicograph, or by the elliptic trammel; or, lastly, by a point on the connecting-rod of a steam engine, we may more aptly apply the term *point path*.

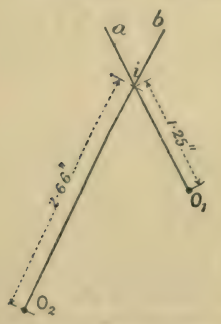

FIG. 14. — EXAMPLE OF TWO BARS PIVOTED AT TWO POINTS, AND MOVED AS STATED IN THE QUESTION.

NOTE.—*Teachers may find it convenient to set Exercises 1 and 2 to their students as class questions. Private students should work out these questions carefully at home. Students should be encouraged to make scale-working models of these problems of cardboard or wood-slip with strong pins or steel needles for the joint centres, and to test by experiment the accuracy of their drawings and calculations.*

* *Eagles' Const. Geom. of Plane Curves*, p. 29.

EXERCISE I., Fig. 14.—Two bars, $O_1 a$, $O_2 b$, are pivoted at O_1 and O_2 respectively. At i they pass through a saddle, which can travel along $O_1 a$ at $\frac{2}{3}$ths the speed at which it can move along $O_2 b$. Trace the locus of i. What is this curve?

Mark off points along $O_1 a$ and $O_2 b$ at distances from i in the proportion of 2 to 5. With O_1 and O_2 as centres, draw intersecting arcs through the points. The curve that passes through the intersections is only approximately circular, as the lengths $O_1 i$ and $O_2 i$ are not quite in the proportion of 2 to 5 to begin with.

EXERCISE II., Fig. 15.—A line P Q, of fixed length, passes through a point O, about which it turns as a fixed centre. A point R, midway between P and Q, is constrained to move along a fixed straight line A B. Trace the paths of P and Q. Take O at 1·2 inches distant from A B, and let P Q = 3·8 inches.

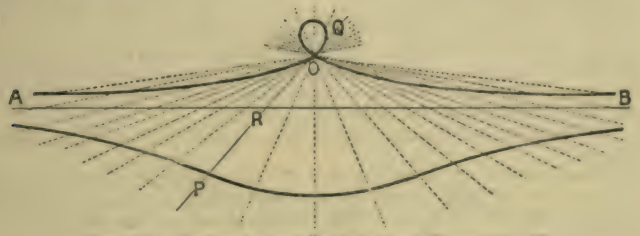

FIG. 15.—DIAGRAM TO ILLUSTRATE EXERCISE II.

Construction.—Through O, as a pole, draw a number of rays—*e.g.*, O P, cutting A B at R. Make R P one-half the given length P Q; also, cut off R Q of the same length on R O produced, if necessary.

Through the double series of points so found draw curves.

These curves are known as the *Conchoid of Nicomedes.*

Lecture I.—Questions.

1. (*a*) State what is required to make the position of any point in space perfectly definite, and also that of any point on a plain sheet of paper. (*b*) Illustrate, by sketches, and describe the methods of fixing the position of any point by oblique, by rectangular, and by polar co-ordinates.

2. Illustrate, by sketches, curves fulfilling (1) kinematic, (2) geometric, and (3) algebraic conditions.

3. Elliptic trammels are frequently used for marking-off the shapes and sizes of manhole doors in boilers, &c. Sketch and describe, as well as show, how such a piece of mechanism is derived.

4. Referring to the opposite figure of Joy's valve gear, as applied to a marine engine of this type, let the length of the stroke of the piston be 48 inches, and the length of the connecting-rod between the crosshead and crank shaft centres be 6 feet. Trace the path described by the centre of the joint J during one revolution of the crank shaft.*

5. Let the length of the locomotive connecting-rod in the opposite figure equal 5½ feet, and the distance of the centre of the Joy valve gear joint be 2 feet from centre of crosshead, whilst the stroke is 24 inches. Trace the curve described by centre of valve gear joint.*

6. Sketch and describe the conicograph. How, and for which purpose, is it used?

7. Define the term "locus of a point." The piston of a vertical engine has a travel of 18 inches, and the connecting-rod is 36 inches in length between centres of bearings. The line of action of the piston cuts the horizontal position of the crank 4 inches from the centre of the crank shaft. Draw the curve of position of the mid point of the connecting-rod for a complete revolution of the crank. Use a scale of one-eighth.

(C. & G., O., Sec. A, 1908.)

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* See the author's *Text-Book on Steam and Steam Engines, &c.*, 16th Edition, for descriptions of these two applications of Joy's valve gear, wherein the motion for working the slide valve is derived from the connecting-rod.

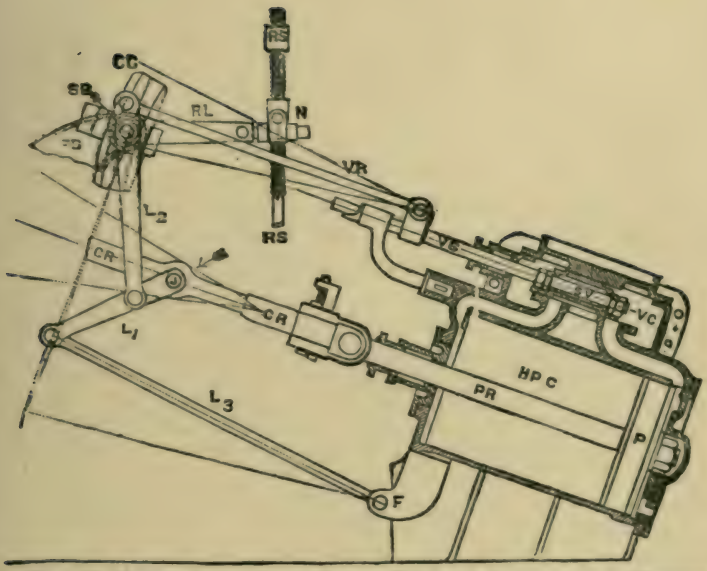


FIG. 15 (Question 4).—JOY'S VALVE GEAR AS APPLIED TO A MARINE ENGINE.

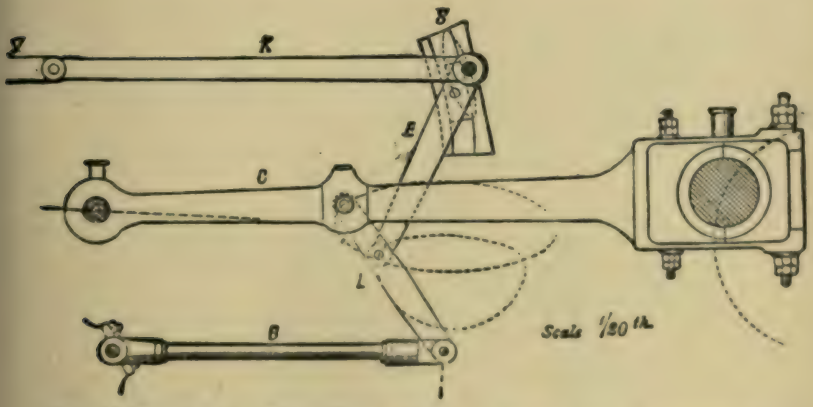


FIG 16 (Question 5).—JOY'S VALVE GEAR AS APPLIED TO A LOCOMOTIVE.

LECTURE II.

CONTENTS.—Point Paths in Mechanisms—Higher and Lower Pairing—Kinematic Pairs—Kinematic Link—Kinematic Chain—Constrained Closed Chain—Inversion of the Chain—The Slider Crank—The Double Slider Crank—Exercises I. to VII.—Questions.

Point Paths in Mechanisms.—The word mechanism is no doubt familiar enough, and every student of engineering has some idea of what it means. Nevertheless, when the question is put in an off-hand way to a class, it generally gets very partial and even contradictory answers. Two such typical answers that come nearest to the mark are: (1) A mechanism is the essential moving part of a machine; (2) a mechanism is an ideal machine. But this leads to a further question, What is a machine?—one that is even more difficult to answer, and not by students only. For instance, Reuleaux, in his *Theoretische Kinematik*, quotes no less than seventeen different definitions of a machine, by as many writers; and Professor Kennedy, in his translation of that work, adds some more.

We may look at machines in general from three different points of view—(1) objects, (2) construction, (3) action; and when we further seek to find what is the general characteristic of all machines, under each of these three heads, the modern theory of machines gives us for answers: (1) To transform natural energy into particular kinds of work by (2) a combination of resistant bodies, (3) whose relative motions are completely constrained.

From our present point of view—viz., the *kinematic*—we have regard only to the two latter heads—viz., (2) the essential form of the pieces which make up the combination of resistant bodies referred to, and (3) their relative movements as determined by the arrangement of the pieces and the nature of the connections between them.

It is with the latter (3) that we are principally concerned in the exercises which follow.

Higher and Lower Pairing.—We have to notice first, that the general characteristic of motion in machines is, that the motions are all constrained—viz., that every point moves in a definite

path, and cannot move in any other than the arrangement of the parts and the nature of the connection between them will admit of. The connection may be (*a*) *line contact—e.g.*, between the teeth of wheels or between cams, wipers, and pallets; (*b*) *surface contact—e.g.*, between a piston and its cylinder or a nut on its screw. *Line* contact between the pieces is called *higher* pairing, and *surface* contact *lower* pairing. The vast majority of pieces in machines are connected in the latter way. When we look into this we naturally expect to find a great variety of ways in which moving pieces may be connected by surface contact. But it is a remarkable fact that there are only three such ways which give complete constrainment to the motion. These are:—

1. The *solid and hollow prism*, constituting what is called the *prism* pair, from their form; or the *sliding* pair, from the fact that they permit only of a sliding motion, or motion of translation. Examples: A piston in its cylinder, a slide block in its guides, a disengaging coupling on a shaft, and a lift valve in its seat.

2. The solid and hollow *revolutes* (solids of revolution), a cylindric pair or turning pair, permitting only of a turning motion or motions of rotation. Examples: The common pin and eye, a shaft axle or spindle in its journal bearings, a loose pulley on a shaft.

3. The common screw and nut, a twisting pair, permitting only of helical motion—*i.e.*, simultaneous sliding and turning, or, more exactly, simple translation combined with simple rotation proportional to the sliding.

Kinematic Pairs.—These constraining surfaces are called kinematic elements, and, as the one is of no use without the other since there must be a pair of them, each of its own kind, they are called kinematic pairs. They are further distinguished as the three kinematic *closed* pairs, since in their case the one working surface completely envelops or encloses the other; hence also the complete constrainment of the motion of adjacent pieces. Thus points (1) on the prism pair can only move in lines parallel to the guiding surfaces; (2) on the revolutes only in circles; (3) on the screw pair only in helices. Paths 1 and 2, as *plane* motions, fall within our present scope; paths 3, as *non-plane* motions, do not. But as the great majority of the point paths in machines are plane motions, this limitation does not restrict us much. In connection with these kinematic pairs, one other point should be noticed. The pairs can be inverted without altering the point paths—*i.e.*, it does not matter which element of a pair is carried by the fixed piece and which by the moving piece, the relative motion of the pair is not altered.

Thus, in the prism pair, the solid prism may move in the hollow one, or the hollow one may move on the solid one, but the motion is one of translation in either case, and the point paths are straight lines. Similarly representing the turning pair by a wheel or an axle, the point paths are circles if either be supposed to move relatively to the other.

Thus the paths which arise from the direct use of the lower pairs are very simple, but other paths can be obtained indirectly by lower pairing, when the pieces so connected are combined. For example, referring to Fig. 8 (Lect. I.), we see a piece Q P', which has its motion determined by two turning and two sliding pairs at Q and R, give elliptic paths at P and P'. Again, in the conicograph the joints 1, 2, 4, and 6, in Figs. 11 and 13 (Lect. I.), are simple turning pairs, joint 3 a turning pair superimposed on a sliding pair, and at joint 5 a sliding pair on a sliding pair, yet the path of point 5 could be made either an ellipse or an hyperbola, according to the relative position assigned to the pairs at 2, 5, and 6 ; and by exchanging the turning pair at 6 of Fig. 13 (Lect. I.) for a sliding pair, as in Fig. 12 (Lect. I.), the path of 5 became a parabola.

Kinematic Link.—A piece may carry a number of kinematic elements, but two are sufficient to constitute it a *kinematic link*. Thus, referring again to the conicograph, Fig. 11 (Lect. I.), each of the bars, 1-2, 2-3, 3-4, 4-1, carries either a pin or an eye—*i.e.*,

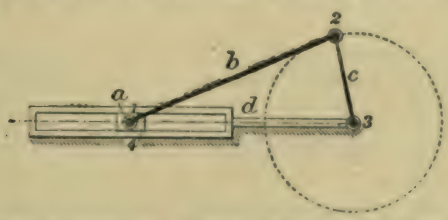


FIG. 1.—LINK MECHANISM AS IN AN ORDINARY STEAM ENGINE.

one of the elements of a turning pair—at each end, while bar 1-3 carries a slot as guide for sliding blocks at 5 and 3, and a forked eye at 1. In each of these cases the one element is of such a kind as to permit of the desired motion of its neighbour element, and to prevent every other kind of motion. Or, referring to Fig. 1 of this lecture, bar *b* carries an eye at both ends, 1 and 2, bar *c* carries a pin at 2 and at 3, bar *d* carries an eye at 3 and a hollow prism guide at 4. Each of these bars, then, in virtue of carrying a pair of elements, forms a kinematic link. But there is a fourth link in Fig. 1—viz., the block *a*, for *a*, in addition

to being a solid prism suitable for movement in the prismatic slot in bar *d*, and thus forming one element of a sliding pair, carries a pin which is the complement of the eye on *b* at 1.

Now, with reference to the possible motions of the pieces, *b* can only move relatively to *a* by turning round 1, and to *c* by turning round 2, *c* relatively to *b* by turning round 2, and to *d* by turning round 3, *d* relatively to *c* by turning round 3, and to *a* by sliding on it; and lastly, *a* can only move relatively to *d* by sliding along in it, and to *d* by turning about 1. It will also be seen that the bar *d*, which in Fig. 1 is the frame of the machine, and the block *a*, are just as much kinematic links as are *b* and *c*, the only parts which would be recognised as links in the common acceptation of the term. So also in the previous example, Fig. 11 (Lect. I.), there is a link which is apt to be overlooked. We may suppose a bar to carry the pins which form the turning centres at 4 and 6. These centres in Fig. 11 (Lect. I.) are fixed, but in carrying these two pins this supposed bar 4–6 carries two elements of the two turning pairs of 4 and 6, and is therefore as much a *kinematic link* as are any of those which actually move. It is the *fixed link* of Reuleaux, and corresponds to the *frame* of the machine of Rankine and the older writers.

Kinematic Chain.—A series of links or combination of pairs of elements, such as either of the foregoing illustrations, constitutes a *kinematic chain.*

Constrained Closed Chain.—Two links joined by any par ticular pair of elements can, as we have just seen, have only the kind of motion permitted by that pair—*i.e.*, the motion of the one is perfectly definite relatively to the other. But, if a third link be added, the motion of the third is indefinite relatively to the first, unless under a certain condition. Definite motions only occur when the links are so arranged that an alteration in the position of any one link relatively to the one next to it is accompanied by a definite alteration in the position of every other link relatively to the first. When a chain satisfies that condition, each link has only one relative motion to each other link, and the chain is then known as a *constrained closed chain.*

For example, referring again to Fig. 11 (Lect. I.), if link 2-3 be added to links 4-1 and 1-2, it is easy to see that while each adjacent pair of links have definite motions relatively to one another, links 4-1 and 2-3 would have no such definite relative motion; but let them take their places as part of the closed chain formed by the whole instrument, then the motion of every link is

perfectly definite relatively to every other. Lastly, let us fix one link of the chain. Then all the links move not only in a definite manner relatively to one another, but in a definite manner relatively to other objects around. The motions are now all of that completely definite and determinate kind which occur in machines. The chain could now represent a machine, so far as its motions are concerned. It is now, in that respect, *the ideal form of a machine*, or, in other words, it is now a MECHANISM.

In connection with the fixing of one link, a very important principle becomes apparent—viz., that a number of mechanisms may be formed out of the one chain. Thus Figs. 1 to 4

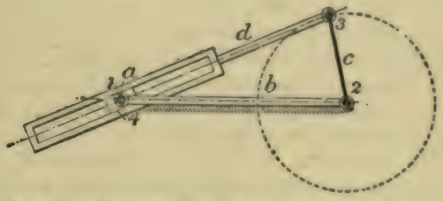


FIG. 2.—MECHANISM OF OSCILLATING STEAM ENGINE.

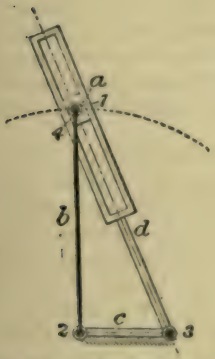


FIG. 3.—MECHANISM FOR QUICK-RETURN MOTION IN MACHINES.

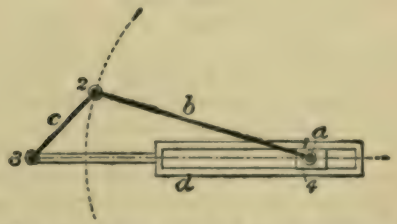


FIG. 4.—MECHANISM FOR TRANSFORMING CIRCULAR INTO RECIPROCATING MOTION, OR *vice versâ*.

show the different mechanisms which may be formed out of the four-link chain of Fig. 1. Fig. 1 includes, as has been already noticed, three turning pairs, and one sliding pair.

1. If we fix the link *d*, we get part of the mechanism of the ordinary type of steam engine. Link *d* would be the frame, consisting of bed-plate, guides, and pedestal bearing, *a* the guide block, *b* the connecting-rod, and *c* the crank.

2. Suppose now that we fix the link *b* instead of the link *d*, we again get a familiar mechanism, Fig. 2, for if we invert the

sliding pair at 1, and make the block *a* move in the guides as it does in Fig. 5, instead of the guides sliding on the block as in Fig. 2, we then see part of the mechanism of the oscillating cylinder engine.

3. In the next place, let the crank *c* now become the fixed link, and we again get a different mechanism, Fig. 3, and one which is also familiar as a means of obtaining a quick return motion in machines.

4. Lastly, if we fix the guide block *a*, we get the mechanism shown in Fig. 4, one which is also used, but which is not so common as the other three.

Inversion of the Chain.—Two or three points are worthy of the student's attention here. For one thing, he will see that the idea of a fixed link is a very important one. It shows the identity of the chain in the variety of the mechanism, and by fixing in turn each link of the chain, we see that we may form as many mechanisms out of one such chain as there are links in it. Another point to be noticed is the reciprocal nature of the relationship between these pairs of elements, which permits of the inversion of the pair without alteration of the relative motion, showing identity where we would otherwise see variety. The alteration by which we change one mechanism into another (using the same chain) in the choice of a fixed link is called *the inversion of the chain.*

The Slider Crank.—The example of a chain, which is used in the present section, is that known in Reuleaux's system as the *cylindric slider crank chain.* The mechanisms derived from it each consist essentially of three parallel cylindric (turning) pairs, and one prismatic (sliding) pair. The chain is very concisely represented by the formula $(C_3''P^{\perp})$, where C and P are the symbols for cylinder and prism respectively; the suffix 3 represents the number of cylindrical pairs of elements; the double accent above the C shows that these pairs are parallel to one another; and the symbol $\perp$ in this connection means that the axis of the prism pair is set at right angles, or normally to the axes of the cylindric pairs.

Referring again to the Figs. 1 and 4, which show the four mechanisms derived from this four-link chain, and taking them in the same order as before, they would be distinguished from one another thus: (1) a turning slider crank $(C_3''P^{\perp})^a$; (2) a swinging block slider crank $(C_3''P^{\perp})^b$; (3) a turning block slider crank $(C_3''P^{\perp})^c$; and (4) a swinging slider crank $(C_3''P^{\perp})^a$; the names being according to the characteristic motion of each, and the letter placed outside the bracket shows on which link the chain has been placed.

NOTE.—*Teachers may find it convenient to set Exercises* I. *to* VII. *to their students as class questions. Private students should work out these questions carefully at home. Students should be encouraged to make scale-working models of these problems of cardboard or wood-slip with strong pins or steel needles for the joint centres, and to test by experiment the accuracy of their drawings and calculations.*

EXERCISE I., Fig. 5.—Link *b* is a coupling-rod, connecting a sliding block *a* with a turning crank *c*. The lengths are $b = 6\frac{3}{4}$ inches, $c = 2\frac{1}{2}$ inches. Find the paths of points on *b*, situated at $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of the length of *b* from centre 1.

This is an example of a *turning* slider crank before referred to as the mechanism of the ordinary type of steam-engine. The piston-rod and piston are kinematically simply an extension of the link *a*, and the cylinder

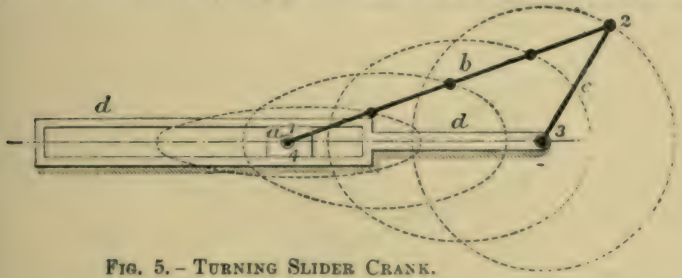


FIG. 5.—TURNING SLIDER CRANK.

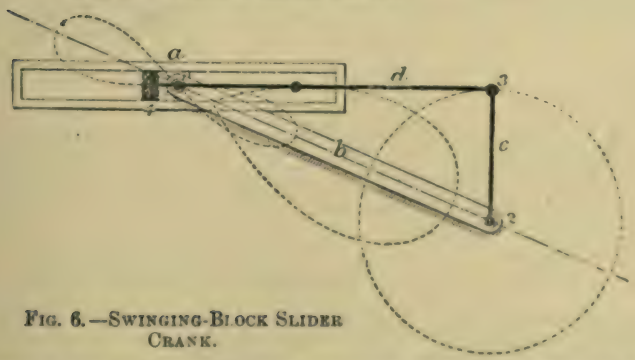


FIG. 6.—SWINGING-BLOCK SLIDER CRANK.

is part of the fixed link *d*. The particular symbol for this steam-engine mechanism is $(C_2''P^4)\,d/a$; *i.e.*, the chain is placed on link *d* and driven by link *a*. For the point paths, assume a number of positions of the crank *c*, find the relative positions of the block *a*, and draw in the centre lines representing the corresponding positions of the connecting-rod *b*. Then mark off the positions of the three points whose paths are to be described. Point 1 describes a straight line, 2 a circle, and the paths of the intermediate points are elliptical, but approximate more to the straight line or the circle, according as the describing points lie nearer to 1 or 2 respectively.

EXERCISE II., Fig. 6.—Link *a* represents the cylinder of an oscillating engine, *d* the piston and piston-rod, *c* the crank, and *b* the frame. The dimensions are: Stroke $2\frac{1}{2}$ feet, distance from centre of piston to centre of crank pin 3 feet 3 inches. Draw the path of (1) the centre of the piston, (2) the middle point of the piston-rod. Scale $\frac{1}{8}$.

A *swinging block* slider crank, the block in this instance being the cylinder and the slider the piston and piston-rod, which are kinematically one link, the one having no motion relatively to the other. The particular symbol in this instance would be $(C_3"P^1)\,b/d$. It is the same mechanism as Fig. 2 by the inversion of the sliding elements on links *a* and *d*—viz., by putting the open prism on *a* and the solid one on *d*. For the point paths, assume a number of different positions of the crank pin centre, and draw lines through them and the centre of oscillation. Measure along these lines for successive positions of the describing points.

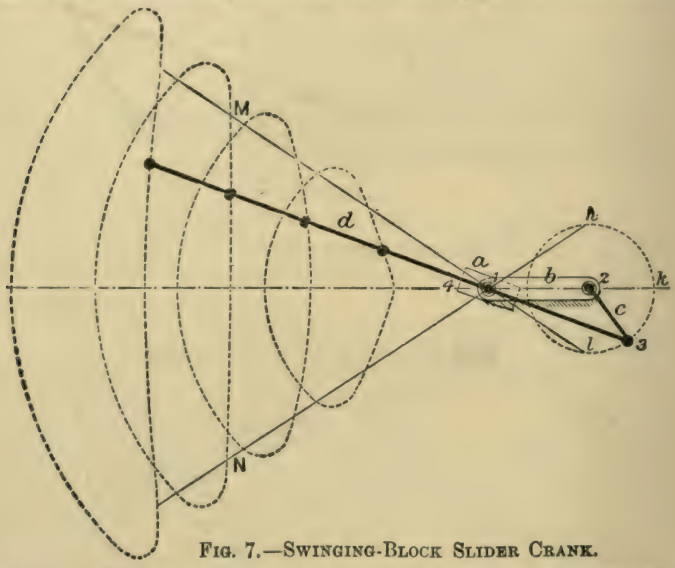


FIG. 7.—SWINGING-BLOCK SLIDER CRANK.

EXERCISE III., Fig. 7.—Link *d* swings about centre 1 and slides in the swinging block *a*. Link *c* is a turning crank, and link *b* is fixed. The dimensions are: Length of $d = 7\frac{1}{8}$ inches; of $c = 1\frac{1}{4}$ inches; distance between centres 1 and 2 = 2 inches. Find the paths of several points on the swinging link *d*.

The same mechanism as the last, viz., a swinging block slider crank, but the paths are those not of points situated between centres 1 and 3, but beyond centre 1. Notice that in Fig. 7 the path of the second point from the outer end of the link is approximately a straight line through a considerable range of the link's motion. Compare with Fig. 15 (Lect. I.). If arc *h k l*, Fig. 7, approximates to the loop on the conchoidal curve, Fig. 15, then it is evident that some point on the link *d* will describe an approximately straight line. (See also Conchoidal Parallel Motions.)

Exercise IV., Fig. 8.—The swinging link b connects the sliding block a with a second swinging link c. The dimensions are: Length of $c = 3\frac{1}{4}$ inches, of $b = 7\frac{1}{2}$ inches, of b between centres 1 and 2 = 3 inches. Find the paths of a number of points lying between centre 2 and the point P.

The same kind of mechanism as in Fig. 4, for although the chain is now placed on link d instead of a, as in that figure, the motions obtained are the same. The point paths on the prolonged part of the link b (Fig. 7) are of the same kind as those which would be obtained on the link c, produced in Fig. 4. This mechanism could also be derived from that of Fig. 1 by reversing the length ratio of the links b and c, or by attaching the link c to some intermediate point of the link b. This alteration of the lengths of the links b and c now reduces the turning crank c to a swinging link, hence the name *swinging* slider crank.

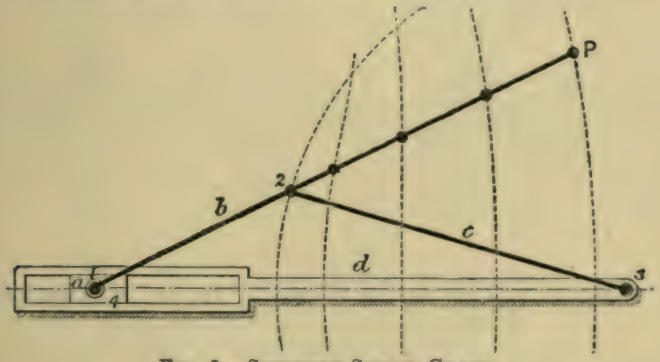


Fig. 8.—Swinging Slider Crank.

The Double Slider Crank.—The double, like the single slider crank, is a four-link chain, but as the name implies, there is now a *double* sliding pair. Instead of three turning pairs and one sliding pair, we have now two of each kind—viz., two parallel cylinder pairs (C_2''), and two pairs of prisms, either ($P_2^{\perp}$) or ($P_2^{\angle}$), according as they are set at right angles or at some oblique angle to one another. There are thus two varieties of this chain, (1) represented by the formula ($C_2''P_2^{\perp}$), in which the slides are set at right angles to one another, hence called the *normal* or double slider crank; and (2) represented by $C_2''P_2^{\angle}$, in which the slides cross obliquely, the *skew* double slider.

Figs. 9 to 12 illustrate the four mechanisms derived from the normal arrangement of the chain, by placing it on each of the links in turn. The same figures may represent the skew arrangement if we suppose the slides set at any angle other than a right angle.

Of the four mechanisms shown, two, viz., those obtained by placing the chain on links d and b, give the same motion with straight line point paths, thus $(C_2''P_2^{\perp})^d = (C_2''P_2^{\perp})^b$, or more

concisely, $(C_2'' P_2^{\perp})^{d=b}$. The other two, viz., $(C_2'' P_2^{\perp})^a$ and $(C_2'' P_2^{\perp})$ give different motions and point paths, of which Fig. 12 illustrates some of those for the former, and the latter had an illustration in Figs. 7 and 8 (Lect. I.).

$(C_2'' P_2^{\perp})^d$, Fig. 9, is distinguished as the *turning* double slider and $(C_2'' P_2^{\perp})^b$, Fig. 10, is the swinging cross block. They are of frequent application in machinery where a circular motion has to be transformed into a straight line motion, or *vice versâ*, and

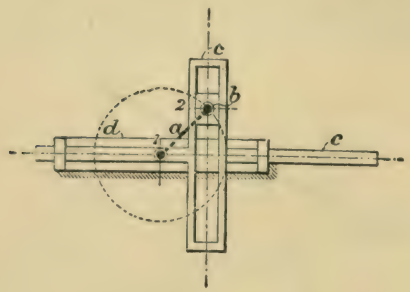


FIG. 9.—TURNING DOUBLE SLIDER MECHANISM.

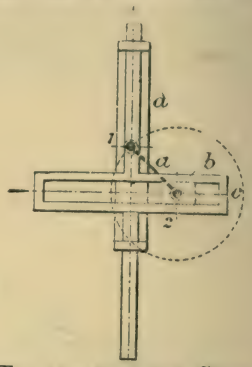


FIG. 10.—SWINGING CROSS BLOCK MECHANISM.

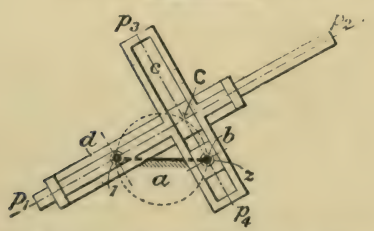


FIG. 11.—SWINGING CROSS BLOCK DOUBLE SLIDER MECHANISM.

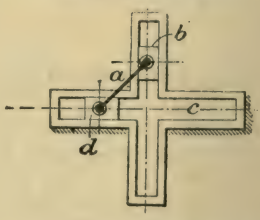


FIG. 12.—SWINGING DOUBLE SLIDER MECHANISM.

the space too limited for the use of a connecting-rod. If *a* moves uniformly, *c* has a simple harmonic motion.

$(C_2'' P_2^{\perp})^a$, Fig. 11, is the turning cross-block double slider. It has two well-known applications in Oldham's coupling for parallel shafts, and the elliptic chuck for turning ovals. In the former, link *a* represents the fixed supports, *b* and *d* the two shafts, and *c* the sliding crosspiece between them. In the latter, link *a* represents the headstock of the lathe, *b* the mandrel, *c* the chuck, and *d* the eccentric ring behind it.

$(C_2{}^nP_2{}^1)^e$ the *swinging* double slider has a well-known application in the elliptic trammel shown in the previous Lecture.

Exercise V., Fig. 9.—Link *a* represents a crank turning about a fixed centre at 1, on a frame *d*. It works a block *b* in a slide *c*, which has itself cross arms sliding in the frame *d*. Draw in dotted outline the range of motion of the cross-piece *c*. Radius of crank, 18 inches. Scale, $\frac{1}{12}$.

Exercise VI., Fig. 10.—Repeat the last exercise for the pieces *c* and *d*, when the block *b* is the fixed link, and the crank *a* turns about centre 2.

Exercise VII., Fig. 13.—Suppose a circular disc to be attached to the cross arms *c*, and that the link *a* is extended to P. Through a hole at P a pencil or other describing point is held against the face of the disc; show the curve which would be traced on it. Distance, 1 to 2 = $4\frac{1}{2}$ inches = 2 to P. The disc, which is shown by a full line, = 18 inches diameter. Scale, $\frac{1}{4}$.

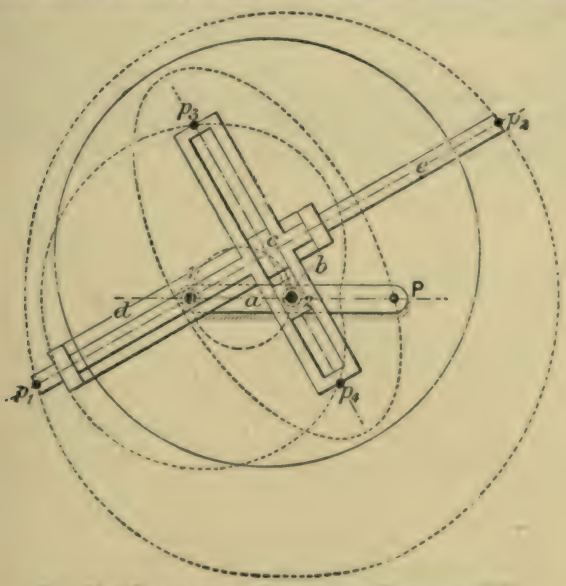


Fig. 13.—Diagram to Illustrate Exercise VII.

The curve would be the same whether we suppose *a* or *c* to be the fixed link, so long as the relative motion of these two pieces remains the same. In the case supposed, *c* is the actual moving link and *a* the fixed one; but for simplicity in drawing the curve, let *a* move and *c* be at rest, as in Fig. 12. The curve traced by P, will then be seen at once to be an ellipse, as in Figs. 7 and 8 (Lect. I.), to which refer for the method of drawing it in. If *c* represents the lathe chuck on which any object to be turned is placed, and P the turning tool held by the tool rest, which, when stationary, may be regarded kinematically as part of the lathe bed represented by the fixed link *a*; then the ellipse, obtained as above, would represent the shape to which the body would be turned.

Lecture II.—A.M.Inst.C.E. Exam. Questions.

1. A simple plane mechanism, consisting of four links, is so constructed that the relative motion of each two consecutive links, which are in contact with one another, is either turning, swinging, or sliding:—Point out the geometrical conditions which must be satisfied for a turning motion to be possible at a joint, and enumerate all the possible associations of the three kinds of motion, which include one or more turning motions. For each of these associations state the number of different machine movements obtainable by inversion. (I.C.E., *Feb.*, 1905.)

2. What do you understand by the terms "lower pair" and "higher pair" as applied to the details of mechanisms? Illustrate your answer by sketches, and give examples of each kind of pairing. Give also an example of the use of lower pairs, and a second example of the use of lower pairs in combination with higher pairs, for the formation of a kinematic chain of elements, which can in each case be used as a machine. (I.C.E., *Feb.*, 1907.)

3. Define the expressions "kinematic chain" and "closed kinematic chain," and state under what conditions the latter becomes a "mechanism." What is meant by the term "inversion of a kinematic chain"? Exemplify your remarks by means of the ordinary crank and connecting-rod mechanism of a steam engine, giving sketches. (I.C.E., *Oct.*, 1907.)

LECTURE III.

CONTENTS.—The Quadric Crank—The Lever Crank, Double Lever, and Double Crank—Exercises I. to VII.—Slider Crank Parallel Motions—Elliptic Parallel Motions—Exercises VIII. to XII.—Conchoidal Parallel Motions—Exercise XIII.—Quadric Crank Parallel Motions—Exercises XIV. and XV.—Questions.

The Quadric Crank.—Another very important chain among mechanisms is the one which consists of four links connected by four parallel cylindric pairs, called the *Quadric Cylindric Crank Chain* (C_4''). There are many varieties of it, depending upon the relative lengths of the links. To enable the student to trace the connection in the exercises which follow, let us first note, as the representative form of the chain, the arrangement in which all four links are of different lengths. Afterwards, we shall consider some modifications of this chain. In the general form, represented by Fig. 1, a is the shortest link, c the next, and b is $> d$. The four mechanisms obtained by placing the chain on each link in turn are :—

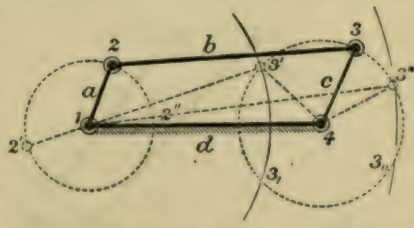


FIG. 1.—LEVER CRANK QUADRIC CHAIN.

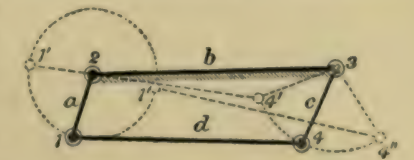


FIG. 2.—LEVER CRANK QUADRIC CHAIN.

(A and B) *The Lever Crank* (Figs. 1 and 2).—By fixing link b or link d, represented by $(C_4'')^{b=d}$. These two mechanisms give the same kind of motion. In each a is a turning link and a crank, c a swinging link and a lever. In the one b is the coupler and d the frame, and in the other the relation is reversed.

(c) *The Double Lever* (Fig. 3).—By fixing link c, represented by $(C_4'')^c$. Links b and d are now both levers with a limited range of movement, and a is the coupler.

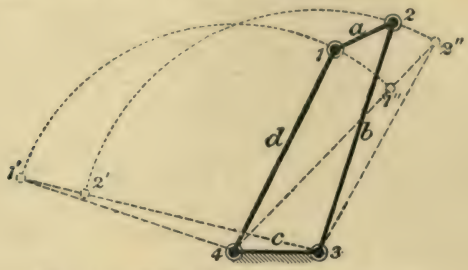


FIG. 3.—DOUBLE LEVER QUADRIC CHAIN.

(d) *The Double Crank* (Fig. 4).—By fixing link a, represented by $(C_4'')^a$. Links b and d are now both cranks, and c is the coupler.

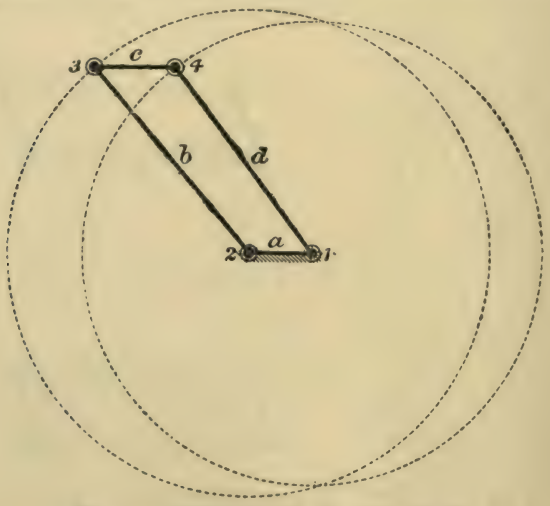


FIG. 4.—DOUBLE CRANK QUADRIC CHAIN.

There are a great number of modifications of the general form, a few of which are illustrated in Figs. 5 to 8. When, for example, opposite pairs of links are equal, the mechanism is a double crank, and if the cranks are set as a and c in Fig. 5,

we have *parallel* cranks; as in Fig. 6, *reverse anti-parallel* cranks; and, as in Fig. 7, *converse anti-parallel* cranks in allusion to the characteristic motion. In Fig. 6 the cranks are always moving in opposite directions, and in Fig. 7, though not parallel, they always move in the same direction. Fig. 8 represents those cases in which adjacent pairs of links are equal, and in the exercises there are several examples with one opposite pair equal—*e.g.*, Fig. 9.

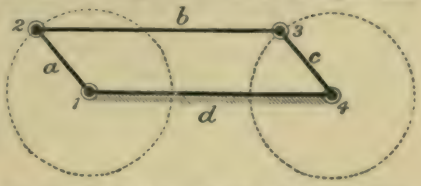


FIG. 5.—PARALLEL CRANK QUADRIC CHAIN.

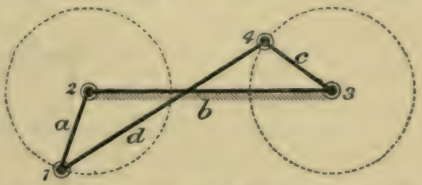


FIG. 6.—REVERSE ANTI-PARALLEL QUADRIC CHAIN MECHANISM.

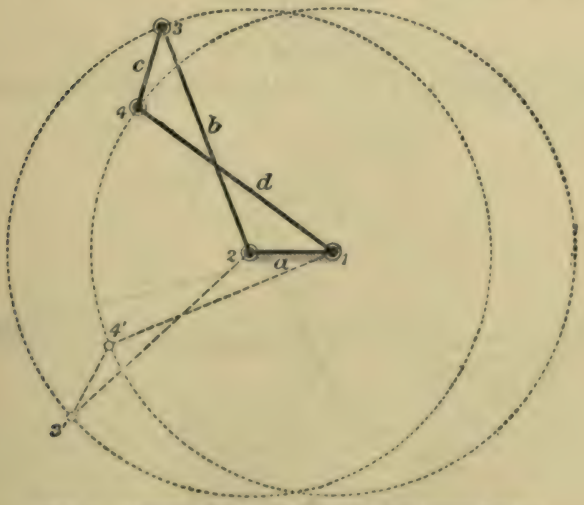


FIG. 7.—CONVERSE ANTI-PARALLEL QUADRIC CHAIN.

NOTE.—*Teachers may find it convenient to set Exercises* I. *to* XV. *to their students as class questions. Private students should work out these questions carefully at home. Students should be encouraged to make scale-working models of these problems of cardboard or wood-slip with strong pins or steel needles for the joint centres, and to test by experiment the accuracy of their drawings and calculations.*

EXERCISE I., Fig. 1.—In a certain beam engine the radii of the beam and the crank are 12 feet 3 inches and 5 feet respectively. The

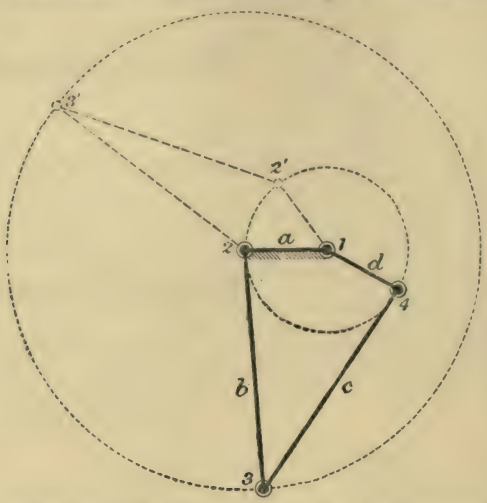


FIG. 8.—QUADRIC CRANK MECHANISM, WITH ADJACENT LINKS EQUAL.

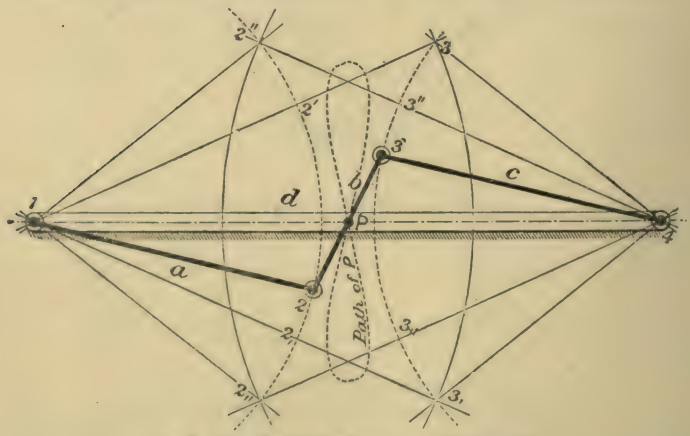


FIG. 9.—WATT'S PARALLEL MOTION.*

*For description of Watt's Parallel Motions, see Professor Jamieson's *Manual of Applied Mechanics*, 9th edition, Lecture XXIV. Also see his *Text-Book on Steam and Steam Engines*, Lecture III.

length of the connecting-rod is 23 feet 6 inches, and the distance between the turning centres of the beam and the crank is 26 feet 9 inches. Show the paths of three points on the connecting-rod, situated at $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of its length from either end of it. Scale, $\frac{1}{48}$ ($\frac{1}{4}$ inch to 1 foot).

EXERCISE II., Fig. 9.—The figure represents Watt's parallel motion. Links a and c are each $3\frac{3}{4}$ inches long, $b = 2$ inches, and $d = 8{\cdot}4$ inches. Trace the paths (1) of a point P in the middle of the coupling link b, and (2) of any other point situated between 2 and 3.

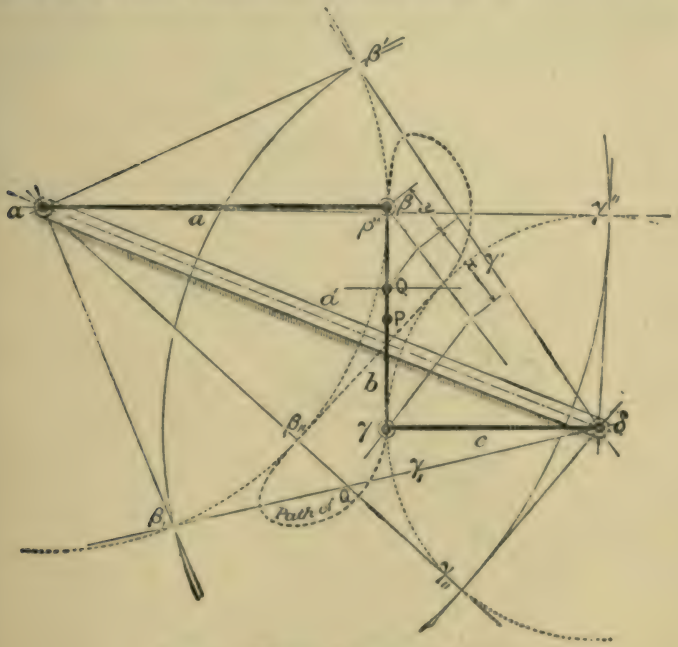


FIG. 10.—WATT'S PARALLEL MOTION.

An example of that particular case of the double lever, where $a = c$ and $> b$. To determine the range of the movements of a and c, it can easily be seen by inspection that in two positions, a and b come into one straight line, as at 1, 2′, 3′, and 1, $2_{,}$, $3_{,}$. In these cases c must then occupy its extreme positions; and *vice versa* by the symmetry of the arrangement, when b and c are in line, as at 4, 3″, 2″, and 4, $3_{,,}$, $2_{,,}$, a is in its extreme positions.

Having drawn the circular arcs, which are the paths of the extremities 2 and 3, of the link b, then, with centres 1 and 4, radius $a + b$ or $c + b$, describe arcs intersecting the paths of 2 and 3, at 2″ $2_{,,}$ and 3′ $3_{,}$ respectively; 1, 2″ and 1, $2_{,,}$ show the range of movement of a, and 4, 3′; 4, $3_{,}$, that of c. Next assume a number of positions of joint 2 in the arc 2″, $2_{,,}$, and find the corresponding positions of 3 in the arc 3′ $3_{,}$ through which to

draw as many positions of the link b. On these mark off the series of positions occupied by the points P and Q, and through them draw the required curves.

Exercise III., Fig. 10.—The figure shows a slightly different form of Watt's parallel motion from the last. Link a represents the beam, c the radius bar, and b the fore link or coupler. The lengths are $a = 4\frac{1}{2}$ inches, $b = 3$ inches, and $c = 2\frac{1}{16}$ inches. Show the path of (1) P, the middle point of the link b, and (2) Q, a point which divides b into two parts x and y, such that $x : y :: a : c$.

An example of the double lever, in which, as drawn in the figure, the adjacent links b and c are very nearly equal.

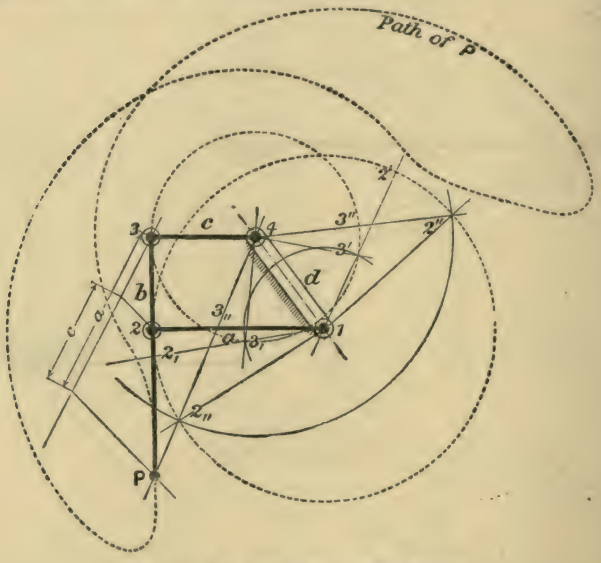


Fig. 11.—Watt's Parallel Motion.

For the range of movement of the levers a and c proceed as in Fig. 9, Exercise II. With centre a radius $a + b$, cut the circular path of γ at γ'' and $\gamma_{,,}$, and with centre δ, radius $c + b$, cut the path of β and β' and $\beta_{,}$. Then $a\,\beta'$, $a\,\beta_{,}$ show the extreme positions of link a, and $\delta\gamma''$, $\delta\gamma_{,,}$ those of link c. Assume a number of intermediate positions of b, and mark off P and Q, as in the last problem. The path of Q will be found to be approximately a straight line through a considerable range. To find distance of Q from β or γ, divide the length of the link b into two parts in the proportion of a to c; or Q may be taken at the intersection of bars b and d in the figure.

Exercise IV., Fig. 11.—The figure represents the modification of Watt's parallel motion used when the radius bar is placed above the beam. Link b is prolonged beyond joint 2 to a point P, such that $3,P : 2,P :: a : c$. Draw the path of P.

Another example of the double lever. The lengths are $a = 3$ inches, $b = 1\frac{5}{8}$ inch, $c = 1\frac{5}{8}$ inch, and $d = 2$ inches. Produce 3 - 2 and cut off 2 - P in the given proportion, as shown in figure. The ranges of movements of links a and c are found in the same way as in Exercise I. With centre 1, radius $a - b$, cut the path of 3 at 3′ and 3, and with centre 4, radius $c + b$, cut the path of 2 at 2″ and 2_n. Then 4,3′ and 4,3, are the extreme positions of link c, 1,2″ and 1, 2_n those of link a.

Exercise V.—Fig. 12 represents the *Roberts* parallel motion. Links a and c, which are equal in length, swing about centres 1 and 4. Find the path of a point P, which is rigidly attached, at distance $a = c$ from centres 2 and 3, to the coupler b. The lengths are, $d = 4{\cdot}4$ inches; $a = c = 2{\cdot}9$ inches; $b = \frac{1}{2}d = 2{\cdot}2$ inches.

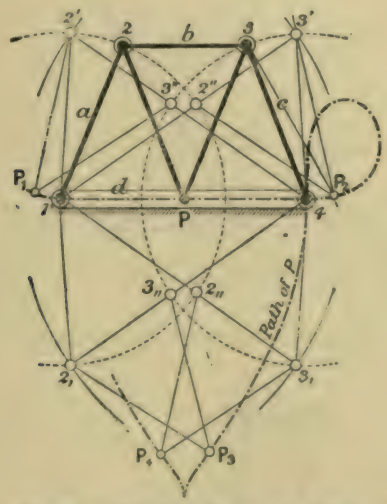


Fig. 12.—Roberts Parallel Motion.

A double lever with one opposite pair of links equal, the triangle 2P3 being simply an extension of the link b. For the construction, find a number of positions of the link b, with its centres on the circular arcs through 2 and 3, described from centres 1 and 4 respectively. The extreme positions can be found at once, as in previous examples, by drawing arcs from centres 1 and 4, with radius $a + b = c + b$, cutting the paths of joints 2 and 3 at 3′ and 3, and at 2′ and 2, respectively. These positions are shown in the figure. The geometrical basis of the arrangement and proportion of the links is shown in connection with a later example.

Exercise VI., Fig. 13.—The *Tchebicheff* parallel motion. Links a and c are swinging levers, and b their coupler. Let $a = c = 5$ inches; $d = 2b = 4$ inches. Show the path of the middle point of b.

The construction is similar to that in the last problem.

EXERCISE VII., Fig. 14.—The mechanism (double crank) now used for feathering float paddle wheels.* Draw (1) as in the last example, the paths of the extremities of the float, or (2) make a skeleton diagram showing a ten float, 20 feet paddle-wheel of the following dimensions: Link d = 1 foot 0 inches; $a = c = 8$ feet 8 inches; $b = 1$ foot 6 inches (between centres); distance, $2-5 = 5\frac{1}{4}$ inches; $5-6 = 1$ foot 4 inches, and $5-7 = 1$ foot 0 inches. Scale, $\frac{1}{48}$ ($\frac{1}{4}$ inch per foot).

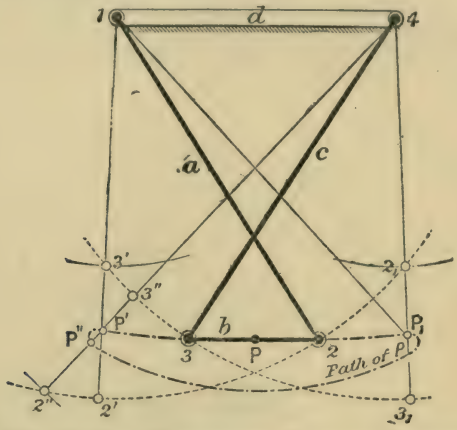


FIG. 13.—THE TCHEBICHEFF PARALLEL MOTION.

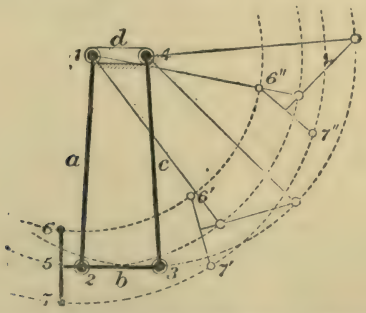


FIG. 14.—DOUBLE CRANK MECHANISM FOR FEATHERING FLOAT PADDLE WHEELS.

Slider Crank Parallel Motions.—Both the slider crank and the quadric crank chain may be illustrated by exercises on the class

* See Lecture XXI. in the 16th edition of Professor Jamieson's *Text-Book on Steam and Steam Engines, including Turbines and Boilers*, for a description and illustration of a Paddle Wheel with Feathering Floats.

of mechanisms called Parallel Motions. In these, it may be explained, the object is to get one or more points to move accurately, or at least very approximately in a straight line through a considerable range of motion, without the direct use of sliding pairs. This has already been illustrated to some extent for the quadric crank chain. In the present Lecture, let us take a look at some of the applications of the slider crank, and in the first place at the geometrical principle upon which these arrangements are based.

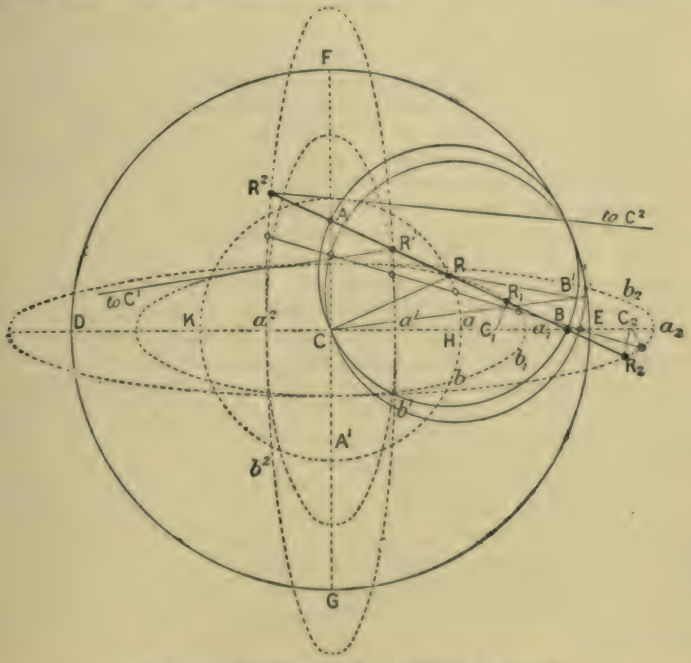


FIG. 15.—SLIDER CRANK PARALLEL MOTIONS

If a circle rolls within a fixed circle of exactly twice its diameter, the path of a point on its circumference (the hypocycloid) is a perfectly straight line. The path of any other point not on the circumference (curtate or prolate hypotrochoid) is an ellipse.

Also, if the extremities of a straight line are constrained to move in two straight lines at right angles to each other, then the path of any point on the line produced is an ellipse, and

the path of the middle point of the line a circle. But, the path of any point between the extremities is an ellipse.

In Fig. 15 these properties are shown in their connection with one another. ABC is a rolling circle, AB a diameter of it. FEG is its guiding circle, of radius equal to AB. As ABC rolls on FEG, the points A and B (the extremities of the diameter AB) trace out FG and DE respectively, two diameters of the guiding circle, which intersect at right angles at C. R, the centre of the rolling circle, traces out the circle KRH of diameter, equal to AB. Any other point between A and B, such as R^1 and R_1, or beyond A and B, such as R^2 and R_2, traces out an ellipse.

It will now be evident to the student, that if we guide two points of the line AB in their proper paths, a third point may be found which will move in a straight line, circle, or ellipse as may be required. For a parallel motion, the third point is required to move in a straight line ; therefore it must be either A or B ; and for a slider crank parallel motion one of the guided points must be either B or A. The other guided point is made to move in an arc of a circle, which is as close an approximation as possible to the actual elliptic path of the point, as shown in the diagram. For a range of angular deviation of AB to either side of its central position, not exceeding 25 degrees, this approximation can be got by finding the centre of a circle, which will pass through the middle position and the two extreme positions of the point. Thus, for example, when B is the one guided point, and R_1 is the other, a point C_1 is found, which is the centre of a circle passing through R_1 a_1 b_1. We have thus, in Fig. 15, in diagrammatic form the lengths for a mechanical arrangement, in which a swinging link AB, guided by a sliding pair at B along the line CE, is so controlled by a link C_1R_1 turning round a fixed centre C_1, that its other extremity A moves very approximately in the straight line FG. In the following diagrams the same letters denote the same relative positions of the points.

(1) Elliptic Parallel Motions.

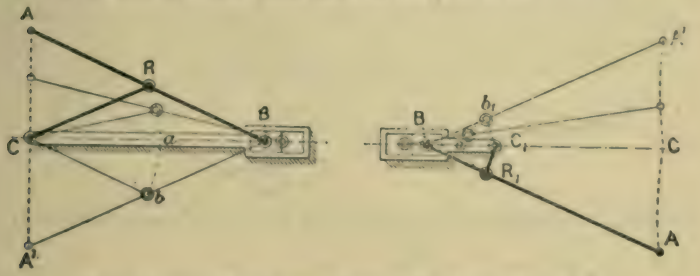
(A) Normal Arrangements.

Exercise VIII., Fig. 15.—Draw the diagram representing the figure which has just been described, supposing the rolling circle to be 3 inches diameter, and $\angle$ ABC $\geqslant$ 25 degrees. Find the paths of the points R^2, A, R^1, R, R_1, B, and R_2, assuming them to be at, say, $\frac{1}{4}$ AB apart.

Exercise IX., Fig. 16.—From the diagram in the last exercise take the proportions for a swinging slider parallel motion, in which B is the one guided point, R the middle point of AB is the other, and A is the point whose path is required.

These proportions (CR = AR = BR) give the grasshopper parallel motion of Scott Russell. Since R moves in a circle, no approximate arc is required, and the path of A is a perfectly straight line, which is of range equal to the diameter of the guiding circle in Fig. 15, or = 2(CR + RA). This case, therefore, differs from the others in the present lesson in being not an *approximate* but an *exact* parallel motion.

EXERCISE X., Fig. 17.—Take next the point R_1 of Fig. 15 at ¼ AB from end B as the second guided point, and find the path of A.



FIGS. 16 AND 17.—SLIDER CRANK PARALLEL MOTIONS.

To get a good approximation to a straight line path, it will be found necessary to limit the *total* range to 25 degrees in this case. The reason is that the actual path of R_1 here occurs at that part of the ellipse where the rate of change of curvature is greatest, and consequently the approximation of the circular arc to the elliptical arc least acceptable.

EXERCISE XI., Fig. 18.—Let the second guided point lie on AB, produced as at R_2, beyond B in Fig. 15. Find the path of A for a total angular deviation of 25 degrees.

FIG. 18.—NORMAL ARRANGEMENT OF ELLIPTIC PARALLEL MOTION.

This case is like the last in respect that the actual path of R_2, like that of R_1, occurs at a similar part of the ellipse, viz., towards the extremity of the *major* axis.

Now, take the case of Fig. 19. Let B be the *one* guided point as before. But, let the other point R' be placed nearer to A. Find, first, the radius of a circular arc which closely approximates to the elliptic path R' *a'* *b'* in this Fig. 19. Then find the path of A.

(B) Skew Arrangements.

In the foregoing examples the points A and B are taken at the extremities of a diameter, so as to move at right angles to each other. This is the more common arrangement, but A and B may be any points on the circle ABC. For example, suppose B to occupy the position B^1, its path is then B^1C, and the guide for its motion must lie along that line. But it is evident that this has no effect on the line of direction of A's motion.

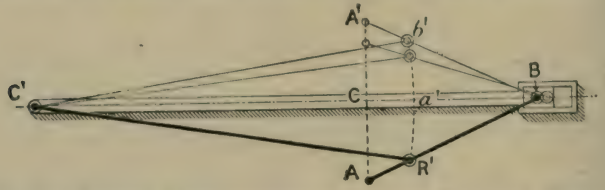


Fig. 19.—Normal Arrangement of Elliptic Parallel Motion.

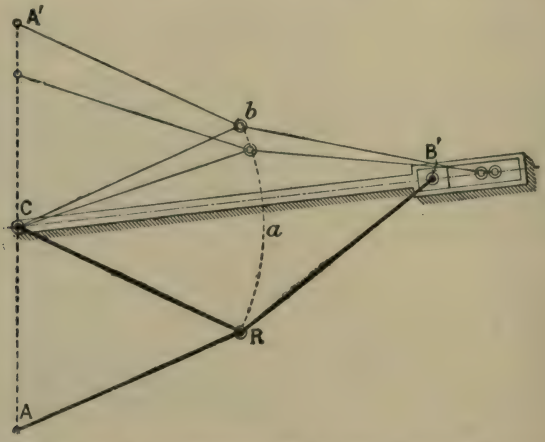


Fig. 20.—Skew Arrangement of Elliptic Parallel Motion.

Exercise XII.—From Fig. 15 take the proportions for a slider-crank arrangement $ACRB^1$, and show the path of A, as in Fig. 20.

(2) Conchoidal Parallel Motions.

Referring to Lecture I., it was there shown (Fig. 15) how to draw the curve known as the conchoid of Nicomedes. In the same way conchoidal curves, traced by various points on the line PQ, may be drawn. The more general form of the problem may thus be stated. Given a fixed point B (Fig. 21), and a straight line RS in the plane of the paper. One

point A on a line PQ is constrained to move along RS. Find the path of a second point—*e.g.*, R^1—on the line PQ, if the latter always passes through B. Several such paths are shown in Fig. 21.

This condition evidently offers itself readily for application as a means of obtaining a parallel motion. For if the point R^1, say, be guided in its conchoidal path, then the point A will move in a straight line. And, as in the case of the ellipse, for a limited range of motion, a circular arc may be found, which approximates sufficiently to the conchoidal arc to make the motion of A practically rectilinear. Thus, in Fig. 21, the four points R^2, R^1, R_1, and R_2, on the line PQ trace out the curves shown, while the line PQ turns about B, and the point A is guided directly in the straight line RS. Conversely, if we guide any of those four points in their

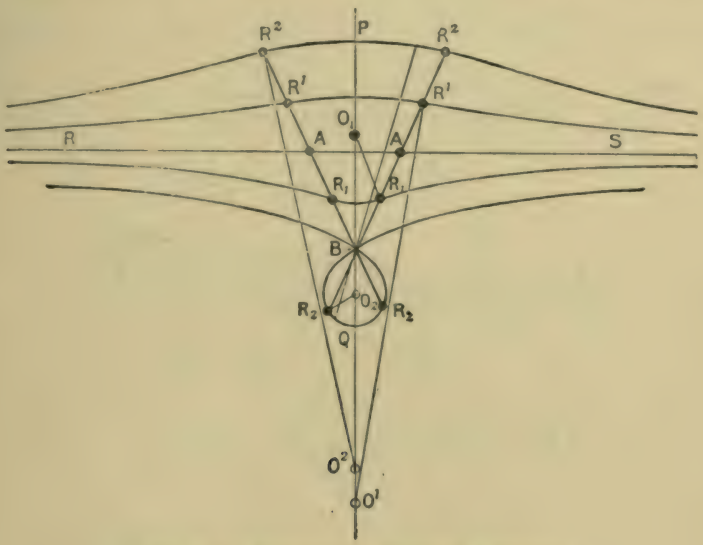


FIG 21.—CONCHOIDAL PARALLEL MOTIONS.

respective curves, R^2R^2, R^1R^1, &c., then A moves without direct guiding in the straight line RS. For the limited range of angular deviation shown (25 degrees from the central position on each side), a circular arc can be drawn, which closely approximates to the conchoidal arc. Thus O^2 is the centre of a circle which passes through the three points R^2, P, R^2. This circle coincides so very nearly with the conchoid through R^2R^2, that the path of A in Fig. 24 is practically a straight line within the range required.

EXERCISE XIII., Fig. 21.—Draw a diagram from which to obtain the proportionate lengths for the four conchoidal parallel motions illustrated by Fig. 7 (Lect. II.), and Figs. 22, 23, and 24 (Lect. III.).

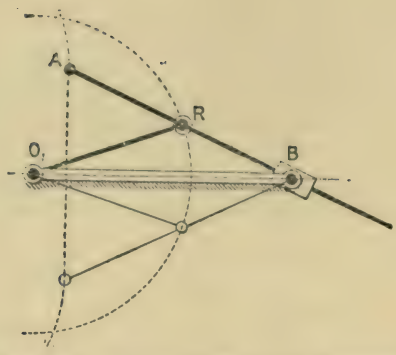


FIG. 22.—CONCHOIDAL PARALLEL MOTIONS.

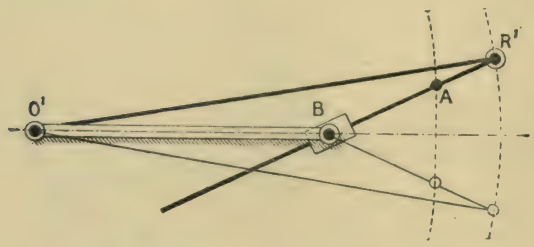


FIG. 23.—CONCHOIDAL PARALLEL MOTION.

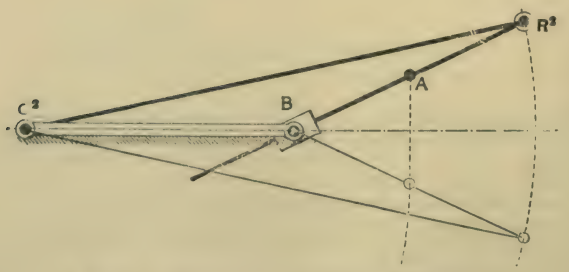


FIG 24.—CONCHOIDAL PARALLEL MOTION.

Quadric Crank Parallel Motions.—Referring to Fig. 15, we have seen that for a slider crank elliptic parallel motion one point on the line A B is guided in an approximate elliptic arc, while one extremity, such as B, is guided in a straight line by a sliding pair. If we could replace this sliding pair by a swinging link of infinite length, B would move as before, and the resultant motion of A would remain the same, but the mechanism would now be a *quadric* instead of a *slider* crank.

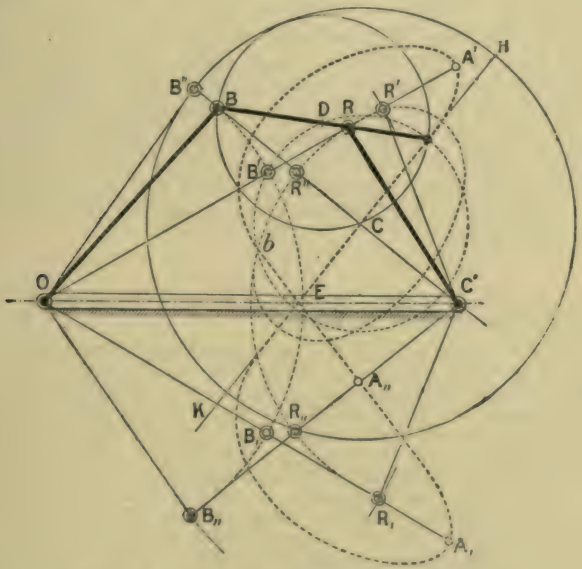


FIG. 25.—QUADRIC CRANK PARALLEL MOTION.

For the very limited range of movement, however, at B, the end of a comparatively short link moves in a near enough approximation to the straight line. Thus, in Fig. 25, the extremity B of the comparatively short swinging link OB, which has been substituted for a sliding pair, moves so very approximately in a straight line from B to B″, and back again, that A moves from A to E in about as good an approximation to the straight line as before (compare with Fig. 19).

In this way we may substitute a quadric crank for each of the slider crank arrangements shown in Figs. 16 to 19. For by finding the centre of a circular arc, which shall closely approximate to the very limited range of straight line motion of the point B in each of these cases, we obtain the length of the third link, which may take the place of the sliding pair. The only difference in the motion of A is that it is now the result of two approximations instead of one.

But a like result may be brought about by assuming for the second-guided point, some point on A B, other than the point B in Fig. 15. Thus, for example, consider Fig. 16 in connection with Fig. 15. The sliding pair at B, Fig. 16, may be replaced by a swinging link, such as C_1R_1, C_2R_2, C^1R^1, or C^2R^2, as obtained from Fig. 15. The same remark applies to the other figures, 17 to 19. The foregoing also applies to the case of the skew slider, in which, as has been before shown, the point B is replaced by some other point—*e.g.*, B^1 in Fig. 15—on the rolling circle ACB, not diametrically opposite to A. An example of this is given in the following exercises, as also one on the double skew slider.

EXERCISE XIV., Fig. 25.—To draw a normal quadric crank parallel motion for a given range of motion m.

Construction.—Draw a line A B = m in length (say 2 feet 3 inches, scale $\frac{1}{12}$). On A B describe a circle. With A as centre, radius $\frac{1}{2}m$ (1 foot 1½ inches), cut the circle at point C. Through C A and C B draw straight lines, and make C E = C A. Divide the line A B into three equal parts, and assume any point R in the middle division. Find the elliptic path of R, while A B moves from the position A B to position C B″. R will then be at R″. Join R R″ and bisect it by a line which intersects B C produced at C′. Then C′ is the one fixed centre, and C′ R is one of the two swinging links required. For the other link, bisect B B″ and intersect the bisection line by a line through C′ E at O. Then O may be the other fixed centre and O B the other swinging link, and the whole figure O B R C′ will represent a quadric crank, of which the produced extremity A of the coupler B R will move from A to E in a very good approximation to a straight line of length m.

The meaning of the construction will be obvious to the student who has intelligently followed the foregoing remarks. For, if with C as centre and radius A B a circle be drawn touching and enclosing the circle A B C, it will then be seen that the points A and B, as extremities of a diameter of a circle A B C, which rolls upon a circle H B″K of radius equal to A B, will describe hypocycloids, which are straight lines (H K and B″ C′) at right angles to each other. Also that R, as a point within the rolling circle A B C, or considered as an intermediate point on the line A B, whose extremities move in straight lines at right angles to each other, describe an elliptic path (the particular case of the hypotrochoid, when the diameter of the rolling circle is equal to the radius of the guiding circle.

Exercise XV., Fig. 26. – To draw a quadric crank parallel motion of the Roberts type of range m.

Draw a line $RS = m$ (say 2 feet 6 inches, to scale $\frac{1}{12}$, and let $AB = 3$ feet 9 inches), and produce it both ways. Bisect it at D and draw $DC \perp RS$. Assume a point C on C D as describing point, and at C set off C A′, C B′, each at 30° with C D and intersecting R S, produced at A and B respectively. Or, assume A and B on the line through R S, making D A = D B, and on it erect an equilateral triangle, of which the vertex C

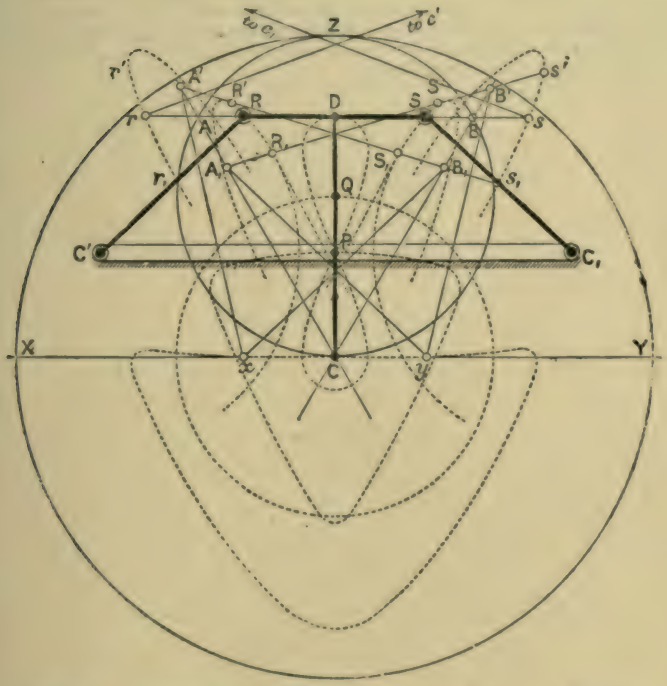


Fig. 26.—Roberts Type of Quadric Crank Parallel Motion.

will be the describing point. Through C draw X Y ∥ R S, and on it cut off Cx, $Cy = \frac{1}{2}m$. With x as centre, radius C A or C B, cut C A produced at A′ and C B at B_1, and with y as centre, same radius, cut C A at A_1 and C B at B′. Join A′ B_1 and A_1 B′, and on these lines cut off A′ R′ and A_1 R_1 each equal to A R. Then find C′, the centre of a circle to pass through R′, R and R_1. The point C′ will be the one fixed centre, and C′ R the one swinging link. C, the other fixed centre, and C, S, the second swinging link, lie symmetrically on the other side of D C.

The construction will be easily understood from previous explanations. Draw a circle through the points A, B, C, and with C as centre, radius equal to the diameter of the circle A B C, draw the outside circle X Z Y. Then C, as a point on the circumference of the circle A B C, describes a straight line, while the latter rolls on X Z Y. R and S, as points within the rolling circle A B C, describe the elliptic paths shown in the figure. Therefore, if R and S be rigidly connected to C, and if they are guided in circular arcs, which closely approximate to the elliptic arcs for the given range of motion, C will move from x to y in a line which approximates to the straight line hypocycloid to a corresponding degree. By a comparison with the double slider of Reuleaux, it will be seen that the Roberts motion only differs from that in the substitution of the points R and S for the points A and B—*i.e.*, that the Roberts motion is an equivalent quadric crank to the Reuleaux double skew slider crank.

Lecture III.—B.Sc. Questions.

1. Describe and prove the exactness of any parallel motion which has only turning pairs. Show, by sketches, how the Watt parallel motion is applied to (*a*) a compound beam-engine, (*b*) a Richard's indicator.

(B.Sc., London, 1904.)

2. In a simple Watt parallel motion the lengths of the levers are 6 inches and 9 inches respectively, and the length of the link connecting the levers is 4½ inches. Find the position of the point on the connecting link, which gives the best straight line motion, and draw the complete path traced out by this point when the levers move through the whole range of their motion. When the levers are horizontal, the connecting link is vertical.

(B.Sc., London, 1905.)

LECTURE IV.

CONTENTS.—Simple Chains of More than Four Links: the Peaucellier Mechanism—Graphic Method of Setting the Peaucellier Compass—Exercises I. and II.—Chains of More than Four Links—Modifications of the Peaucellier Cell—Exercises III. to VIII.—Compound Chains—Exercises IX. to XIV.—Questions.

Simple Chains of More than Four Links: the Peaucellier Mechanism.—We shall now take up a few examples of point paths in simple chains of more than four links. One very interesting case is that of the Peaucellier compass and straight line motion. Before describing the mechanism, let us look at the underlying geometrical principle. The general form of the problem is that in which the pole may be any point either within or without the circle. On drawing a few examples with the pole in different positions, we find that the results all agree in one respect—viz., that the inverse curve is always a circle. Referring to Fig. 1, we find, that when the pole F is taken *within* the circle, the inverse circle is concave towards the first circle; but if taken *without* the circle, it is convex to it. These results lead us to infer the existence of an intermediate case, for we notice, that the curvature grows less and less the nearer the pole is taken to the circumference, either within or without the circle. We find, in fact, that when the pole is taken neither within nor without, but on the circumference itself, that the curvature vanishes and the inverse locus becomes a straight line. These three typical cases are shown together in the one diagram, Fig. 1. F is the pole, F Q any line in all positions passing through F, P a point on the line F Q moving in a circle, Q a second point on the line F Q moving in such a way with reference to the motion of P that the product F P × F Q is always the same.

P_1 and Q_1 represent the first case, when F is within the circle described by P_1; P_2 and Q_2 the second case, when F is outside the circle described by P_2; P_3 and Q_3 the third case, F being a point on the circumference of the circle through P_3.

Now, if we were to seek for a method of describing such a curve by a mechanical arrangement, four conditions would

have to be taken into account:—(1) That F and C are fixed points; (2) that P moves in a circle about C; (3) that the describing point Q must in all positions be in one straight line with P and F; (4) that Q keeps at such distances from P, that the product F P × F Q is constant.

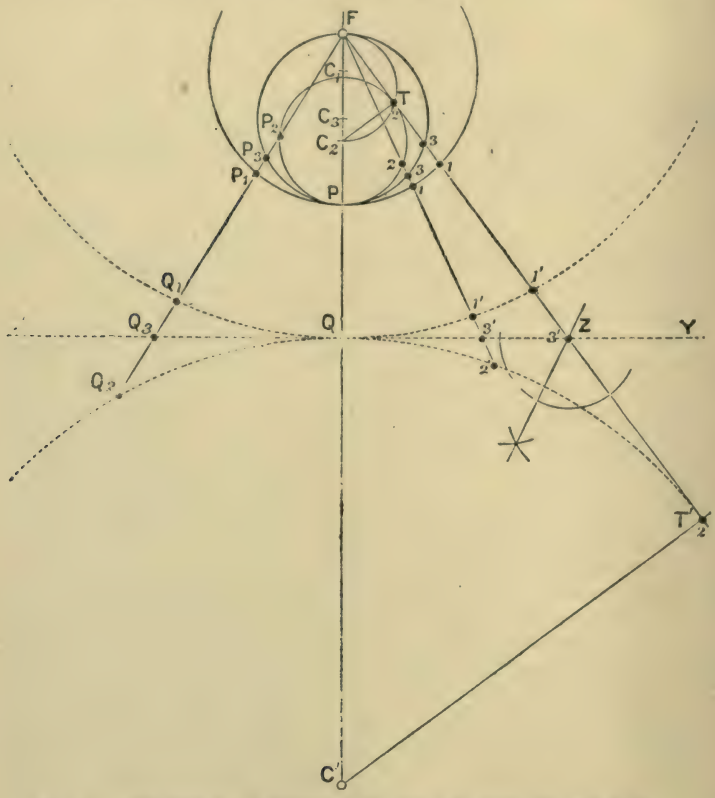


FIG. 1.—GEOMETRICAL PRINCIPLE OF THE PEAUCELLIER COMPASS.

Condition (1) is complied with by making F C the fixed link of the mechanism, (2) by making C P a radial link, (3) and (4) by attaching a jointed rhombus to P and connecting the two wing joints of it to F, by swinging links of equal length.

We thus arrive at the universal or compound compass invented in 1864 by M. Peaucellier, captain of engineers in the French

Army. It has had two applications—(*a*) that of drawing very flat circular arcs, but more important (*b*) it was the first

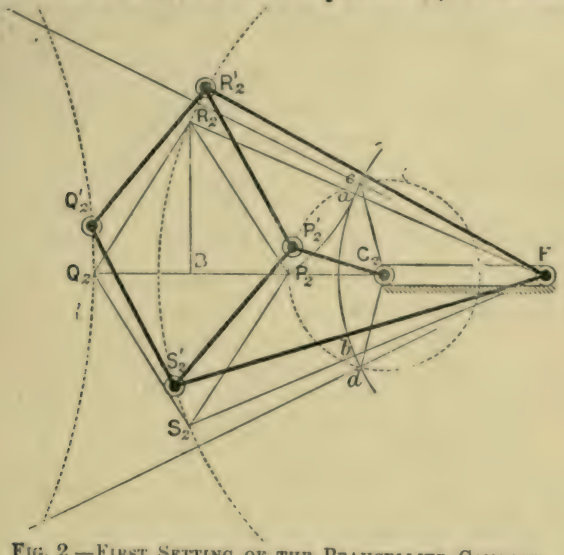


FIG. 2.—FIRST SETTING OF THE PEAUCELLIER COMPASS.

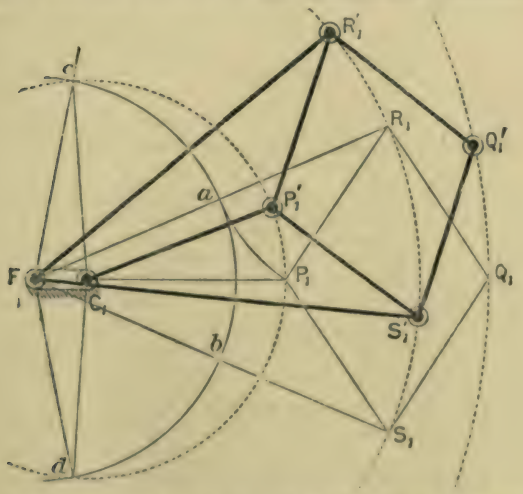


FIG. 3.—SECOND SETTING OF THE PEAUCELLIER COMPASS.

linkage which solved the problem of converting circular motion into absolutely perfect straight line motion without the use of guides (sliding pairs). Four settings of it are shown in Figs. 2, 3, 5 and 6. F lies without the circle described by P in Fig. 2, within it in Fig. 3, and on it in Figs. 5 and 6. In these figures the same letters are used as in the diagram Fig. 1, to show the connection between the diagram and the figures of the mechanisms. It remains to be shown that the joint Q in these mechanisms describes the curve which is the inverse of that described by P, with reference to the fixed point F as pole—*i.e.*, to show that $FP \times FQ = c$. For example, in Fig. 2, let $FR_2 = FS_2 = L$, and $P_2R_2 = P_2S_2 = l$. Also let,

$$\angle R_2 F Q_2 = \theta \text{ and } \angle R_2 P_2 Q_2 = \angle R_2 Q_2 P_2 = \varphi.$$

From R_2 drop a perpendicular R_2B upon FQ_2.

$$FP_2 = FB - P_2B = L\cos\theta - l\cos\varphi.$$
$$FQ_2 = FB + BQ_2 = L\cos\theta + l\cos\varphi.$$
Hence $$FP_2 \times FQ_2 = L^2\cos^2\theta - l^2\cos^2\varphi = L^2\cos^2\theta - l^2(1-\sin^2\varphi).$$
But $$BR_2 = L\sin\theta = l\sin\varphi.$$
$\therefore$ $$FP_2 \times FQ_2 = L^2(\cos^2\theta + \sin^2\theta) - l^2 = L^2 - l^2.$$

Now, in the same mechanism neither L nor l vary in length; thus the product $FP_2 \times FQ_2$ is constant—*i.e.*, Q_2 describes the inverse of P_2.

Graphic Method of Setting the Peaucellier Compass.—For setting the compass to draw arcs of given radius R, the following simple graphical construction gives the lengths of the fixed link and of that which couples it to the jointed rhombus. Referring to Figs. 2, 3, and 4, let $L + l$ have the same meaning as in the above paragraph, and let a be the length of the fixed link in each case, and let r be that of the radial link C_1P_1 or C_2P_2, while R is the radius of the arc required to be drawn.

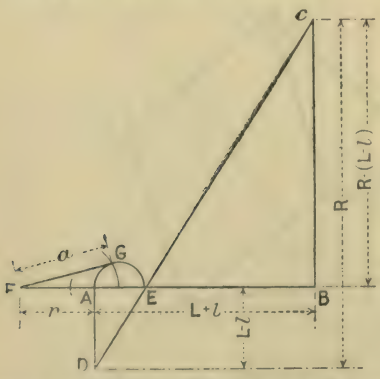


FIG. 4.—GRAPHICAL METHOD FOR SETTING THE PEAUCELLIER COMPASS.

In Fig. 4, make $AB = L + l$. Set off perpendicularly to one side $BC = R - (L - l)$ and to the other, $AD = L - l$. Join DC,

cutting A B at E. On A E describe a semicircle. Produce E A to any convenient point F, and from F draw F G, a tangent to the semicircle. Then the ratio of A F to F G is the required ratio for C_2P_2 to C_2F, in Fig. 2, and the ratio A F to F G is that required for C_1P_1 to C_1F, in Fig. 3. Or more simply thus :—Choose F, in Fig. 4 conveniently, *i.e.*, so that, for example, $P_2C_2 + C_2F$, in Fig. 2, may be within the limits of the mechanism, and make $P_2C_2 = FA$, and $C_2F = FG$.

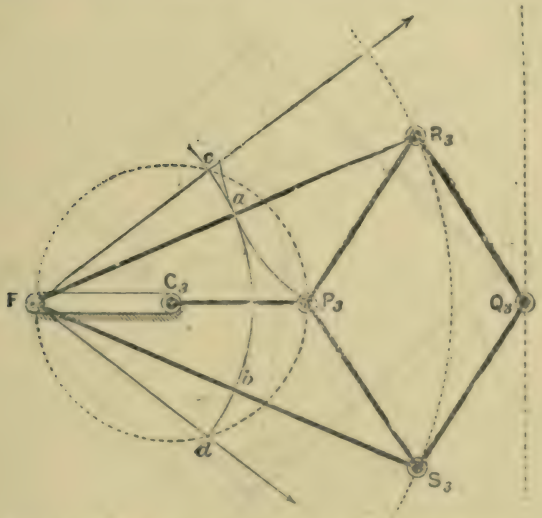


FIG. 5.—THIRD SETTING OF THE PEAUCELLIER COMPASS.

NOTE.—*Teachers may find it convenient to set Exercises* I. *to* XI. *to their students as class questions. Private students should work out these questions carefully at home. Students should be encouraged to make scale-working models of these problems of cardboard or wood-slip with strong pins or steel needles for the joint centres, and to test by experiment the accuracy of their drawings and calculations.*

EXERCISE I., Fig. 5.—From the diagram (Fig. 1) take the proportions of a Peaucellier straight line motion, scale $\frac{3}{4}$, and make the jointed rhombus the same size as in the last two problems . Then, find the locus of Q_3 and its range of motion.

The construction is similar to that shown in the last two problems.

EXERCISE II., Fig. 6.—Take the lengths of the fixed link F C and of the lever C P, the same as in the last problem, but let them be placed inside the rhombus, and make the sides of the latter twice the length of those in Exercise I. Find the path of Q_3 and its range of motion.

The construction for the path and range are again similar, but they may not be quite so obvious. For a second position of Q_3 assume any point P'_3 on the circle through P_3. With P'_3 as centre and radius $= P_3R_3$, or P_3S_3, cut the circle through R_3 and S_3 at R'_3 and S'_3. Then, with R'_3 and S'_3 as centres, and the same radius, draw arcs intersecting at Q'_3. Q'_3 should be on a line P'_3F produced.

For the range, produce R_3F, and mark off on it $R_3a = R_3Q_3$. Then with F as centre, radius Fa, cut circle through P_3 at c and d. Lines drawn through cF and dF intercept the range of Q_3's motion on the straight line locus, which is its path.

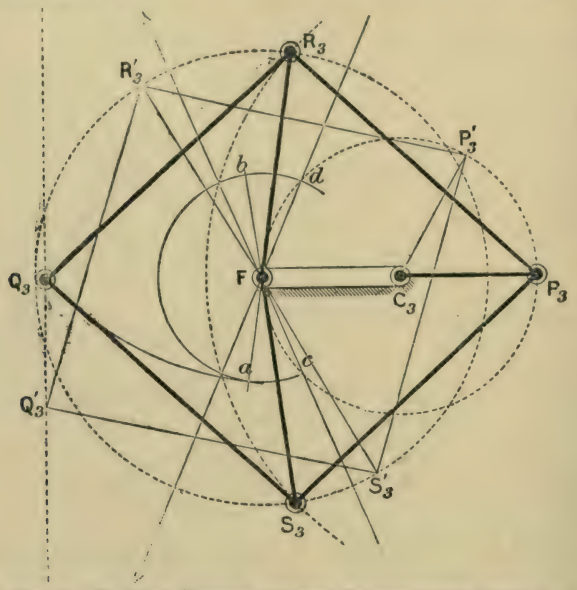


FIG. 6.—FOURTH SETTING OF THE PEAUCELLIER COMPASS.

Chains of more than Four Links—Modifications of the Peaucellier Cell.—The Peaucellier cell, by which is meant the jointed rhombus P R Q S, and the two long radius rods F R and F S of Figs. 3, 5, and 6 may be looked upon as being built up of the two simple quadric cranks known as the "kite" and the "spear-head" respectively, represented in Figs. 7 and 8. For, if we superimpose the long links of a spearhead upon those of a kite of equal size—*i.e.*, of equal length of link—we get at once the form of the Peaucellier cell, shown in Figs. 5 and 9. Similarly, if the short links of the spearhead be laid upon the short links of the kite, we get the form of the cell shown in Figs. 6

and 10. Other combinations, in whole or in part, are shown in the succeeding Figs. 11, 14, and 16. Thus, in Fig. 11, the short links of an equal kite and spearhead are joined end to end, and in Fig. 12 this combination is repeated, with the difference that the spearhead is half the size of the kite. In Fig. 15—the Kempe cell—two kites, the one twice the size of the other, are shown combined. In each case the long links are twice the length of the short ones. A long link of the small kite can therefore be exactly fitted on to a short link of the large kite, as

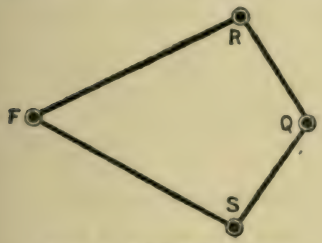


FIG. 7.—KITE MECHANISM.

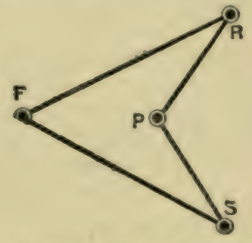


FIG. 8.—SPEARHEAD MECHANISM.

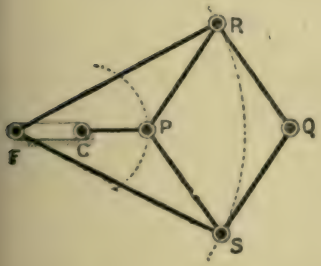


FIG. 9.—COMBINATION KITE AND SPEARHEAD MECHANISM.

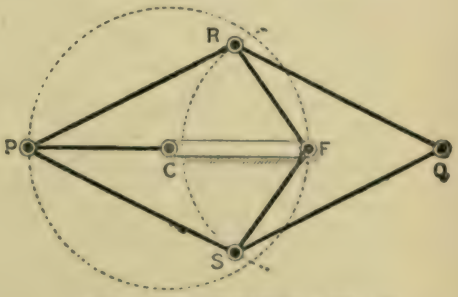


FIG. 10.—COMBINATION KITE AND SPEARHEAD MECHANISM.

shown. Again, Fig. 13, which shows a double cell, may be looked upon as being made up either of two equal spearheads joined end to end, or of two kites made to overlap towards the shorter links. Also, in Fig. 14, we have Fig. 11 repeated in a different form—the Hart cell. For, if we bisect the links by drawing a line F T, the contra-parallelogram W R M N will be seen to be a half kite F R Q, and a half spearhead F W P, joined together end to end at F. This may be made more evident by drawing in the other halves of these figures, viz., F S Q and

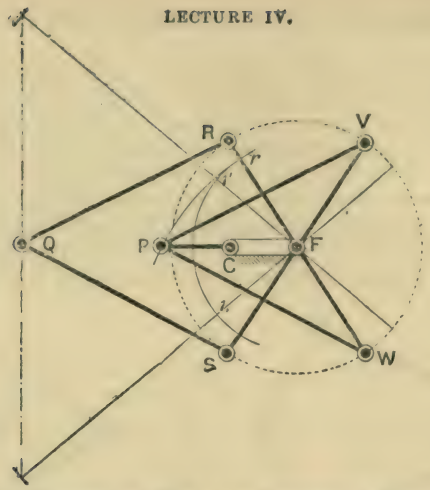


FIG. 11.—COMBINATION KITE AND SPEARHEAD MECHANISM.

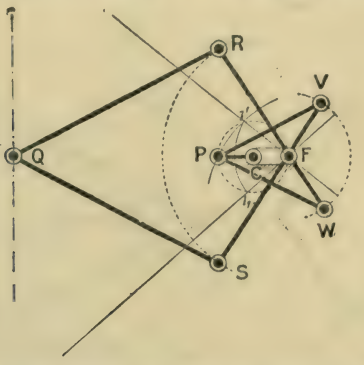


FIG. 12.—COMBINATION KITE AND SPEARHEAD MECHANISM.

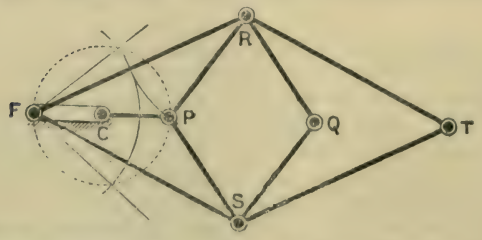


FIG. 13.—COMBINATION KITE AND SPEARHEAD MECHANISM.

F V P, as has been done in Fig. 14. The latter figure represents a simplification of the Peaucellier mechanism, inasmuch as the same resultant motion is now given with the use of six links instead of eight. This property of the contra-parallelogram was discovered by Mr. Hart, of Woolwich, in 1874,* viz., that if a line be drawn across the links of the contra-parallelogram, so as to intersect them at four points, at the same relative distances apart as are the four joints F P Q T in the double cell (Fig 13), then both sets of points have the same properties—(1) that in all positions of the mechanism the four points remain on one straight line ; (2) that the product F P × F Q remains constant.

Lastly, if the four links of the contra-parallelogram be each bent to the same angle at the four bisection points just mentioned, we get the Sylvester-Kempe cell (Fig. 16), the straight line F P Q T of Fig. 14 becomes a parallelogram of constant angles and of constant area, so that the product of two adjacent sides is constant.

Exercise III., Figs. 7 to 10.—Draw a "kite" and a "spearhead" such as shown in Figs. 7 and 8, making the long links, say, 3 inches, and the short ones 1½ inch in length, centre to centre. Next make a tracing of each, and apply Fig. 8 to Fig. 7, so as to get the results shown in Figs. 9 and 10. Prick through the copies on to the drawing paper, and add the fixed link F C and the crank C P.

Exercise IV., Fig. 11.—Trace again the kite and spearhead of the last exercise, and apply them so as to get Fig. 11. Prick through a copy, and add the links F C = C P. Fix the link F C, and find the path of the point Q and its range of motion.

The range of motion is obtained in the same manner as was shown in the last lesson. Thus, with C as centre, describe a circle passing through P and F. On W R cut off W r = W P. Next with centre F radius F r, describe an arc cutting the circle last drawn at 1′ and 1_1. Then C 1′ and C 1_1 are the extreme positions of the link C P, and lines drawn from F through 1′ and 1_1 intercept the range of motion on the path of Q.

To obtain other positions of Q on its path, assume various points on the arc 1′ P 1_1 as successive positions of P, and with each one in turn as centre, radius P V or P W, cut the path circle of V or W. Then draw lines from the points so found through F for the corresponding positions of R and S. With these last as centres, radius R Q or S Q, draw arcs which will intersect at points on the path of Q.

The path is a straight line, for the same reason as was shown in the case of the Peaucellier straight-line motion—viz., that F P × F Q is of constant value in all positions of the mechanism, or, as it might be otherwise stated, that the product of the heights (F Q and F P) of the combined kite and spearhead (F R Q S and F P V W) is the same for all positions of the combination.

Exercise V., Fig. 15.—The Kempe cell. Draw a kite in which the shorter arms are half the length of the longer, and on it place a similar

* See a lecture on "Linkages," by A. B. Kempe, B.A.

kite of half the size, one long arm of the latter being placed on a short arm of the former as shown. Show that if the points G and B be made to approach each other, that the imaginary line joining them is always perpendicular to the triple-jointed link C D E. Make the latter, say 3½ inches long.

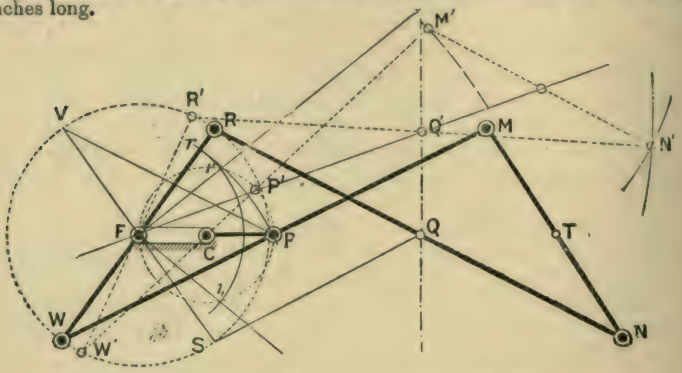


FIG. 14.—THE HART CELL MECHANISM.

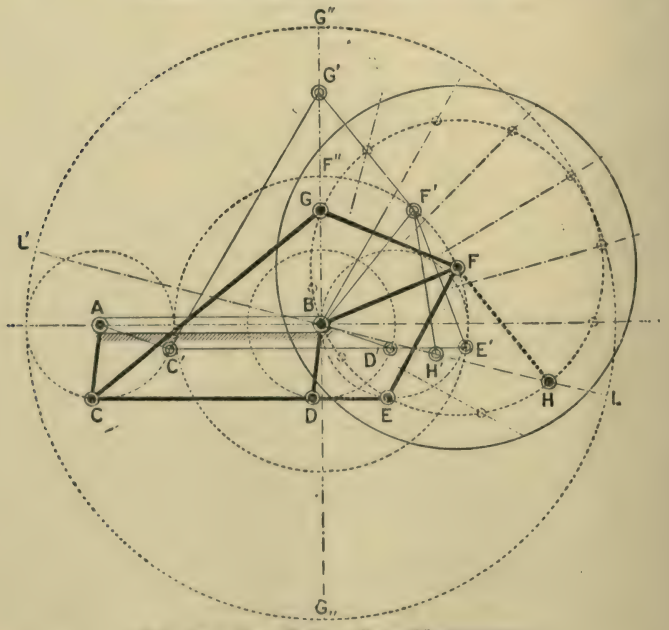


FIG. 15.—THE KEMPE CELL MECHANISM.

Fix any one link, say *e.g.* C D E, then the motions of all the other links become determinate. On drawing lines between G and B in successive positions, it will be found that G B is always perpendicular to C E.

EXERCISE VI., Fig. 15.—Show that if the inner joint B be fixed and the triple-jointed link C D E be made to move parallel to any given fixed line, *e.g.* C E, then the opposite joint G will move in a straight line path, which is a perpendicular to C E and of range equal to double the length of the long links C E or C G.

Firstly, to obtain the required motion introduce a fixed link B A parallel to C E, and a crank A C equal and parallel to the link B D.

The paths of the joints C, D, E, and F are circles, and can therefore be drawn in at once. For the path of G, assume other positions of the joint F, *e.g.* F′ on its circle. Then with F′ as centre, radius F E, cut E's path at E′. Next, with E′ as centre, radius E C, cut C's path at C′. Lastly, with F′ and C′ as centres, radii F G and C G respectively, draw arcs intersecting at G′, which is a second point on the path of G.

For the range, evidently when F is at F″, G will then occupy the extreme position G″, and if the mechanism be so constructed that the various parts clear one another, it will be found that G may move from G″ to $G_{\prime\prime}$, a distance equal to twice the length of the longest links.

If a circle be drawn from F as centre, passing through G, B, and E, and a second circle be drawn from B as centre, with twice the radius, it will touch and enclose the first, and the student will at once recognise in the straight line path of G a hypocycloid, described by G as a point on the circumference of the circle G B E, when it rolls upon that described from B, the latter having a radius equal to the diameter of G B E. And whereas, both in the slider and in the quadric crank parallel motions, only a part of the diameter of the guiding circle was available for the range of motion, this mechanism may be constructed so that the whole diameter may be traversed by the describing point.

EXERCISE VII., Fig. 15.—Given the mechanism A B C D E F G, with A B as fixed link. Let it be required that the path of the describing point should make any given angle α with the axis of the fixed link.

Construction.—Draw B L′ at the given angle α with A B. From F as centre, draw the circle G B E. Produce L′ B to cut this circle at H. Join F H; then, if F H represents a bent prolongation of the link G F—*i.e.*, if F H is rigidly attached to G F—H will move in the line L′ B L.

As shown in the note to the last problem, G describes a straight line hypocycloid passing through the point B. By the construction in the present problem H is a point on the circumference of the same rolling circle, and therefore when G moves along G″ B $G_{\prime\prime}$, H must move in the line L B L′. In the same way points may be obtained with paths making any angle with A B, and if it should be required that the same mechanism should be able to describe any path, we have only to substitute a disc on centre F for the bent link G F H. Then the tracing point may be placed in any hole on the circumference of the circle G B E.

EXERCISE VIII., Fig. 16.—Draw again the contra-parallelogram W R M N of the last exercise, and bisect the sides at the points F P Q T. Draw next each of the four links as if bent to a right angle opposite these middle points to right and left alternately, as shown in Fig. 16, at points f, q, t, p. Show the path and range of motion of the point q, if pf be bisected at c and cf be assumed as a fixed link.

First for the range of motion. On W p cut off W f_1 = W f, and with f as centre, radius $f'p$, cut circle c at p' and p_1. Then cp', cp_1 are the extreme

positions of cp. For the range of movement of Wf: On pW cut off $pd = pc$, and, with c as centre, dW as radius, intersect circle f at W_0 and W_1. Then fW_0 and fW_1 are the extreme positions of fW.

For the path of q assume any position of p on the arc $p' p p_1$, such as p_0. With p_0 as centre, radius pW, cut circle f at W_0 say. Then, with p_0 and W_0 as centres, radius pM and WM respectively, describe intersecting arcs at M_0. Next find R_0 by making arc $W_0 R_0$ = arc W R. With R_0 as centre, radius R N, describe an arc; and from M_0 as centre, radius M N, draw an intersecting arc at N_0. Lastly, with R_0 and N_0 as centres, radius qR or qN, describe arcs intersecting at q_0, which is a second point on the locus of q.

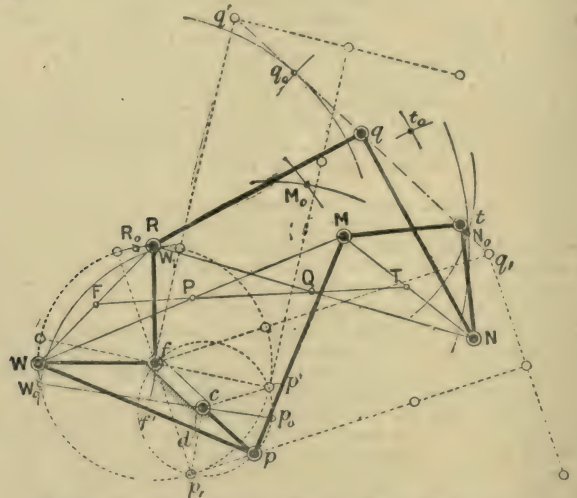


FIG. 16.—THE SYLVESTER-KEMPE CELL MECHANISM.

Compound Chains.—In the last lesson some of the figures—*e.g.*, 11, 12, and 15—showed examples of the way in which chains are combined. Of all the previous examples, one characteristic would be noticed—viz., that no link carried more than two elements. These, then, were examples of *simple* chains; but, in coming now to more complex forms, we distinguish a *compound* chain as one in which one or more links carry more than two elements. Thus the Peaucellier parallel motion (Figs. 9 and 10) is an example of the former, while that of Kempe (Fig. 15) illustrates the other. It should also be noticed that all the examples hitherto given have belonged to what Professor Kennedy calls the *first order* of those mechanisms which have only lower pairs. In all of them, if one position of the mechanism be either given or assumed, then other positions can be

found by mere straight line and circle constructions. But, in some of the examples which follow—*e.g.*, the compound double kite (Fig. 22)—this cannot always be done. Such cases, therefore, Kennedy assigns to a second or higher order of lower-paired mechanisms, and deals with them by the process of "fitting in"* in those links which have not been otherwise determined.

Having given a number of typical examples of point paths in simple chain mechanisms, we shall pass on now to some examples of compound chains.

EXERCISE IX., Fig. 17.—Draw Watt's parallel motion A C F E, compounded, as in Fig. 17, with B D F. Show first the range of movement of the various links in the mechanism thus obtained; secondly, the range of motion through which the upper table T T moves in a horizontal plane; or that the centre of the side table S S moves horizontally in an approximate straight line. Dimensions from Exercise II. (Lect. III.).

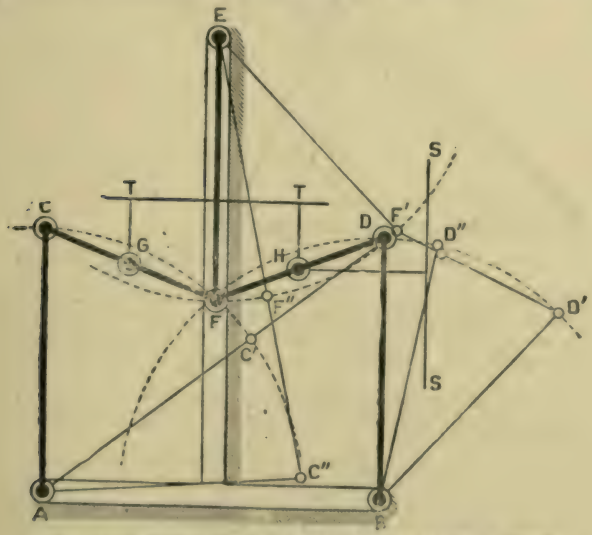


FIG. 17.—COMBINATION OF WATT'S PARALLEL MOTION.

For the total range of movement, draw first the circular paths of C, D, and F. From centre A, radius A C + C F, draw an arc intersecting the path of F at F', and from centre F', radius F D, cut path of D at D'. Then E F' and B D' represent the extreme positions of E F and B D to the right-hand side. But A C' is not the extreme position to that side of A C. For that, draw from centre E, with radius E F + F C, an arc cutting the path of C at C''. Then A C'' is the extreme right-hand position of A C.

* *Mechanics of Machinery*, § 58, p. 441.

For the second part of the question, draw part of the path of the points G and H.

EXERCISE X.—Draw the Tchebicheff parallel motion to the dimensions given in Exercise VI. (Lect. III.), and add two links, E G and F G, as in Fig. 18, making E G = C D and F G = ⅓ B C. Then find, as in the last exercise, (1) the range of movement of the various links, (2) the range of motion of the table T T in a horizontal plane.

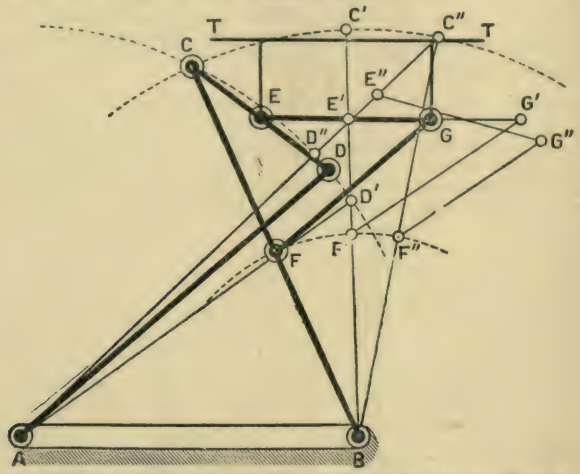


FIG. 18.—COMBINATION OF TCHEBICHEFF PARALLEL MOTION.

With centre A, radius A D + D C, cut the path of C at C''. Then B C'' is the extreme position of B C to the right hand. With centre B, radius B C − C D, cut the path of D at D'; then, A D' is the extreme position of A D to the right hand. The positions of the other links are also shown in the figure.

For the second part of the question, draw as much of the paths of E and G as is necessary to determine the portions which are approximately straight lines.

EXERCISE XI.—Fig. 19 represents the mechanism of a model which was shown at the Vienna Exhibition of 1873,* by the Imperial Technical School of Moscow. Draw to the dimensions given on the figure, and show the range of movement of the parts and the path of the guided point G. Scale, ½.

It will readily be seen that the mechanism consists of the Tchebicheff parallel motion A B C D, compounded with the grasshopper parallel motion of Scott Russell, E G F H.

When the latter is a slider crank, the upper end of the arm E G moves in a straight line, but in the present case the path of G can be only approximately rectilinear, because it depends upon the motion of E, which is itself only an approximation. Notice that the stroke of the piston P is in length equal to *twice* the stroke of the crank.

* Illustrated in *Engineering*, vol. xvi., p. 284.

EXERCISE XII.—Fig. 20 represents the principal mechanism of a steam engine, shown at the Glasgow Exhibition, 1888. Draw to the dimensions supplied on the sketch, and determine the path of the crosshead F. Scale $\frac{1}{12}$.

On examination, B C D E F will be recognised as a normal quadric crank parallel motion (see Lecture III.). The point F describes a straight line of length equal to the stroke of the engine.

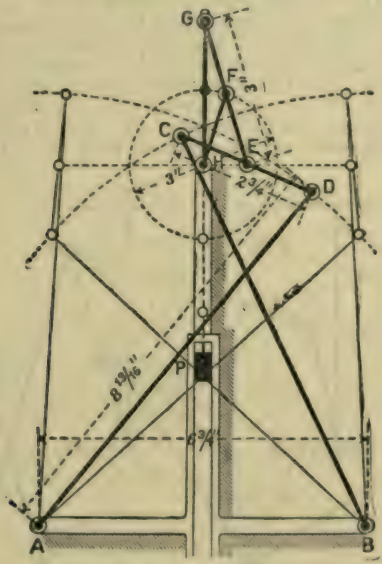


FIG. 19.—MODEL MECHANISM.

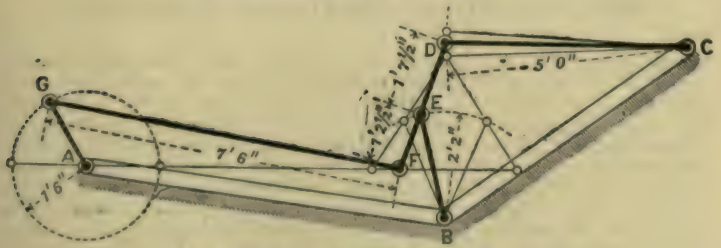


FIG. 20.—STEAM ENGINE MECHANISM.

EXERCISE XIII., Fig. 21.—Draw the contra parallelogram W R N M W (dimensions, say, W M = R N = 3½ inches, and W R = M N = 1¾ inches); then represent these links each as if bent to a right angle at the points p, f, q, and t respectively, in the same manner as in Exercise V., Fig. 16.

On fq, pt, sides of the rectangle $pfqt$, draw the equal and parallel isosceles triangles fdq, pct, and on fd draw the rectangle $fTTd$. The lines pc, ct, and dq represent additional links, and $fTTd$ represents a frame or table carried on the joints f and d. Try the link pc, and determine the range of motion of the mechanism, and show the point paths of all the joints.

The Sylvester-Kempe quadriplane applied to the purpose of giving a limited motion in a horizontal plane to a table TT.

The range of motion is determined by that of the quadric crank $pMtc$. Draw first the circular paths of M and t. With c as centre, radius ct + tM, cut M's path at M′. Then $pM't'c$ is the extreme position of the crank to the left hand. For the corresponding positions of the other links proceed as follows :—With centre t', radius tN, and centre M′, radius MN, draw intersecting arcs. This determines N′. Next with centre M′, radius MW (MpW—one piece) cut the path of W at W′. Then from W′ as

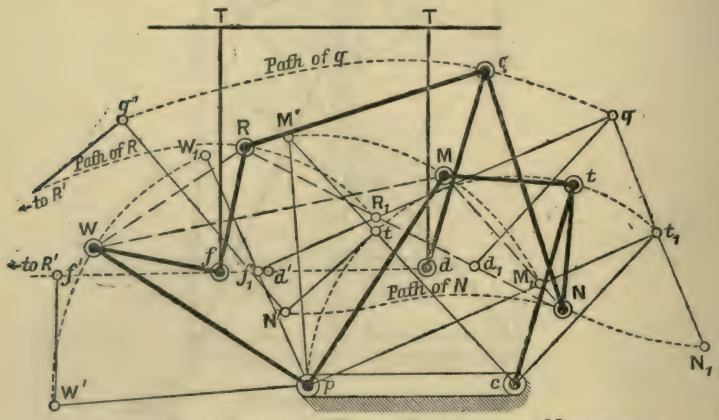


FIG. 21.—THE SYLVESTER-KEMPE QUADRIPLANE MECHANISM.

centre, radius WR, and from centre N′, radius NR, draw arcs intersecting at R′ (to left of diagram). Also from R′ and N′, radius Rq = Nq, draw arcs intersecting at q'. And from W′ and R′, radius Wf = Rf, draw arcs intersecting at f'. Lastly, from q' and f', radius fd = dq, draw arcs intersecting at d'. These points, when connected by the proper lines, show the extreme position of the mechanism to the left hand. The extreme position at the other end of the range is determined in a precisely similar manner. From centre p, radius pM+Mt, cut the path of t at t_1; then, $p\,M_1\,t_1\,c$ is the extreme position of the quadric crank to the right hand. For the remainder of the figure, proceed as before. The paths of W, M, and t are circles. For the paths of f, R, q, d, and N assume various positions of t intermediate between t' and t_1, and find the corresponding positions of the other points in the same way as f', R′, &c., were determined above.

EXERCISE XIV., Fig. 22.—Fig. 22 represents a compound double kite of the Kempe type. Given the left-hand figure ABCDEF, of the same

dimensions as in Ex. V., draw (1) the right-hand figure D G H J K L; (2) show the range of motion of the compound mechanism, link A B being supposed fixed; (3) draw the path of the centre of each joint.

(1) Produce C D to K, making D K = D C. Similarly, make D J = D E. Then, with centres D, J, and K, radii D G = D B, K L = E F, and J H = C A respectively, draw arcs of circles. This is evidently as far as we can get towards determining the position of the compound link G L H by ordinary methods of construction. For the exact position of G, L, and H we may either assume from symmetrical considerations that ∠ K D G = ∠ E D B; or, preferably, we may proceed without making any assumption by the "fitting" process, as follows:—Use a strip of paper, cut to a straight edge, apply the latter to the axis of the fixed link, and make

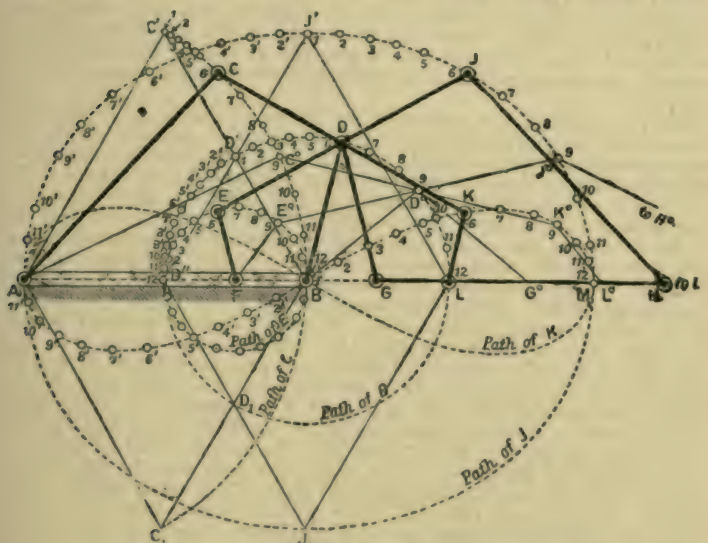


FIG. 22.—COMPOUND DOUBLE KITE MECHANISM.

marks opposite the points A, F, and B. Now turn the strip end for end, and adjust it among the circular arcs drawn from D, J, and K, until the three points marked lie on the three arcs. G, L, and H, as so determined, will be found to lie on the axis of A B produced, and this completes the figure of the required compound linkage.

(2) The range of motion: First, for the extreme positions of the kite A B C D E F, we see that when D is at L, C and E are both at B. Thus the whole figure has collapsed into the straight line A L, and that is evidently as far as it can get to the right hand. In other directions, we see that A C can get no further, either above or below A B, then bring B D and D C into one straight line. Hence with centre B, radius B D + D C, cut the path of C at C' and C,. The kite will then be represented in outline by A C' D' B and A C, D, B respectively. Secondly,

for the extreme positions of the other half D G H J, we see that when D is at L, C and E are at B, and J and K will therefore be at M. Also it will be found, on drawing arcs from these new positions of D, J, and K as centres, that G, L, and H fit in on the line A B produced. The whole compound linkage has, therefore, now fallen into the straight line A B produced, and this is evidently its extreme position to the right hand, H being at a distance J H from the point M. Returning from this position it will be noticed that J reaches its maximum distance from the centre line A M at J′; *i.e.*, when C is at C′, D at D′, and E at D″. Again, the extreme position of D G H J to the left will evidently be reached when D is at D″. This sends E round to B and takes C down to the same point. Now, when C, D, and E are all on the line A B, so will J and K. Both these points will be at A, and on drawing arcs and fitting in G L H as before, it will be found that G H now coincides with A B. This, then, is the extreme of the range to the left hand. G H L thus moves through a range of three times its own length, and as will be seen further, on working out the paths, that it moves throughout this range in one straight line.

(3) The paths of the centres of the joints. Those of C, D, and E, as circles, can be drawn in at once. For the paths of the other joints, we may conveniently begin with C at C′, which, as we have before seen, is its uppermost position, and assume various positions of C on the arc C′ C_1. Next find the corresponding positions of D and E on their respective circles, and having drawn in the links C D and E D, produced them to J and K in each case. The corresponding positions of the various joints are marked by the same numbers 1, 2, 3, &c., on the diagrams. Lastly, from the various positions of D, J, and K, draw arcs, as before described, for fitting in G L H. Thus, for example, let C^0, which is No. 9 on the arc C′ C_1, be the assumed position of C. With C^0 as centre, radius C D, cut the path of D at D^0, and with D^0 in turn as centre, radius D E, cut the path of E at E^0. Draw $C^0 D^0$, $E^0 D^0$, and produce them to K^0 and J^0 respectively, making $D^0 K^0 = D^0 C^0$ and $D^0 J^0 = D^0 E^0$. Then with D^0, J^0, and B^0 as centres, radii D G, J H, and K L respectively, draw arcs. Lastly, with a strip of paper, as before explained, find the relative positions of G^0, L^0, and H^0 by "fitting." The path of J will be found to be an ellipse, of which A B is the semi-major axis, and the minor axis is equal in length to C′ C_1, the chord of the arc C′ C_1, which subtends the angular range of the link A C. The path of K is a lemniscoid curve of length equal to the major axis of J's path, and of breadth equal to the diameter of the circle described by E.

Lecture IV.—A.M.Inst.C.E. and B.Sc. Exam. Questions.

LECTURE V.

CONTENTS.—Motion and Position in a Plane—Instantaneous or Virtual Centres, Centrode, and Axode—Permanent Centre—Relative Motion of Two Bodies in a Plane—Three Virtual Centres lying in a Straight Line—Relative Linear Velocities of Points in Mechanisms—Relative Angular Velocities—Relative Angular Velocity of Two Non-Adjacent Links—Examples I. to III.—Questions.

Motion and Position in a Plane.—Two chief classes of problems arise in connection with the *Science of Kinematics*, the first dealing with the position and motion of a particle, and the second treating of similar questions relating to rigid bodies.

Motion is defined as change of position. And, it is known, if the position of the point or body considered is known for every instant. The position of a point or of a body can only be defined in relation to another point or body, as the case may be, whose position is fixed, or, in other words, whose change of position may be neglected. Position and motion are then purely relative. In stating the position of a point or body we must then refer to some other point or body, and in considering the motion of a point or system of points, such motion can only be imagined with reference to a second point or system of points, supposed to be fixed.

In the case of plane motion, this reference system is usually taken to be the surface on which is drawn the diagram, representing the motion of the body considered. In order to define the plane motion of a plane figure, with regard to a plane, it is sufficient to know the motion of any two points in the figure with reference to the plane.

In most cases, problems arising in the kinematic study of machines are found to involve the consideration of *plane motion* only.

A rigid body having *plane motion* moves in such a way, that all planes originally parallel to a certain fixed plane (that of motion) remain parallel thereto during the whole movement of the body, while any point whatever in the body moves in a plane either parallel to or coincident with the plane of motion. A body moving in this manner will have no motion of translation

in a direction normal to the plane of motion, and the position of the body, with respect to the plane of motion, will agree exactly with the position of its projection on the plane of motion. Hence, in considering the plane motion of rigid bodies, we need deal only with the *kinematics of plane figures*, and all propositions relating to the plane motion of plane figures will be applicable to that of rigid bodies.

It is not, in general, so necessary to trace out the whole motion of a body as to know what is its *instantaneous motion* at some given stage of its movement. By this term is meant, the

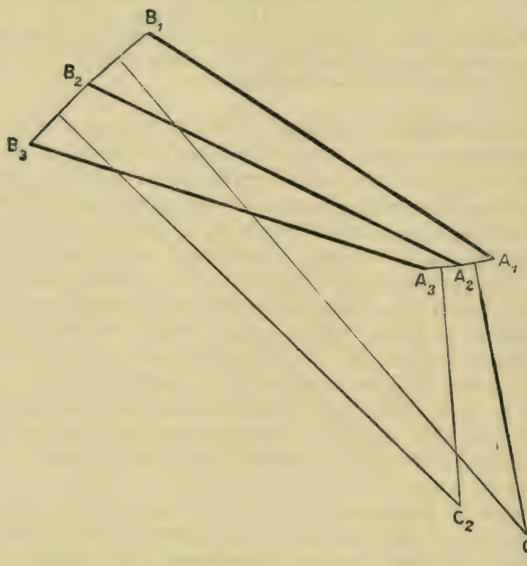


FIG. 1.—ILLUSTRATING INSTANTANEOUS CENTRE, CENTRODE, AND AXODE.

change of position executed by the body in a very small period of time. The manner in which these small changes of position follow one another, must now be considered for the case of plane motion.

Instantaneous or Virtual Centres, Centrode and Axode.—Let $A_1 B_1$, $A_2 B_2$, represent two successive positions of a plane figure, as defined by the position of two points A_1 B_1 in Fig. 1, at the beginning and end of an interval of time which is very small as compared with the whole period of motion.

Join $A_1 A_2$, $B_1 B_2$, and bisect the lines $A_1 A_2$, $B_1 B_2$, by straight

lines perpendicular to $A_1 A_2$, $B_1 B_2$, and intersecting at C_1. Then it is plain, that $C_1 A_1 = C_1 A_2$ and $C_1 B_1 = C_1 B_2$, and if the point A_1 had described a very small circular arc with centre C_1, its new position would have been A_2, and its path would have been indistinguishable from the line $A_1 A_2$. The actual infinitesimally small change of position of the point A_1 is therefore the same as if it had been rotated in the plane of motion around an axis perpendicular to the plane and passing through C_1, and similarly for the point B_1. Thus, knowing the change of position of two points in the rigid figure considered, we say that the actual instantaneous motion of the body $A_1 B_1$ has been equivalent to a *virtual rotation* about the centre C_1. During the next instant, the instantaneous motion may be around some other point C_2 indefinitely near to C_1, and so on. The point C_2 corresponds to the movement from $A_2 B_2$ to $A_3 B_3$. Thus to every part of the motion of $A_1 B_1$, with regard to the plane, there corresponds a certain point C in the plane, about which an equivalent virtual rotation has taken place. Such points, as C_1, C_2, &c., are called the *instantaneous* or *virtual centres* of $A_1 B_1$ with regard to the plane. The locus of C_1, or the curve described by the point C on the plane, is known as the *centrode* of $A_1 B_1$ with regard to the plane, and, in general, it forms a continuous curve.

In the case of a rigid body having plane motion, it would be more correct to consider the equivalent rotation as taking place about a *virtual axis* perpendicular to the plane of motion, of which the points C_1, C_2, &c., are the successive traces on the plane of motion. Such a virtual axis would then describe a surface in space, this surface being known as the *axode* of the body with regard to the plane of motion. For most cases of plane motion, we are content to simplify matters by considering the *centrode* instead of the *axode*. However, in more complex forms of motion the axode becomes of great kinematic importance. It is, in every case, what is called a ruled surface—*i.e.*, a surface described by successive positions of a straight line in space.

Permanent Centre.—Referring again to the plane motion of $A_1 B_1$ in Fig. 1, let us inquire what happens if our construction fails? This will occur if the bisectors of the lines $A_1 A_2$ and $B_1 B_2$ are parallel, in which case the successive positions of $A_1 B_1$ are also parallel to one another, and the motion of the body, or of the figure it represents, is one of *simple translation in a straight line*. The *virtual centre* for such motion as this is then at an *infinite distance*, and we may regard any plane motion of translation in a straight line as equivalent to a rotation about an infinitely distant centre. Again, suppose that one of our reference points A_1 does not change its position at all, then, it

is easily seen that $A_1 B_1$ has now simple rotation about A_1, and during the continuation of this motion we have no longer a virtual but a *permanent centre*. It may happen, that the lines bisecting $A_1 A_2$ and $B_1 B_2$ are coincident. A little consideration will show that in this case, the point C_1 is at the intersection of $A_1 B_1$ and $A_2 B_2$, produced if necessary, since the triangles $A_1 B_1 C_1$ and $A_2 B_2 C_2$ must be equal in every respect, and, as before, a simple rotation about C_1 would suffice to move $A_1 B_1$ into the new position $A_2 B_2$.

It is thus shown, that in every case, *the motion of a plane figure in a plane, may be regarded as equivalent to a simple rotation about some permanent or virtual centre.* The position of the centre in the plane will be fixed for simple rotation, or will be at an infinite distance in the case of simple translation. Such a virtual centre is, in general, neither fixed nor at an infinite distance, but changes its position as the body moves, and its locus in the plane is the centrode of the body with reference to the plane. Only rigid bodies or figures can have centrodes, since we assume that the position of our reference line $A_1 B_1$ in the body or figure remains unchanged throughout the motion, and we represent a rigid body by the line joining the two points in question. It has also been seen, that the centrode of a body with regard to the plane of motion, is a curve described on that plane by the virtual centre of the body.

Relative Motion of Two Bodies in a Plane.—Let $A_1 B_1$ and $C_1 D_1$ in Fig. 2 be the original position of two bodies in a plane. Let $C_1 D_1$ remain fixed, then $A_2 B_2$, $A_3 B_3 \ldots A_6 B_6$ will represent successive positions of $A_1 B_1$, the motion from $A_1 B_1$ to $A_2 B_2$ corresponding to a rotation about a virtual centre I_1, and so on. The curve $I_1 \ldots I_5$ is then the centrode of $A_1 B_1$ with regard to $C_1 D_1$.

Next, we plot the positions $C_2 D_2 \ldots C_6 D_6$ which $C_1 D_1$ would occupy, supposing that the relative motion were the same as before, but that $A_1 B_1$ now remain fixed. For example, in the figure, $C_4 D_4$ has the same position relative to $A_1 B_1$ that $A_4 B_4$ has to $C_1 D_1$, and so on for all the other positions. We now find the series of virtual centres I_1, $N_2 \ldots N_5$ by the construction previously explained, and see that these centres lie on another curve touching the first at I_1, and forming the centrode of $C_1 D_1$ with regard to $A_1 B_1$. Remembering that this curve is attached to, or rather described on, the body represented by $A_1 B_1$, suppose that $C_1 D_1$ remains fixed, while $A_1 B_1$ moves from $A_1 B_1$ to $A_2 B_2$ with the centrode attached—*i.e.*, $A_1 B_1$ rotates instantaneously about I_1. If the movement is imitated by tracing $A_1 B_1$ and the curve I_1, $N_2 \ldots N_5$ on paper, and placing

$A_1 B_1$ in the position $A_2 B_2$, it will be found that N_2 coincides with I_2. When $A_1 B_1$ is at $A_3 B_3$, then, I_3 and N_3 coincides, and so on. Such successive coincidences can only occur if the curve $I_1 N_5$ rolls on the curve $I_1 I_5$.

In the same way, if we trace $C_1 D_1$ and the curve $I_1 \ldots I_5$, and let $C_1 D_1$ occupy its successive positions, we find that the points coincide as before, the curve $I_1 \ldots I_5$ now rolling on $I_1 N_5$.

It is evident, therefore, that the two centrodes corresponding to the relative motion of the two bodies $A_1 B_1$ and $C_1 D_1$ always touch at a point, which is the virtual centre for the instant considered, and we may represent such relative motion by the rolling on one another of a pair of centrodes. Further, the student will find, that from the form of these centrodes we can determine the relative motion of the two bodies.

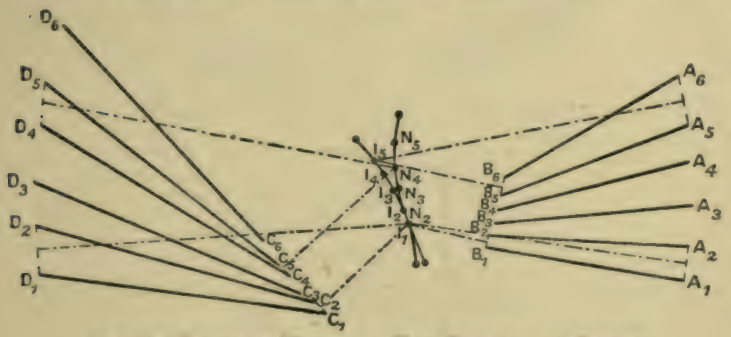


FIG. 2.—RELATIVE MOTION OF TWO BODIES IN A PLANE.

Three Virtual Centres lying in a Straight Line.—On examination of any particular case, we shall see that the various virtual centres in a mechanism having plane motion are arranged in threes, each three lying in a straight line, whatever be the position of the mechanism, as in Fig. 3.

As this property is of very great value in determining the positions of the virtual centres for complex mechanisms, we will give the proof of this statement here. Consider any three of the bodies, or links forming the kinematic chain or mechanism, and let us call them *a*, *b*, *c*. Denoting the virtual centre of *a* with regard to *b* by O_{ab}, and remembering that this is the same point as the virtual centre of *b* with regard to *a*, we have for the three bodies considered, the three virtual centres O_{ab}, O_{ac}, O_{bc}. First consider *b* as being fixed, then any point in *a* has a simple motion of rotation with regard to the point O_{ab}, so that, for

example, the point O_{ac} is moving instantaneously and relatively to b in a direction at right angles to the line $O_{ac} \ldots O_{ab}$.

Again, with reference to the point O_{bc}, any point in c, such as O_{ac}, must be moving instantaneously and relatively to b in a direction at right angles to the line $O_{ac} \ldots O_{bc}$.

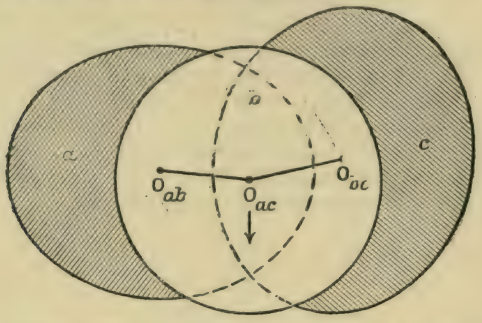


FIG. 3.—THREE VIRTUAL CENTRES LYING IN A STRAIGHT LINE.

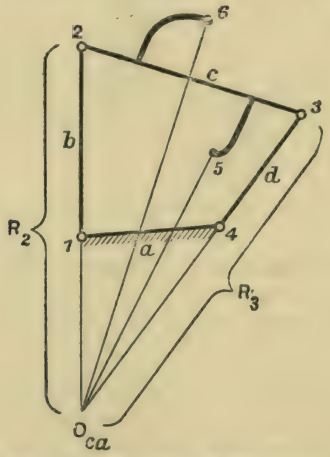


FIG. 4.—FOUR-BAR MECHANISM.

Thus the point O_{ac}, regarded as a point in a, is moving in a line perpendicular to $O_{ac} \ldots O_{bc}$, b being considered as fixed in each case. O_{ac} cannot have two separate directions of instantaneous movement at the same instant, hence the lines $O_{ac} \ldots O_{ab}$ and $O_{ac} \ldots O_{bc}$ are both perpendicular to the same

line. They cannot be parallel, since they both pass through O_{ac}, and they therefore coincide in direction—*i.e.*, the points O_{ab}, O_{ac}, O_{bc} lie on one straight line.

Relative Linear Velocities of Points in Mechanisms.—Having found the virtual centre of any two bars of a mechanism, it is not difficult to find the velocity of any point in one bar relatively to any point in the other, since their linear velocities vary directly as their virtual radii, as in Fig. 4.

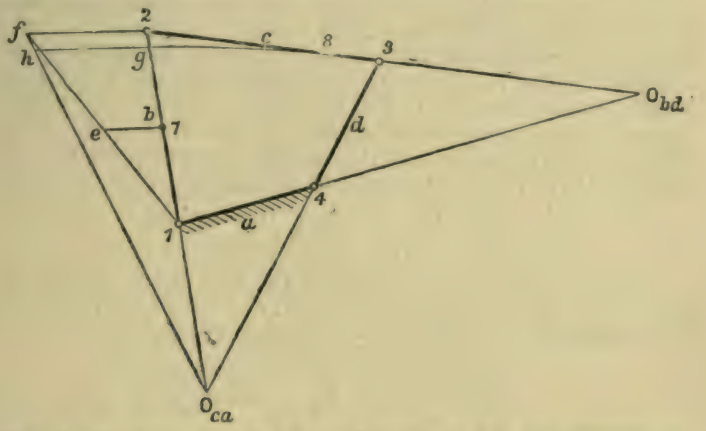


FIG. 5.—TO SHOW THAT RELATIVE VELOCITIES ARE NOT AFFECTED BY THE SHAPE OF THE BARS.

In the four-bar mechanism shown, let the bar a be fixed. To find the relative velocities of the points 2, 3, 5, and 6, we have—

$$\frac{\text{velocity of 2}}{\text{velocity of 3}} = \frac{O_{ca} \ldots 2}{O_{ca} \ldots 3} = \frac{R_2}{R_3}.$$

Similarly,

$$\frac{\text{velocity of 2}}{\text{velocity of 5}} = \frac{O_{ca} \ldots 2}{O_{ca} \ldots 5} = \frac{R_2}{R_5},$$

and

$$\frac{\text{velocity of 5}}{\text{velocity of 6}} = \frac{R_5}{R_6}.$$

Thus, the relative velocities are not affected in the slightest degree by the *shape* of the bars.

Suppose we wish to find the velocity of the point 7 on the bar b, relatively to the point 8 on the bar c in Fig. 5.

Denoting all radii from the virtual centre O_{ba} as r_7, r_8, &c., and from O_{ca} as R_7, R_8, &c.

$$\frac{\text{Velocity of 7}}{\text{Velocity of 2}} = \frac{r_7}{r_2}; \text{ and } \frac{\text{velocity of 2}}{\text{velocity of 8}} = \frac{R_2}{R_8}.$$

$$\therefore \quad \frac{\text{Velocity of 7}}{\text{Velocity of 8}} = \frac{r_7 \times \text{velocity of 2} \times R_2}{r_2 \times \text{velocity of 2} \times R_8} = \frac{r_7 \times R_2}{r_2 \times R_8}.$$

It is often more convenient to arrive at the above result by a graphical construction, since it involves less measurement and no arithmetical multiplication. Thus, from the point 7 set off a line $7e$ to a convenient scale *in any direction* to represent the velocity of point 7. From O_{ba} draw a line through e, and from 2 draw a line $2f$ parallel to $7e$; the length of this line will then represent the velocity of the point 2 to the same scale as the length of line $7e$ represents the velocity of point 7, for the two triangles $O_{ba}.f.2$ and $O_{ba}.e.7$ are similar; therefore—

$$\frac{2f}{7e} = \frac{O_{ba}.2}{O_{ba}.7} = \frac{r_2}{r_7} = \frac{\text{velocity of 2}}{\text{velocity of 7}}.$$

Join O_{ca} to f. From the centre O_{ca} and radius R_8, set off $O_{ca}g = R_8$, and draw gh parallel to $2.f$; then $gh = V_8$, for the two triangles $O_{ca}hg$ and $O_{ca}f2$ are similar; therefore—

$$\frac{gh}{2f} = \frac{O_{ca}.g}{O_{ca}.2} = \frac{R_8}{R_2} = \frac{\text{velocity of 8}}{\text{velocity of 2}},$$

$$\text{And,} \quad \frac{\text{velocity of 7}}{\text{velocity of 8}} = \frac{7e}{2f} \times \frac{2f}{gh} = \frac{7e}{gh}.$$

The same graphical process can be readily applied to all cases of velocities in mechanisms.

Relative Angular Velocities.—There is no greater difficulty in solving problems connected with relative angular velocities, than we have experienced in connection with relative linear velocities. Since all points in a rotating bar or two points of different bodies having the same radius and equal linear velocities, have the same equal angular velocity at each instant; therefore the angular velocities of any two bodies having plane motion are proportional directly to the linear velocities of any two of their points having the same radius, and inversely to the radii of any two of their points having the same linear velocity.

$$\left.\begin{array}{l}\text{Angular velocity of a}\\ \text{point on a rotating}\\ \text{body or link}\end{array}\right\} = \frac{\text{linear velocity of the point}}{\text{radius of point from permanent centre}}.$$

$$\omega = \frac{2\pi r n}{r} = \frac{v}{r}.$$

By aid of the above relations we can find the angular velocity of any link in a mechanism, having given that of any other link, or to find the relative angular velocity of any two links. Let us take the four-bar mechanism in Fig. 6, and compare the angular velocities of the bars b, c, and d relatively to a, if the angular velocity of b relatively to a is given. When comparing the angular velocities of links b and c relatively to a, they have a common point O_{bc}, which has the same linear velocity relatively to a.

Let the linear velocity of B or O_{bc} when regarded as a point on the bar $C = V_c$, and its radius about $O_{ca} = R_c$.

Let the linear velocity of B or O_{bc} when regarded as a point on the bar $AB = V_b$, and its radius about $O_{bc} = r_b$.

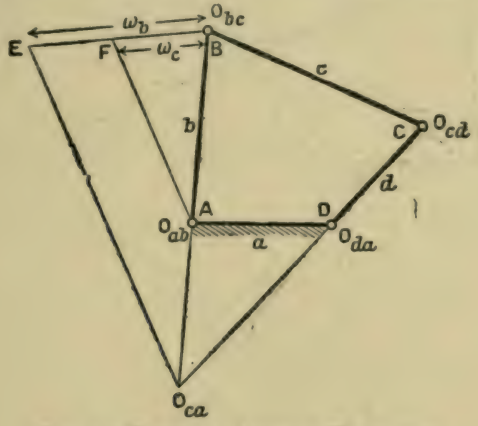


FIG. 6.—RELATIVE ANGULAR VELOCITIES.

Then $V_b = V_c$, since we are considering the linear velocity of the same point—

i.e., $$\omega_b r_b = \omega_c R_c,$$

Or, $$\frac{\omega_b}{\omega_c} = \frac{R_c}{r_b} = \frac{O_{ca} \cdot B}{O_{ab} \cdot B}.$$

This may be easily obtained by graphical construction, thus:—Set off a line B E in any direction from B, whose length on some scale is equal to ω_b. Join O_{ca} . E; from O_{ab} draw O_{ab} F parallel to O_{ca} E. Then B F = ω_c, because the two triangles B . F . O_{ab} and B . E . O_{ca} are similar.

Hence, $$\frac{BE}{BF} = \frac{O_{ca}B}{O_{ab}B} = \frac{\omega_b}{\omega_c}.$$

To Find the Relative Angular Velocity of Two Non-adjacent Links.—When finding the relative angular velocity of two non-adjacent links, such as b and d in Fig. 7, we proceed exactly as before. We take their common point O_{ba}, and draw through it any line whatever on which to set off a segment O_{bd} G for the given angular velocity of b. Join O_{da} to G, and then draw O_{ab} H parallel to O_{da} G. Then O_{bd} H, to the same scale as used for O_{bd} G, is the angular velocity required :—

Hence,
$$\frac{O_{bd}\,H}{O_{bd}\,G} = \frac{\text{angular velocity of } d}{\text{angular velocity of } b}$$

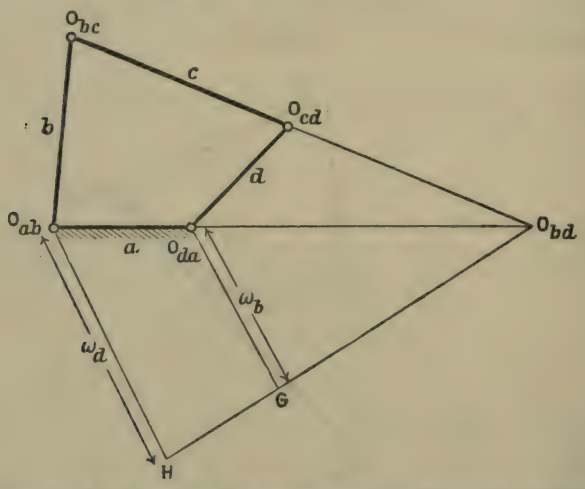


FIG. 7.—TO FIND THE RELATIVE ANGULAR VELOCITY OF TWO NON-ADJACENT LINKS.

When link c is parallel to a, the virtual centre O_{bd} is at infinity, and the angular velocity of b is then equal to the angular velocity of d.

EXAMPLE I.—What is meant by the instantaneous centre of a body moving in a fixed plane? A rigid body has a plane motion. Three points on the body A, B, and C, in the same plane, are such, that A B = 3 feet, B C = 2 feet, and A C = 2·6 feet. At a certain instant it is known, that the point A has a velocity of 4 feet per second in the direction from A to C, and that the point B is moving in the direction from C to B. Show how the velocity of any other point on the body

may be obtained, graphically or otherwise, and determine the values for the velocities of B and the point midway between A and B. (I.C.E., *Feb.*, 1899.)

ANSWER.—Let A B C in Fig. 8 be the triangular figure formed upon the rigid body by joining the three given points.

We know that the velocity of the point A is 4 feet per second, and moving in the direction from A to C; also, that the point B is moving from C to B. Show how to find (i.) the velocity of point B; and (ii.) velocity of a point D midway between A and B.

Whatever be the motion of a figure in a plane at any instant, it is always possible to find a point in the plane such that a rotation about it shall, for the instant, be the same as that motion. This point is termed the *virtual* or *instantaneous* centre of the piece moving in the fixed plane.

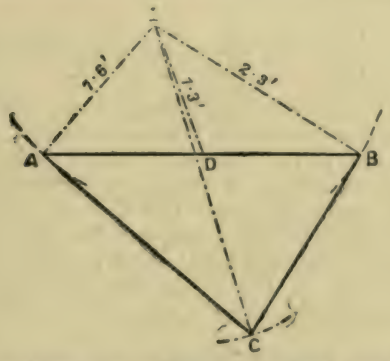


FIG. 8.—REPRESENTING THE POSITION OF THE THREE POINTS ON THE RIGID BODY GIVEN BY THE QUESTION.

(i.) When a figure is moving in any way whatever in a plane, the normals to the directions of motion of all its points,—that is, the virtual radii of all its points, pass through one point, which is the virtual centre for its motion. Hence, at the points A and B draw the virtual radii A I, B I, or lines at right angles to the tangents A C, C B, since they represent the directions or paths along which the points are moving at the instant. The point I of intersection of the normals is then the virtual or *instantaneous centre* of motion of the figure.

We now find the velocity of the point B arithmetically by measuring the lengths of I B and I A, *to any scale*, and multi-

plying the given velocity of the point A by the ratio between them—*i.e.*, by $\frac{\text{virtual radius I B}}{\text{virtual radius I A}} = \frac{V_B}{V_A}$, &c.

$$\therefore \quad V_B = V_A \cdot \frac{I\,B}{I\,A} = V_A \cdot \frac{R_B}{R_A} = 4 \times \frac{2{\cdot}3}{1{\cdot}6} = 5{\cdot}75 \text{ feet per second.}$$

(ii.) The velocity of the point D midway between A and B,

$$V_D = V_A \cdot \frac{I\,D}{I\,A} = V_A \cdot \frac{R_D}{R_A} = 4 \times \frac{1{\cdot}3}{1{\cdot}6} = 3{\cdot}25 \text{ feet per second.}$$

(iii.) The velocity of any other point on the body, such as the point C, may be found from the method under consideration, by drawing the virtual radius I C, and finding the value of the ratio $\frac{I\,C}{I\,A}$ as indicated above.

$$\therefore \quad V_C = V_A \cdot \frac{I\,C}{I\,A} = V_A \cdot \frac{R_C}{R_A} = 4 \times \frac{2{\cdot}94}{1{\cdot}6} = 7{\cdot}35 \text{ feet per second.}$$

EXAMPLE II.—A B is a link, 3 inches long, rotating in a vertical plane about A as centre. C B is a second link, 15 inches long, pivoted to the first at B. The end C moves to and fro along the horizontal line through A. Find (i.) the instantaneous centre of motion of C B when the angle C A B is 30°, (ii.) the relative velocities of C and B in that position. Hence show clearly how to plot a curve of velocities for C. (I.C.E., *Feb.*, 1907.)

ANSWER.—C A B, in Fig. 9, is the mechanism drawn to scale, where A B is the short 3-inch link rotating in a vertical plane.

It will be seen from the diagram, that it represents a very familiar case and one of frequent occurrence, since it resolves itself into finding the velocity with which the piston of an ordinary direct-acting engine is moving comparatively to the velocity of the crank-pin.

(i.) *The Instantaneous Centre of Motion.*—It has been shown, that the motion at any instant of a rigid body in a plane, can be expressed as a motion of rotation about an axis at right angles to the plane of the motion. This axis being called the *instantaneous axis* of rotation, and the point of intersection of the instantaneous axis with the plane is termed the *instantaneous centre.*

The connecting-rod C B being considered rigid, the two points C and B are, at the instant under consideration, each rotating about the *instantaneous centre* of rotation of the connecting-rod.

But the motion of B is, at the instant, in a direction at right angles to A B. The point B may, therefore, be considered to rotate about any point in A B, or A B produced, since all circles drawn through B, and having their centres on A B, or A B produced, will have the same tangent at B. Similarly, the point C moving at the instant in the direction C A, may be considered to rotate about any point on the line C I, drawn through C at right angles to C A. Thus, if both C and B are at the instant rotating about the same point, that point must be I, the point of intersection of A B and C I. The point I is therefore the *instantaneous centre* of rotation of the connecting-rod. From

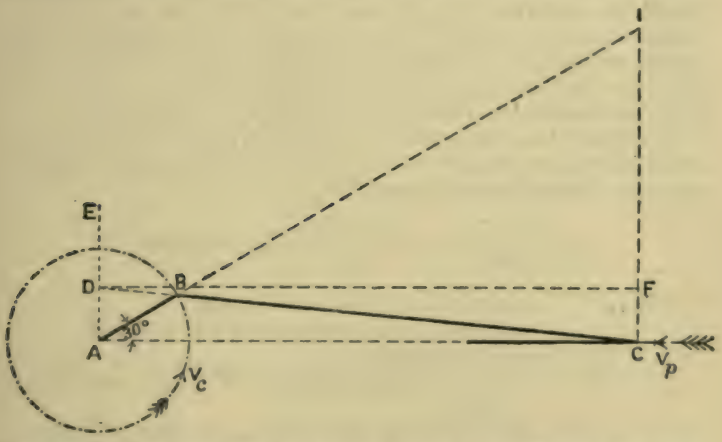


FIG. 9.—TO FIND THE INSTANTANEOUS CENTRE AND RELATIVE VELOCITY OF PISTON TO CRANK-PIN.

previous illustrations, &c., it will be perfectly clear that the instantaneous centre changes its position from instant to instant as the crank A B revolves.

(ii.) *The Relative Velocities of B and C.*—Let V_c and V_p respectively be the linear velocities of the crank-pin B and piston or crosshead at C, at the instant. Then, since B and C are points on the same rigid body rotating about I, their linear velocities are proportional to their radii from the centre of rotation.

$$\therefore \quad \frac{\text{Speed of piston}}{\text{Speed of crank}} = \frac{V_p}{V_c} = \frac{I\,C}{I\,B} \quad . \quad . \quad (1)$$

In some positions of the crank-pin the point I will move off the paper, so that the relation in Eq. (1) cannot always be used.

Draw A E at right angles to the line of stroke C A, and let the connecting-rod (produced if necessary) cut A E at D. Then, evidently, the triangles I B C and A B D are similar, and the lengths of their corresponding sides are proportional.

That is, $$\frac{I C}{I B} = \frac{A D}{A B}, \text{ and } \frac{V_P}{V_C} = \frac{A D}{A B};$$

$$\therefore \quad V_P = V_C \frac{A D}{A B}. \quad . \quad . \quad . \quad . \quad (2)$$

(iii.) *To Plot a Curve of Velocities for the Point C.*—If the speed of the crank-pin V_C is constant, and since A B is of constant length, we get from Eq. (2) that the velocity of the piston or crosshead is *proportional to* the intercept A D. Therefore, if the length of the crank represents the uniform velocity of the crank-pin, then the length of the intercept A D, measured to the same scale, represents the velocity of the piston or crosshead C.

EXAMPLE III.—Show that in an ordinary reciprocating engine the acceleration pressure p when the crank has passed through an angle θ from the dead centre is

$$p = \frac{w V^2}{g R}\left(\cos\theta + \frac{\cos 2\theta}{n}\right) \text{ approximately.}$$

Where V = linear velocity of the crank-pin.
n = ratio of length of connecting-rod to crank.
R = radius of crank.
w = weight of reciprocating parts per square inch of piston.
p = acceleration pressure per square inch of piston.
(I.C.E., *Oct.*, 1908.)

ANSWER.—In many problems connected with the dynamics of the steam engine, it is necessary to obtain an analytical expression for the acceleration of the piston in terms of the crank angle, which, although not mathematically exact, is sufficiently near the true value for all practical purposes.

Let the mechanism for an ordinary reciprocating engine be as shown by Fig. 10, and let the crank angle θ be measured from the inner dead centre. The displacement of the piston from the point of mid-stroke is E A (where C_2 E is made equal to $C_2 B_2$), or, the displacement of the piston from the back end of the cylinder is B_1 E = x. If the obliquity of the connecting-rod be neglected, the displacement would be B_1 F. Hence, the actual displacement may be assumed as made up of two parts :—
(i.) The primary displacement, B_1 F, due to the motion of the

crank; and (ii.) the secondary displacement, F E, due to the obliquity of the connecting-rod.

In (i.) the primary displacement is equal to

$$B_1 F = A B_1 - A F = (R - R\cos\theta) = R(1 - \cos\theta). \quad . \quad . \quad (1)$$

And, in (ii.), the secondary displacement is equal to

$$F E = \frac{(F B_2)^2}{(C_2 E + C_2 F)} \quad . \quad . \quad . \quad (\textit{Euclid}, \text{ Book IV., Prop. 13.})$$

$$\therefore F E = \frac{(F B_2)^2}{2 C_2 B_2} = \frac{(R\sin\theta)^2}{2l} = \frac{R\sin^2\theta}{2n}, \quad . \quad . \quad . \quad . \quad . \quad (2)$$

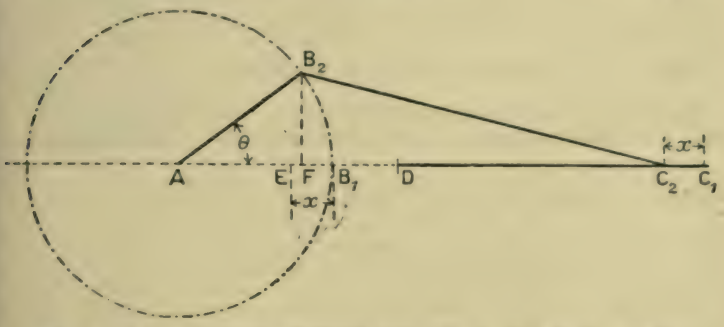


FIG. 10.—ACCELERATION PRESSURE PER SQ. IN. OF PISTON IN TERMS OF CRANK ANGLE.

where R and l are the respective lengths of the crank radius and connecting-rod, and $n = \frac{l}{R}$.

Therefore the *total displacement* of the piston $B_1 E$ is given approximately by the expression—

$$x = B_1 F + F E = R(1 - \cos\theta) + \frac{R\sin^2\theta}{2n}.$$

$$x = R\left\{(1 - \cos\theta) + \frac{\sin^2\theta}{2n}\right\}. \quad . \quad . \quad . \quad . \quad (3)$$

The *velocity* of the piston or crosshead is given by the expression—

$$v = \frac{dx}{dt} = \omega R\left(\sin\theta + \frac{\sin 2\theta}{2n}\right), \text{ since } \omega = \frac{d\theta}{dt}. \quad (4)$$

The *acceleration* of the piston, assuming the crank-shaft to rotate uniformly, so that $\frac{d^2\theta}{dt^2} = 0$, is given by

$$\alpha = \frac{d^2x}{dt^2} = \omega^2 \mathrm{R}\left(\cos\theta + \frac{\cos 2\theta}{n}\right). \quad . \quad . \quad . \quad (5)$$

In each of these expressions, the first term represents the primary effect—*i.e.*, the effect of the crank arm alone, when the obliquity of the connecting-rod is neglected. And, the second term represents the secondary effect—*i.e.*, the very approximate effect of the obliquity of the rod. At the two ends of the stroke the expressions are exact, since they make the accelerations at the back and front ends $\omega^2 \mathrm{R}\left(1 \pm \frac{1}{n}\right)$.

But, Force = mass × acceleration.

If w = weight of reciprocating parts per square inch of piston, and p = accelerating pressure or force per square inch of piston,

Then, $$p = \frac{w}{g} \times \alpha = \frac{w}{g} \times \omega^2 \mathrm{R}\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$\therefore \quad p = \frac{w}{g}\frac{\mathrm{V}^2}{\mathrm{R}}\left(\cos\theta + \frac{\cos 2\theta}{n}\right), \text{ since, } \mathrm{V} = \omega\mathrm{R}.$$

LECTURE V.—QUESTIONS.

1. Name the two chief classes of problems in connection with the *Science of Kinematics*. Define motion, and state how you would draw a diagram of the plane motion of a point on a moving body to scale.

2. State concisely what is meant by the terms, "instantaneous motion," "instantaneous or virtual centres," "centrode," and "axode." Illustrate these by a line diagram, and describe shortly the positions and curves which are represented by these terms.

3. Show, by aid of a figure, that the motion of a plane figure in a plane may be regarded as equivalent to a simple rotation about some permanent or virtual centre, in *every* case.

4. Illustrate, and describe concisely, the case of the "relative motion of two bodies in a plane."

5. Prove, by aid of a figure, that the various "virtual centres" in a mechanism, which has only plane motion, are arranged in threes, and that each three lie in a straight line, whatever the position of the mechanism.

6. Show by diagram and ratios of the linear velocities, as well as by graphical construction, how to find the linear velocity in *one* bar relatively to the velocities of other points in the other bars in the case of a four-bar piece of mechanism.

7. Reproduce Fig. 6 of this lecture, and compare the angular velocities of the bars b, c, and d relatively to the bar a, if the velocity of b is given relatively to that of the bar a.

8. Show graphically how you would ascertain, by aid of a scale diagram, the "instantaneous centre" and the relative velocity for a reciprocating engine piston to that of the crank-pin.

9. Given that the crank-pin circle of a reciprocating steam engine has a radius of 1 foot and that it makes 100 revolutions per minute. If the ratio of the length of the connecting-rod to crank radius be as 3 to 1, and the weight of the reciprocating parts are equal to 5 lbs. per square inch of the piston, then find the acceleration pressure per square inch of the piston.

LECTURE V.—A.M.INST.C.E. AND B.SC. EXAM. QUESTIONS.

1. To two parallel shafts, A and B, are keyed two crank arms, A C and B D, and their extremities are connected by a coupling-rod C D, so that the mechanism forms a four-bar chain. Show that the angular velocity-ratio transmitted between the shafts is, at any instant, inversely proportional to the segments into which the coupling-rod C D divides the line of centres A B. (I.C.E., *Feb.*, 1905.)

2. The cranks of a four-bar chain are 1 foot and 2 feet radius respectively, the coupling-rod is 2 feet 8 inches long, and the fixed bearings of the cranks are 4 feet apart. The longer crank, turning with uniform angular velocity, makes 60 revolutions per minute. Find graphically (i.) the linear velocity of the crank-pin of the smaller crank; (ii.) the linear velocity of the crank-pin of the smaller crank relative to the velocity of the other crank-pin; (iii.) the angular velocity of the coupling-rod; for the configuration of the gear where the coupling-rod subtends an angle of 60° between the longer crank and the centre line joining the fixed bearings, and the coupling-rod produced cuts the centre line produced externally. (I.C.E., *Feb.*, 1907.)

3. In a four-bar chain one of the cranks revolves at a uniform angular velocity. Show how the angular velocity and acceleration of the second crank may be graphically determined in any position. (B.Sc., London, *July*, 1907.)

4. The links of a quadric chain are, in sequence, a, b, c, and d, b being the fixed link. Plot the links according to the following dimensions:—$a = 1{\cdot}0$ inch, $b = 5{\cdot}4$ inches, $c = 2{\cdot}0$ inches, $d = 5{\cdot}0$ inches. Find a sufficient number of positions of the virtual centre of link d to be able to sketch approximately the path of the virtual centre. What will be the motion of c whilst a is making a complete revolution? (I.C.E., *Feb.*, 1908.)

5. If in a four-link mechanism link a is 2 feet long, b the fixed link 4 feet, c 1½ foot and d 3 feet, sketch the mechanism to scale, and show what will be the path of the join of a and d while c rotates. Also find graphically the path of the virtual centre between b and d. (I.C.E., *Oct.*, 1908.)

LECTURE VI.

CONTENTS.—Hooke's Coupling or Universal Joint—Double Hooke's Joint—Aggregate Motion—Examples I. and II.—Epicyclic Trains of Wheels—Epicyclic Train for Drawing Ellipses—Examples III. and IV.—Sun and Planet Wheels—Sun and Planet Cycle Gear—Cams—Heart Wheel or Heart-shaped Cam—Cam for Intermittent Motion—Quick Return Cam—Cam with Groove on Face—Cylindrical Grooved Cam—Example V.—Pawl and Ratchet Wheel—La Garousse's Double-acting Pawl—Reversible Pawl—Masked Ratchet—Silent Feed—Counting Wheels—Geneva Stop—Counting Machine—Watt's Parallel Motion—Parallel Motion—Questions.

IN this Lecture we shall examine a few more of the many devices for transmitting circular motion and for converting it into rectilinear motion, or *vice versâ*, together with other miscellaneous mechanisms.

Hooke's Coupling or Universal Joint.—This is a contrivance sometimes used for connecting two intersecting shafts. Each of the shafts ends in a fork, F_1, F_2, which embraces two arms of the crosspiece, O. The four arms of this cross are of equal length. As C_1 rotates, F_1 and F_2 describe circles in planes perpendicular to their respective axes. Since these planes are inclined to each other the angular velocity of C_2 at any instant is different from that of C_1, but the mean angular velocities are equal to one another, because at one instant C_2 goes faster than C_1, and at another slower. This joint will not work when the two shafts are inclined at 90°, or any smaller angle, to each other.

HOOKE'S JOINT.

Double Hooke's Joint.—The variable velocity ratio obtained with a Hooke's joint may be obviated by the use of two joints instead of one. The forks are connected by an intermediate link, C_2, which must be carried on corresponding arms of the two crosses, as shown in the next figure. If the intermediate shaft be equally inclined to the other two shafts, the irregularities caused in the motion by its transmission through the first coupling are exactly neutralised by the equal and opposite ones caused by the second joint. The first and third shafts, therefore, revolve with the same

velocity at every instant. The double joint works equally well whether the two extreme axes are inclined as shown in the figure, or are parallel to each other but not in line.

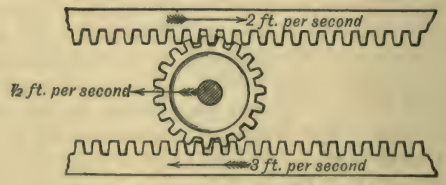
DOUBLE HOOKE'S JOINT.

Both the single and double Hooke's joint are, as a rule, used only for light work, such as for astronomical instruments.*

Aggregate Motion.—The motion of a piece of machinery is not always a simple one, but is very often the resultant or *aggregate* of several independent motions impressed upon it simultaneously.

Thus, the motion of a screw working in a fixed nut is the aggregate of the circular motion of the cylinder and the axial motion caused by the thread. These two together, give a helical motion to any point on the screw. Weston's differential pulley block, which the student has already studied, forms a very good example of aggregate motion. Here, the actual motion of the load is the resultant of two opposite and nearly equal motions imparted by the two parts of the chain supporting it.

In some printing machines, the following arrangement is adopted in order to double the horizontal motion obtained from a crank and connecting-rod. The end of the connecting-rod carries a pinion which runs between two racks. One of these racks moves horizontally between guides while the other is fixed. The motion of the movable rack is composed of that of the connecting-rod end plus that due to the rotation of the pinion. Sometimes, as in Example II., two wheels of different sizes are keyed together on the connecting-rod end, one gearing with a fixed rack and the other with a movable one. In this way the travel of the rack may be made greater, or less, than the diameter of the crank-pin circle in any desired proportion. In all these cases, where the several impressed motions are in parallel directions, the resultant is simply their algebraic sum.



PINION DRIVEN BY TWO RACKS.

EXAMPLE I.—A toothed wheel runs in gear with two parallel racks, one above and the other below it, the wheel being free to run between the racks. If the upper rack

* Recent applications of this device are found in the steering gear for the Admiralty torpedo-boat destroyers, and the Jamieson-M'Whirter Mechanical "Speed Indicating Apparatus" for Electric Tramway Cars, &c. (Patent 3,696 of 13th February, 1902).

has a velocity of 2 feet per second in one direction, and the lower rack a velocity of 3 feet per second in the opposite direction, what is the velocity of the centre of the wheel? (S. & A. Exam., 1887.)

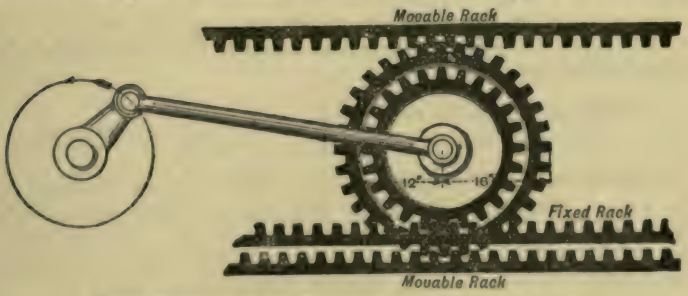
ANSWER.—When a wheel rolls along a road with a velocity, v, it is clear, that the point on the rim which is at any instant touching the ground is for the moment at rest, while the point on the rim vertically over the centre will be moving with a velocity twice that of the centre of the wheel; that is, its velocity will be $2v$.

Now, suppose the lower rack fixed. Then, from what has been said, it is evident that the wheel will run between the racks with a velocity equal to half the velocity of the upper rack. That is, the wheel will be moving to the right with a velocity of 1 foot per second.

In the same way, if we suppose the upper rack fixed the wheel will move to the left with a velocity of $1\frac{1}{2}$ feet per second.

When both racks move, as shown, then the wheel will have a velocity equal to half the difference of their velocities; that is, the velocity of the wheel will be $= 1\frac{1}{2} - 1 = \frac{1}{2}$ foot per second, and in the same direction as the lower rack.

EXAMPLE II.—A crank, 12 inches long, is attached by a connecting-rod to the axis of a spur wheel 24 inches in diameter, which runs upon a *fixed horizontal rack*. On the axis of the spur



RACK DRIVEN BY A CRANK AND MOVABLE PINION.

wheel, and locked to it, is a second spur wheel, 32 inches in diameter, which gears with a *free horizontal rack sliding in guides*. Find the travel of the rack in inches for each revolution of the crank. (S. and A. Exam., 1890.)

ANSWER.—There are two answers to this question, according as we consider the two racks (the fixed and movable racks) to be on the same, or on the opposite, sides of the axis of the spur wheels. In either case, the motion of the movable rack is made up of two

motions—one due to the motion of the axis of the wheels, and the other due to the circular motion of the wheels about their common axis. Suppose the crank to make one-half turn from the inside dead point to the outside dead point in the direction shown by the arrow. Then, evidently, the axis of the wheels will be moved towards the left by an amount equal to twice the length of the crank = 2 × 12 = 24 inches. Also, any tooth on the smaller wheel will turn through an arc of the circumference equal to 24 inches. Any tooth on the larger wheel will turn through an arc of its pitch circle equal to $\frac{16}{12} \times 24 = 32$ inches. Hence, the exact motion of the movable rack consists of a motion of 24 inches along with the axis of the wheels, and another of 32 inches due to the turning of the wheels about their common axis.

First, suppose the racks to be on opposite sides of the axis. Then, from an inspection of the figure, it is clear that the motion of the movable rack will be the sum of these two separate motions.

$$\therefore \text{ Motion of movable rack per half turn of crank } \Big\} = 24 + 32 = 56 \text{ inches.}$$

During the other semi-revolution, the rack moves back the same distance—*i.e.*, it moves 112 inches in all.

Secondly, when the racks are on the same side of the axis, it is equally clear that the motion will be equal to the difference of the two separate motions.

$$\therefore \text{ Motion of movable rack per half turn of crank } \Big\} = 24 - 32 = -8 \text{ inches.}$$

This shows that the rack moves in the opposite direction to the axis. The whole motion in this case per revolution of the crank is 8 inches each way or 16 inches in all. In this latter case it is evident that the axis of the wheels must be guided horizontally in order to keep them in gear with the racks.

Epicyclic Trains of Wheels.—We sometimes find that the axes of some of the wheels in a train are not fixed, but rotate around another axis. Such trains are called **Epicyclic Trains.*** The movable axes are fixed to an arm, called the **Train Arm**, which rotates about that axis. Epicyclic trains are used in those machines where some motion is required which it would be difficult, or inconvenient, to obtain with an ordinary train. For instance, they are used in the "Cordelier," or rope making machine, in order to twist the fibres of the strands in one direction while the strands themselves are being twisted together

* See Greek derivation of *Epicyclic* in footnote on p. 184, Vol. I.

in the opposite direction. If this were not done the fibres would be getting untwisted while the strands were being twisted, and a useless rope would result. By putting a little extra twist on the fibres, the rope will be hard and firm and will not tend to untwist.

The student has already studied an application of the epicyclic train in Lect. VIII., Vol. I.—viz., the Rotatory Dynamometer. In this case the train is one of bevel and not of spur wheels as in the other examples we will consider here.

The motion of any wheel in an epicyclic train is an aggregate motion; for, the wheel has a certain angular velocity due to its rotating about its own axis, and another caused by the rotation of that axis along with the train arm. In this case also, the resultant motion is the algebraic sum of the several parts.

Let, N_D = Number of revolutions of driver in unit time relative to some fixed point

„ N_F = Number of revolutions of follower in the same time relative to the same fixed point.

„ N_A = Number of revolutions of the arm in the same time relative to the same fixed point.

„ e = Value of the train = $\left\{\begin{array}{l}\text{Number of revolutions of follower in a given time relative to the arm.} \\ \hline \text{Number of revolutions of driver in the same time relative to the arm.}\end{array}\right.$

Care must be taken to give e its proper sign; for, e is negative if the driver and follower rotate in opposite directions relative to the arm, and positive if in the same direction.

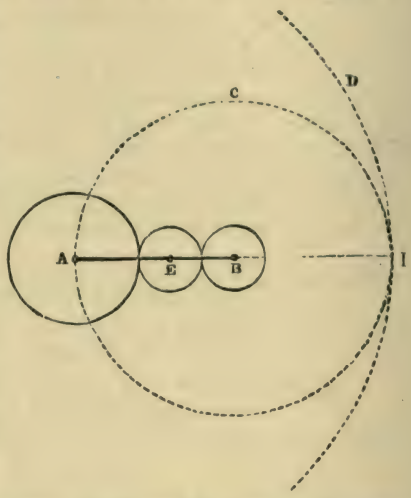
Now, since the driver rotates N_D times in unit time and the train arm N_A times, their relative motion will be $(N_D - N_A)$ turns. Similarly the number of revolutions of the follower relative to the arm will be $(N_F - N_A)$.

$$\therefore \qquad e = \frac{N_F - N_A}{N_D - N_A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (I)$$

It should be noted that if the driver and follower rotate in opposite directions, one of them must be considered the positive, and the other the negative, direction.

Epicyclic Train for Drawing Ellipses.—The figure shows the wheel work of an instrument for tracing ellipses by means of rolling circles. Suppose the circle A C I to roll inside a circular

arc, D I, of twice its radius. Then, as is proved in Lecture XI. in connection with the hypocycloid, the point, I, will move along the straight line I A (A being the centre of the arc, D I), and it is manifest that B (the centre of A C I) will describe a circle round A. Any other point in B I, or B I produced, must, therefore, trace out an ellipse. It is inconvenient in practice to roll the circle, A C I, inside D I, and the same result may be obtained by aid of an epicyclic train. Let A be a wheel *fixed* at the centre of the arc, D I, and B another, of half its size, concentric with the rolling circle, A C I. These are connected by the train arm, A B, which rotates about A, and an idle wheel, E. B I is a tracing arm, rigidly fixed to the wheel, B. Let the train arm, A B, now rotate. A, the driver, does not rotate; therefore, $N_D = 0$, and $e = \frac{A}{B} = 2$.



EPICYCLIC TRAIN FOR TRACING ELLIPSES.

Then, from equation (I) we get:—

$$e = \frac{N_F - N_A}{N_D - N_A}, \quad \therefore 2 = \frac{N_F - N_A}{0 - N_A}. \quad \therefore N_F = -N_A.$$

Or, the tracing arm, B I, and train arm, A B, rotate in opposite directions with equal velocities. This is obviously the same as if the tracing arm were a radius of the imaginary rolling circle, A C I, rolling inside the arc, D I. We can thus see that if we put a tracing point on B I, it will trace out an ellipse.

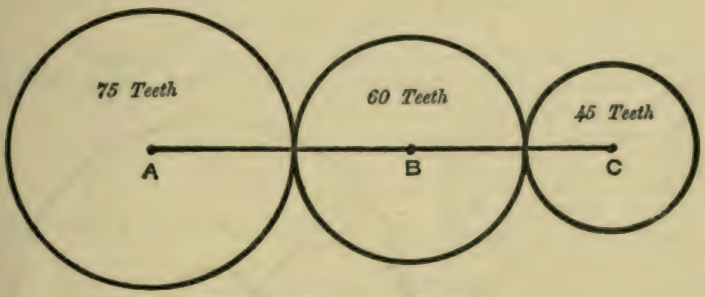
EXAMPLE III.—An epicyclic train consists of three wheels, A, B, C, taken in order, and in gear with each other. The first wheel, A, has 75 teeth, B has 60 teeth, C has 45 teeth; also, the driver, A, rotates three times in a minute, and the arm rotates four

times in a minute, and in the opposite direction. How many rotations do B and C make respectively per minute?

ANSWER.—The arrangement is shown diagrammatically in the above figure.

(1) *To find the motion of* B.—Here, using the same notation as in the text, we have :—

$$N_D = 3\,;\ N_A = -4\,;\ \text{and } e_1 = -\frac{75}{60} = -\frac{5}{4}.$$



EPICYCLIC TRAIN OF WHEELS.

From equation (I) we get :—

$$e_1 = \frac{N_{F1} - N_A}{N_D - N_A}, \qquad \therefore\ -\frac{5}{4} = \frac{N_{F1} + 4}{3 + 4},$$

$$\therefore \qquad N_{F1} = -\frac{35}{4} - 4 = -12\tfrac{3}{4}.$$

That is, the follower, B, rotates $12\frac{3}{4}$ times per minute, and in the opposite direction to the driver, A.

(2) *To find the motion of* C.—In this case, $e_2 = \frac{75}{60} \times \frac{60}{45} = \frac{5}{3}$.

As before, $\qquad e_2 = \frac{N_{F2} - N_A}{N_D - N_A}, \qquad \therefore\ \frac{5}{3} = \frac{N_{F2} + 4}{3 + 4},$

$$\therefore \qquad N_{F2} = \frac{35}{3} - 4 = 7\tfrac{2}{3}.$$

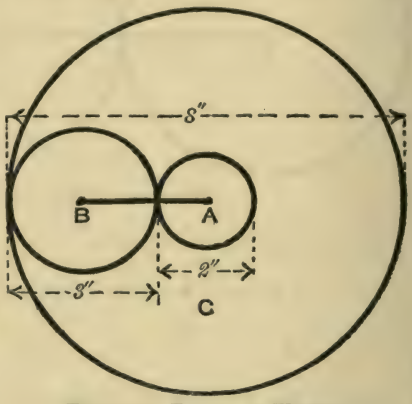
Whence, C turns $7\frac{2}{3}$ times per minute, in the same direction as A.

These results may be arrived at otherwise as follows :—Suppose the wheels and arm rigidly connected, so as to move as one piece; then, let the arm turn for one minute, so as to receive its four negative turns; each of the wheels will then also receive four

negative turns. Now, suppose the arm to be fixed, and let the wheel, A, receive seven positive turns, so as to cancel the four negative turns already given, and leave the nett motion of three positive turns, as required by the question.

The effect on B will be to give it $7 \times \frac{75}{60} = 8\frac{3}{4}$ negative turns, and on C to give it $7 \times \frac{75}{60} \times \frac{60}{45} = 11\frac{2}{3}$ positive turns; hence, the total motion of B in one minute will be $-4 - 8\frac{3}{4} = 12\frac{3}{4}$ negative turns, and of C, $-4 + 11\frac{2}{3} = 7\frac{2}{3}$ positive turns.

EXAMPLE IV.—What is an epicyclic train of wheels? Two spur wheels, A and B, whose diameters are 2 and 3 respectively, are in gear with an annular wheel, C, whose diameter is 8. The wheels A and C have a common axis, but B is carried by an arm centred on the axis of A. If A make five revolutions while C makes one revolution, both in the same direction, find the angle described by the arm during this time. (S. & A. Exam., 1888.)



EPICYCLIC TRAIN OF WHEELS.

ANSWER.—An epicyclic train of wheels is one in which the axes of the wheels are not fixed in space, but are attached to a rotating frame or bar, in such a manner that the whole train of wheels can derive motion from the rotation of the bar.

Using the same letters as before, and calling A the driver and C the follower, we have :—$N_D = 5$; $N_F = 1$; and $e = -\frac{2}{3} \times \frac{3}{8} = -\frac{1}{4}$.

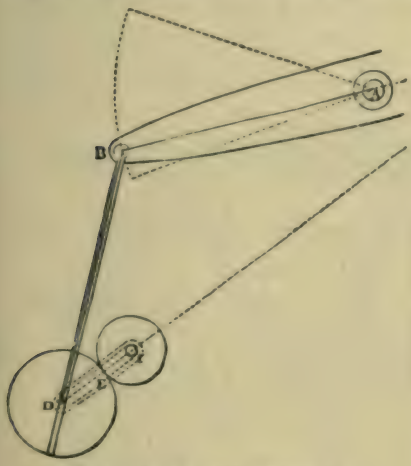
Then, from equation (I), $e = \dfrac{N_F - N_A}{N_D - N_A}$.

$$\therefore \quad -\tfrac{1}{4} = \frac{1 - N_A}{5 - N_A}.$$

$$\therefore \quad N_A = 1\tfrac{4}{5} \text{ turns.}$$

That is, the train arm has made $1\frac{4}{5}$ turns in the same direction as A and C.

Sun and Planet Wheels.—This device was invented by Watt to convert the oscillatory motion of the beam in his engines into the circular motion of the flywheel. As will be seen from the first figure, it consists of a wheel, D, rigidly fixed to the connecting rod, D B, and kept in gear with another wheel, C, by the link, D E C. The wheel, C, is keyed to the flywheel shaft. As the beam oscillates up and down, the connecting-rod pulls D up one side of C, and pushes it down the other. It thereby causes C to rotate, and with it the flywheel and shaft. The student will easily see that the wheels, D and C, form an epicyclic train, of which the link, D E C, is the train arm. We may, therefore, apply the formula already given for epicyclic trains, to find how often the flywheel revolves for each up and down movement of the beam. Doing this, and assuming D and C to be of the same size, we get:—



SUN AND PLANET WHEELS.

N_D = Number of revolutions of the driver, D, for each up and down movement = 0.

N_F = Number of revolutions of the follower, C, for each up and down movement.

N_A = Number of revolutions of the arm, D E C, for each up and down movement = 1.

$e = -1$.

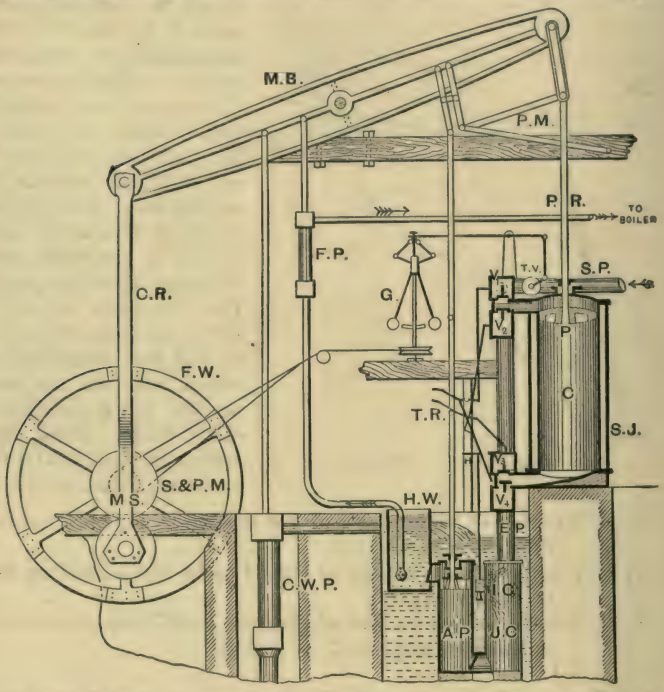
From equation (I), $\quad e = \dfrac{N_F - N_A}{N_D - N_A}$; *i.e.*, $\quad -1 = \dfrac{N_F - 1}{0 - 1}$.

Or, $\qquad N_F = 2$.

From this we see, when the wheels are equal, that the flywheel goes round twice while the connecting-rod goes once up and down. If the wheel D be twice as large as the wheel C, we would find, in the same way, that C went round three times during this period. The following figure shows these sun and planet wheels as applied to Watt's double-acting steam engine.*

* See Index of the Author's *Text-Book on Steam and Steam Engines.*

Sun and Planet Cycle Gear.—The illustration on next page (which has been kindly supplied by the makers of this gear, the "Elliptic" Cycle Syndicate of Grantham) shows a recent interesting application of the sun and planet wheels. Both wheels are elliptic in this case, and, therefore, give a variable velocity-ratio. By means of this gear, the pedals, which travel in an elliptical path, are caused to move at a uniform speed. The velocity-ratio



WATT'S DOUBLE-ACTING STEAM ENGINE, SHOWING SUN AND PLANET MOTION (S. & P.M.) AND PARALLEL MOTION (P.M.).

between the cranks and driving wheel has a double variation at each revolution of the cranks, which move more quickly at the top and bottom positions. The quicker movement caused by the gearing is counteracted by the quick vertical movement of the pedals, due to, and governed by, the position at which the crank-pin bearing is attached to the pedal bar. The result is a regular and uniform movement of the pedals in an elliptical path; the train value being 2 : 1.

It is claimed by the inventor, that a bicycle having an elliptical pedal path, with a uniform movement, has a great advantage over a circular pedal path, as the pressure can be applied continuously, whilst in the circular path nearly one-quarter of the travel at top and bottom is horizontal, and, therefore, not in an effective direction.

The shape of the planet wheel governs the double variation, and, consequently, must always be an ellipse, but the sun wheel may be made eccentric, and with half the number of teeth of the elliptical wheel, the value of the train then being 3 : 1.

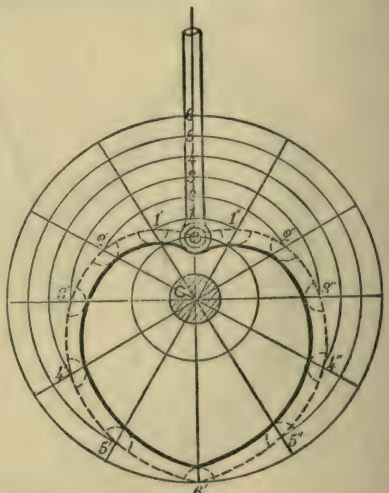
HARRISON'S ELLIPTIC CYCLE GEAR.

Cams.—Cams are usually of the form of discs or cylinders. They rotate about an axis, and give a reciprocating motion to some point in a rod by means of the form of their periphery or surface, or by grooves in their surface.

The cam generally revolves uniformly round its axis, whilst the reciprocating motion may be of any nature, depending on the shape of the cam, and may be in a plane inclined at any angle to the axis of rotation. In the following examples, uniformity of rotation is assumed in the case of the cam, and the motion of the reciprocating piece takes place in a plane perpendicular to the axis.

Heart Wheel or Heart-shaped Cam.—Suppose that it is required to give a uniform reciprocating motion to a bar moving vertically between guides, and in a line passing through, C, the centre of motion of the cam plate.

Let the sliding bar be at its lowest position, as shown, and when in its highest position let its extremity be at the point 6. The distance thus moved is called the *travel* and will be passed over during one-half revolution of the cam. The required curved outline may be obtained in the following manner:—With centre, C, describe circles passing through the extreme positions of the end of the rod. Divide the travel into, say, six equal parts at the points 1, 2, 3, &c. Divide the semi-circumference into the same number of equal parts by radial lines C 1′, C 2′, &c. Then with centre, C, draw the concentric arcs 1, 1′; 2, 2′; &c., intersecting these radii in the points 1′, 2′, 3′, &c. The dotted line drawn through these points will represent the required curve.



HEART-SHAPED CAM.

If the end of the sliding bar rests on this curve it is clear, that for equal angles turned through by the cam, the bar will move outwards through equal distances, and consequently, will have uniform linear motion imparted to it. The return motion will evidently be obtained by the similar and equal curve 1″, 2″, 3″, &c., on the opposite side of the cam.

A cam so formed would impart the required motion to a *point*. If the end of the sliding bar be provided with a roller in order to diminish the friction, then the shape of the cam must be altered so that the centre of the roller shall move over the outline of the cam as traced above. To accomplish this we must draw a curve inside the original one by describing small arcs with centres on the original curve as at 1′, 2′, 3′, &c., and with a radius equal to that of the roller, and then by drawing a smooth curve touching these arcs, as shown by the heavy line in the figure.

Cam for Intermittent Motion.—Sometimes the motion imparted by a cam is intermittent. For instance, a common form of lever punching machine is fitted with a cam which gives the punch an upward movement, then a period of rest, and finally a downward movement during each revolution. As an example of this, let us set out a cam to impart vertical motion to a bar, so that the latter shall be raised uniformly during the first half revolution, remain at rest during the next one-sixth, and descend uniformly during the remainder of the revolution.

As before, suppose the reciprocation to be in a line passing through, C, the centre of motion of the cam plate. Then, with centre, C, draw circles passing through the extreme positions of the end of the bar. Divide the circumference into three parts corresponding to the periods of one-half, one-sixth, and one-third

Cam Giving an Interval of Rest.

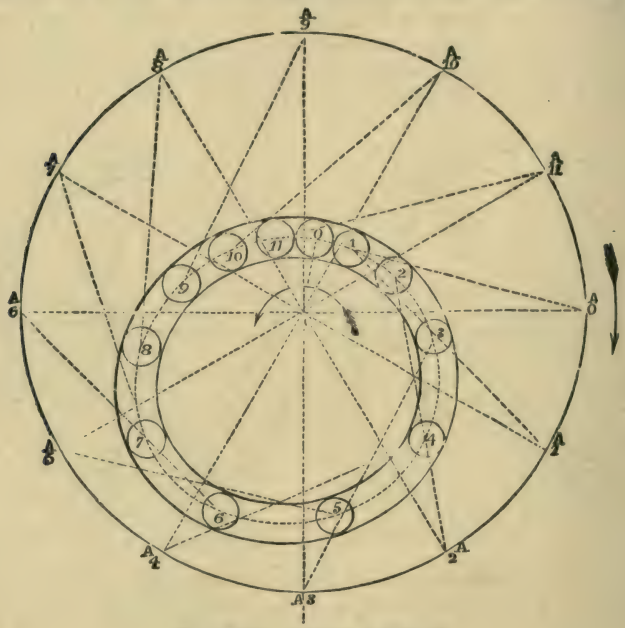
Cam Giving a Quick Return.

revolution, by drawing radial lines making angles of 180°, 60°, and 120°. Since the motion is to be uniform, divide the travel into a convenient number of equal parts, say twelve; and the circumference into the same number of equal parts by radial lines. Draw the concentric arcs 2, 2″; 4, 4″; &c., and 3, 3′; 6, 6′; &c., as shown. The curves through the points so determined will give the required motions. The interval of rest will evidently be given by the circular portion from 12″ to 12′. The complete outline is represented by the heavy line in the diagram.

Quick Return Cam.—The student will readily understand from the right-hand figure, that if two-thirds of a revolution be occupied in raising the motion bar and the remainder in lowering the same, the return stroke will be performed in half the time of

forward stroke. The curves of this cam are found in the same way as in the previous examples.

Cam with Groove on Face.—When the reciprocating bar has to be pulled as well as pushed by the cam, it is evident that the cams already considered would not drive it, but leave it at its extreme position. In such a case the periphery of the cam plate is not used, but a groove is cut in its face, as shown by the accompanying figure. The end of the rod carries a pin which works in this groove. The rod, therefore, gets pushed out by the inner face of



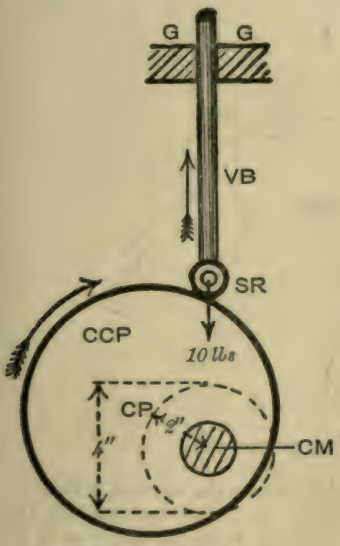
CAM WITH GROOVE ON FACE.

the slot and pulled in by the outer face. The central dotted curve is obtained in precisely the same way as before, and the two full curves are drawn parallel to it at a distance on each side of it equal to the radius of the pin.

Cylindrical Grooved Cam.—This differs from the above in that its rim is cylindrical and long. A groove is cut around its cylindrical surface, but it is not made circular. Parts of it are spiral, and so act on a pin like a screw. This gives a motion to the bar parallel to the axis about which the cam rotates.

EXAMPLE V.—A vertical bar, moving in guides, is driven by a circular cam plate having a centre of motion in the centre line of the bar. The distance from the centre of motion to the centre of the plate is 2 inches, and the bar exerts a pressure of 10 lbs. when rising, but falls by its own weight. Find the work done in 100 revolutions of the plate.

ANSWER.—Since the distance between the roller, S R, and the centre of the plate, C P, remains constant as the plate revolves,



CIRCULAR CAM PLATE.

INDEX TO PARTS.

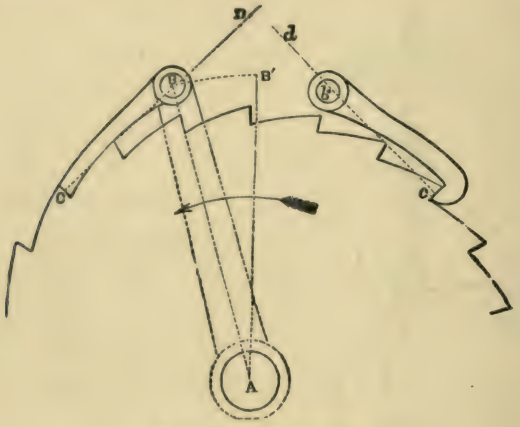
G	represents	Guides.
V B	,,	Vertical bar.
S R	,,	Slipping roller.
C C P	,,	Circular cam plate.
C P	,,	Centre of plate.
C M	,,	Centre of motion.

it is evident that the bar will move as if it were actuated by a crank of length equal to the distance between C M and C P, and a connecting-rod of length equal to the radius of the plate. Hence the stroke of the bar will be 4 inches, or $\frac{1}{3}$ foot—*i.e.*, twice the length of the equivalent crank. Neglecting friction, the work done in raising the bar by one revolution of the plate, will be :—

$$10 \times \tfrac{1}{3} \text{ (ft.-lbs).}$$

$\therefore$ *Work done in* 100 *revolutions* $= 100 \times 10 \times \frac{1}{3} = 333\dot{3}$ ft.-lbs.

Pawl and Ratchet Wheel.—A toothed wheel which is acted upon by a vibrating piece, termed a *click* or *pawl*, is called a *ratchet wheel*. Ratchet wheels are made in many different forms, and are used for a variety of purposes. For instance, clocks and watches are usually provided with ratchet wheels to allow the spring or weight to be wound up, without disturbing the rest of the works, and they are used to drive the feeding arrangements of many machines. When, as in the latter case, the click or pawl drives the ratchet wheel, it is carried on a vibrating arm. In the first

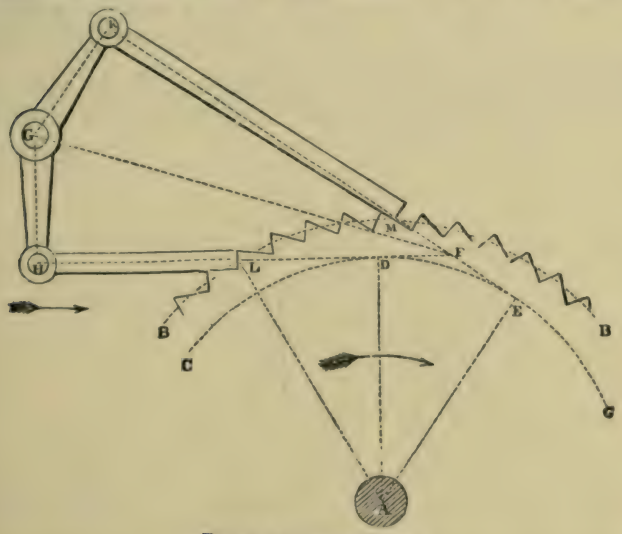


PAWL AND RATCHET.

figure, A B is the vibrating bar which drives the ratchet wheel, by means of the click, B C, and teeth, C *c*, when moving in the direction shown by the arrow. When A B moves back to A B′, the click slides over the top of the next tooth and drops behind it. It is then ready to drive the wheel through the space of another tooth when A B again moves forward. While the pawl is moving back from B to B′, the wheel is prevented from moving with it by another *pawl* or *detent*, *b c*. In this case, the vibrating bar is on the same axis as the ratchet wheel; but this is not always so, as will be seen from the next example. The reactions between the teeth and the pawl have to keep them in contact with each other. The resultant pressure of the teeth on the pawl must therefore be such, that its moment tends to turn the pawl towards A, the centre of the ratchet wheel. This condition evidently is satisfied if C D, the direction of the resultant pressure at C, passes between A and the axis, B, about which the pawl

turns. Similarly, the moment of the resultant pressure on the detent must tend to turn it towards A, but its direction, *d c* (not *c d*), must lie outside A *b*, because this detent ends in a hook. Both pawls might have been like B C, which acts by pushing, or both hooks, which act by pulling, like *b c*. The pawls are pressed against the ratchet by their own weight, or by springs, according to circumstances. When a ratchet wheel is used only to prevent the recoil of the axis on which it is fixed, the vibrating arm is, of course, not required, and only the detent is used.

La Garousse's Double-Acting Pawl.—This is a pawl which advances the ratchet wheel at each stroke. As will be seen from

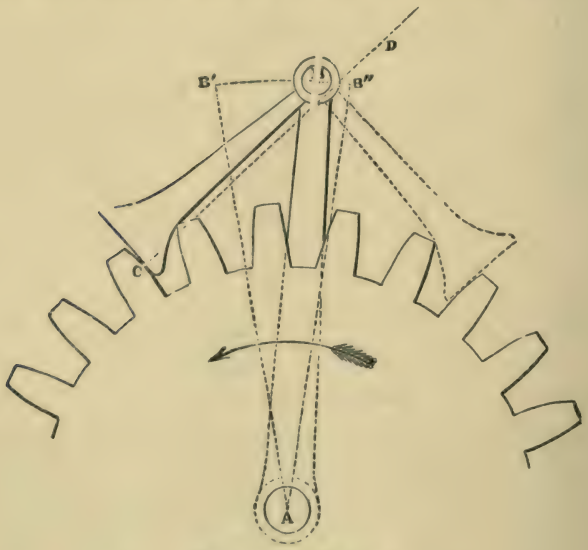


DOUBLE-ACTING PAWL.

the diagram, it is composed of two clicks, K M and H L, carried by an arm, K G H, which vibrates about its centre, G. While the arm is turning in the direction of the straight arrow, H L advances the ratchet by the space of half a tooth while K M retires another half, and, therefore, drops behind the next tooth. During the return of K G H, K M drives the ratchet while H L moves back. It will thus be seen that no detent is required, and that the motion of the ratchet is nearly continuous. The pawls may be hooks, when, of course, the teeth will be modified to suit. The positions of the clicks and arm may be found in the following manner:—Draw any convenient radius, A L, of the pitch circle, B B, and from it set out

the angle, L A D, equal to the desired mean obliquity of the clicks. Draw L D perpendicular to A D, and describe the circle C C with radius A D; the directions of the clicks at mid-stroke will be tangents to this circle. Make angle D A E equal to an *odd* number of times half the pitch angle of the teeth, and draw E M, the tangent, at E. Let this intersect the tangent L D at F, and the pitch circle in M. Draw F G, bisecting angle M F L, and take G, any convenient point in it, for the centre of the rocking shaft. Lastly, make G H and G K perpendicular to H L and K M respectively. Then, K G H is the position of the vibrating arm, and H L and K M the lengths of the two clicks, and their positions at mid-stroke. The *effective* stroke of the clicks is half the pitch of the teeth, and the *total* stroke as much greater as may be necessary to ensure their clearing the teeth.

Reversible Pawl.—The next figure shows a form of click used in the feed motion of shaping and other machines. The ratchet



REVERSIBLE CLICK.

wheel is here an ordinary toothed wheel, and the click, B C, is so shaped as to be able to drive it either way. When the click is in the position shown in full lines, it drives the ratchet wheel in the direction of the arrow. When the wheel is required to rotate

the other way, the click is lifted over to the dotted position; and, if it be desired to stop the feed motion without stopping the machine, the click is put in an upright position. A portion of the pin at B, which turns with the click, is triangular in section. A spring presses on this part and so keeps the click in any one of its three positions. The ratchet wheel is keyed to A, the axis of the screw which moves the slide carrying the cutter, and the friction between this screw and its nut is sufficient, without any detent, to prevent the ratchet from moving back. The vibrating arm, A B, which carries the click is driven by a small eccentric or crank. The pawl may, of course, be made to move the rachet more than one tooth at a time by adjusting the angle through which A B vibrates.

Masked Ratchet.—In numbering machines it is often necessary to print the same number twice, as in cheques and their counterfoils. The ratchet which shifts the type wheels must, therefore, be moved at every alternate back-stroke of the printing machine. This may be accomplished by putting a second ratchet, running free on the shaft, alongside the driving one and making the pawl broad enough to move both. The second ratchet has the same number of teeth as the other, but its teeth are made alternately deep and shallow. It is also a little larger than the driving ratchet, so that the pawl passes over the top of the teeth of the latter, without moving it, when in a shallow tooth. Next stroke the pawl drops into a deep tooth. This allows it to catch the

INDEX TO PARTS.

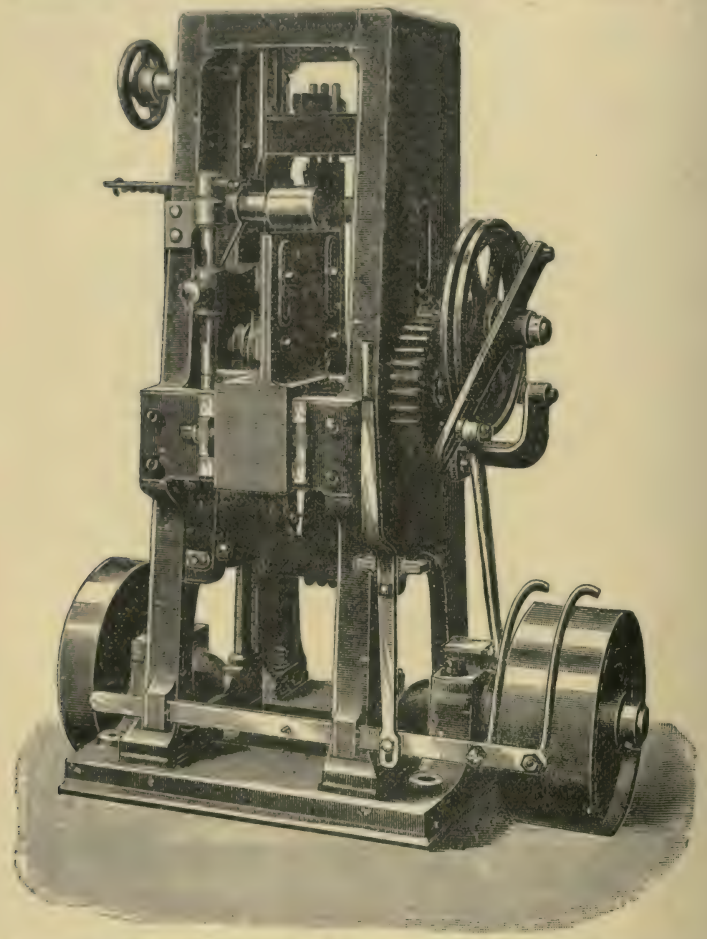
G W for Grooved wheel.
E C ,, Eccentric cam.
L ,, Lever.
E D ,, Eccentric detent.
E R ,, Eccentric rod.

WORSSAM'S SILENT FEED.

teeth of the main ratchet and so shift the type wheel. This arrangement is called a *masked ratchet.*

Silent Feed.—A ratchet wheel is always more or less noisy in

action, and the wear caused by the sudden drop of the pawl is considerable. To avoid this, a friction catch is sometimes substituted for the pawl and a grooved wheel for the toothed one.



VERTICAL SAWING MACHINE, BY JOHN M'DOWAL & SONS OF JOHNSTONE, SHOWING SILENT FEED.

The pawl and ratchet then becomes a *silent feed.* The action of this arrangement will be easily understood by a reference to

the figure.* E C is an eccentric cam which is tapered at its edge to fit the groove in the grooved wheel, G W. It can turn on a pin carried by the lever, L. When E C moves as shown by the arrow, the friction causes it to turn about its axis, and, since the axis is not concentric with the circular part of its rim, it gets wedged in the groove. Hence, for the rest of the stroke, the lever carries G W round with it. At the beginning of the return stroke, E C turns in the opposite direction, and so gets released from the groove. A detent, E D, precisely similar to E C, but carried on a fixed arm, prevents the wheel from moving backwards. The lever, L L, is worked by an eccentric, and the length of its stroke may be adjusted by altering the position of the end of the eccentric rod, E R, in the slot. The second illustration shows a sawing machine, with this feed motion at the right-hand side.

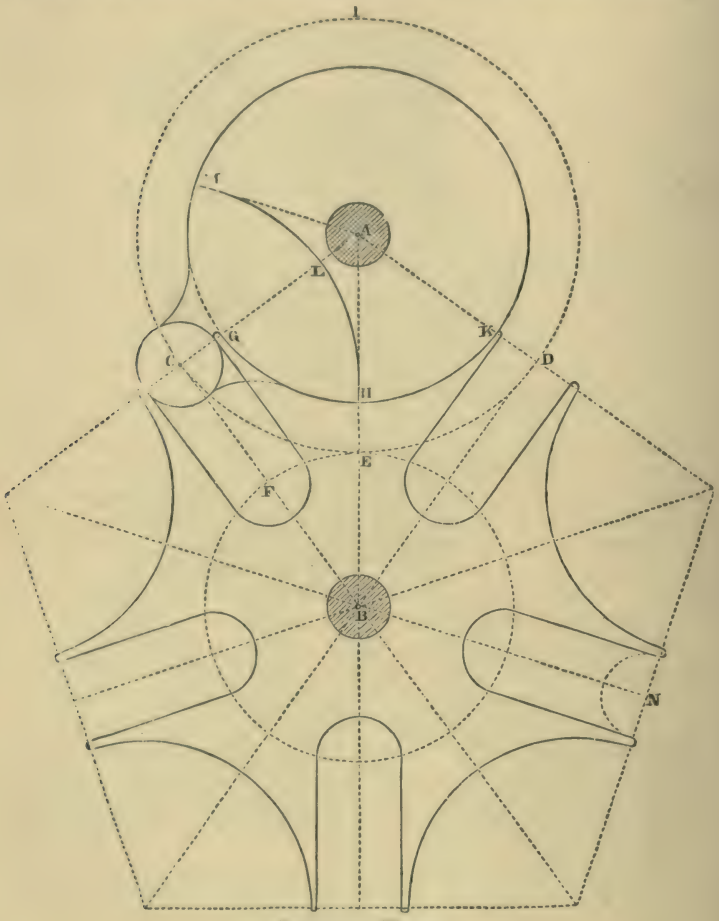
Counting Wheels.—In counting machines, the wheel carrying the figures for the tens must turn through one-tenth of a revolution while the units wheel shifts from 9 to 0, and it must remain at rest at other times. The same is true of the wheels for the hundreds and tens, and so on. The most obvious way to do this is to put ten teeth on the follower on the tens shaft, and only one on the driver on the units shaft. Every time the units wheel passes a certain point it will, therefore, shift the tens wheel by one tooth. The teeth on the follower are usually pins, and a roller is pressed between them by a spring. This roller serves to bring the wheel to its exact position, and to lock it there.

Another device is shown in the accompanying figure, which avoids the shock that always takes place in the first arrangement. Here, A is the shaft whose revolutions require to be counted, and B the centre of the counting wheel. The wheel fixed to the shaft, A, carries a pin, C, which moves the counting wheel by gearing with the sides of the slots. While the pin is in a slot, the horns, G, K, &c., pass through the part, M G H L, of the driving wheel, which is cut for the purpose. After the pin has left the slot, the curved part, G H K, bears on the convex arc, H K M, and so locks the counting wheel in the position shown in the figure. The figure shows only five slots, but there may be ten, or any other required number. When used for counting, there would, of course, be ten.

The following construction may be used for finding the proportions of the various parts:—Join A B, and set out angles C B A, A B D, &c., each equal to $\frac{360^\circ}{2n}$, or $\frac{180^\circ}{n}$, where n is the number of

* We have to thank Messrs John M'Dowal & Sons, of Johnstone, the makers of this machine, for these two figures

slots required. Draw A C and A D perpendicular to B C and B D respectively, and complete the regular polygon, of which C A and A D are the halves of two adjacent sides. In the figure, this is a



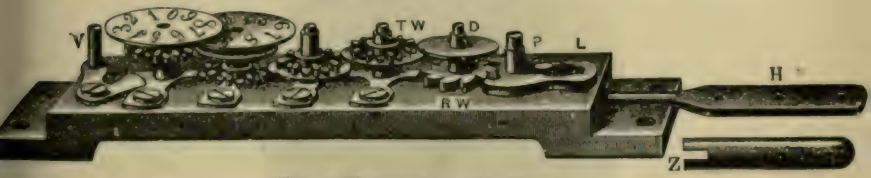
COUNTING WHEEL.

regular pentagon. Then each alternate line radiating from B, such as B C or B D, is the centre line of a slot, and the others, as B A, the centre lines of the circular arcs, G H K, &c. With centre

A and radius A C, describe the circle C E D, to represent the path of the centre of the pin, and another circle, with B as centre, to touch it. Then, at the points where this latter circle cuts the centre lines of the slots, make semicircles of the same radius as the pin, C. The sides of the slots may then be drawn parallel to their respective centre lines to touch these semicircles. The arcs, G H K, are drawn with radius equal to A H, and their centres at the angles of the polygon, so as to be concentric with H K M when in gear with it. The arc, M L H, has its centre in A C, and its radius equal to B G or B K.

Geneva Stop.—This is a modification of the above, used to prevent watches being overwound, and such like purposes. This is effected by filling up one of the slots, as shown by the dotted circle at N. The pin on A is arrested when N reaches C or D. The same thing would result from filling up one of the hollows like G H K. It is obvious that the shaft, A, can make one complete turn for every slot, except the stopping one, and a complete turn all but the angle, G A K, for that one.

Counting Machines.—The accompanying illustration shows a very good form of counting-machine (with three of its dials removed)



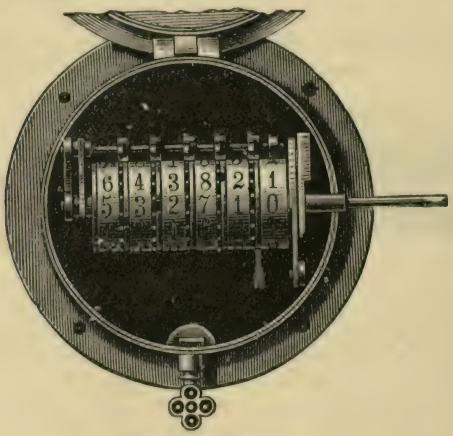
THE "UNIVERSAL COUNTER.

which works on a modification of the first of the above-mentioned methods.* The driving mechanism consists essentially of a short lever, L, which can oscillate about the pin, P, and which drives the ratchet wheel, R W, by two projections on its end. Below this lever, L, there is a circular plate from which a pin projects into the hole in L. This pin, not being concentric with the circular plate, will cause L to oscillate when the plate rotates. The rod, Z, is attached to the back of this plate when the instrument is used to count revolutions, and the lever, H, to its edge, as shown in the figure, when used to count oscillations. The ratchet wheel spindle carries the first or units dial, and also a disc, D, having a pin projecting downwards from its lower edge. The next spindle, to which the tens dial is attached, has a toothed

* This and the following figure were kindly supplied by the makers of these instruments, Messrs. Schäffer & Budenberg of Glasgow, &c.

wheel, T W, with twenty teeth. These teeth are alternately broad and narrow, and the wheel is locked by the disc, D, gearing between two consecutive broad teeth. At the proper time, the pin on D comes into gear with a narrow tooth, and, at the same time, a notch on the edge of the disc allows one broad tooth to pass round. The tens spindle, therefore, makes one-tenth of a revolution. The same arrangement is adopted for the other dials. The case of the instrument has windows which allow only one figure on each dial to be seen at a time. In order that the dials may be easily set to zero, their spindles are each mounted on separate levers, which are locked in their places by the bent lever to which V is attached. Pulling V to the left frees these levers, and permits them to be so turned as to put the toothed wheels out of gear with their respective discs, when the dials may be set to zero.

Another counter by the same makers is shown in the second

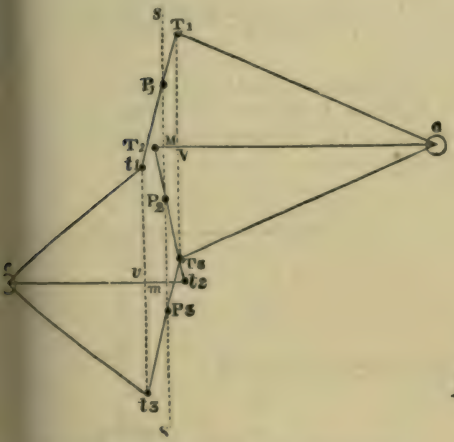


HARDING COUNTER.

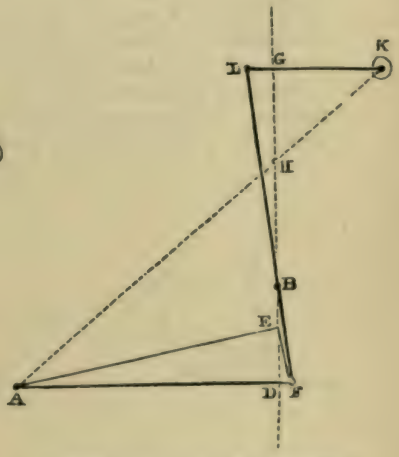
figure. In this instrument the dials are cylindrical and all run loose on one shaft. To the right-hand side of each (except the first) is fixed a number of pins, and to the left (except on the last) two only. Above the dials, between each pair, a set of little toothed wheels is mounted loosely on a secondary spindle. These toothed wheels gear with the pins on the dials, and every alternate tooth is also broad enough to gear with the side of the right-hand dial, which locks them in the same way as the disc in the "Universal" counter. The two pins on the left of a dial come into

gear at the proper time, and a slot on the side of the dial allows the little pinion to turn two teeth. This is just sufficient to cause the next dial to turn through one-tenth of a revolution. The same thing is true of each of the other dials. This instrument is driven in a similar way to the previous one.

Watt's Parallel Motion.—Referring to the illustration of Watt's double-acting engine, previously given in this Lecture, the student will notice that the beam and piston-rod are connected by a set of links. This system of links has been called *Watt's Parallel Motion*. The first part—viz., that for guiding a point in a straight line—is usually termed a "parallel motion," although this term properly belongs to the portion which makes certain other points travel in paths parallel to that of this guided point. The next figure will



WATT'S APPROXIMATE STRAIGHT-LINE MOTION.



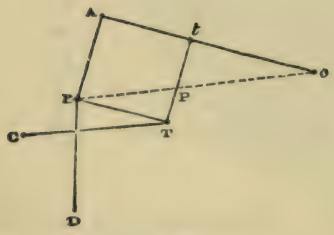
CONSTRUCTION FOR LENGTHS OF LINKS.

serve to show the principle on which an approximate rectilinear motion is obtained. Part of the beam of the engine is shown in three different positions, C T_1, C T_2, and C T_3. The point, T, in it is connected by the link, T t, to the end of a lever or radius rod, c t, pivotted at c. In their mid positions, C T_2, c t_2, these two levers are usually parallel to each other, and perpendicular to the line P_1 P_2 P_3. The point, T, describes an arc of a circle round C, and t round c. As these arcs curve in opposite directions, we should expect some intermediate point on the link, T t, to curve in neither direction, but to describe an approximate straight line.

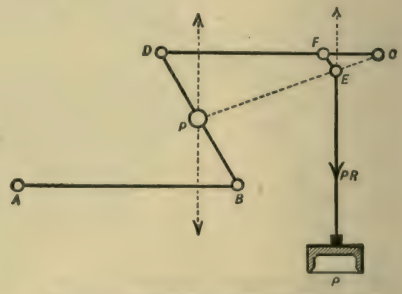
This point, P, may be found by making $\frac{P\,t}{P\,T} = \frac{C\,T}{c\,t}$. The actual path of P is like the figure 8, and the parts which cross are very nearly exact straight lines for a short distance on either side of the crossing.

Prof. Rankine gives the following construction for the lengths of the links in his *Machinery and Millwork* :—Let A be the centre of the beam, G D the centre line of the piston-rod's motion, and B the mid position of its end. Draw A D perpendicular to G D. Make D E equal to one-fourth of the stroke, and join A E. Draw E F perpendicular to A E, and meeting A D in F. A F is the length of the beam. If G be the point where the radius rod cuts G D, draw G K at right angles to G D, and make D H equal to G B. Join A to H, and F to B, and produce A H and F B to meet G K in K and L. Then, F L is the connecting link, K L is the radius rod, and B is the point on the link, F L, to which the piston-rod must be attached.

Parallel Motion.—We will now consider the parallel motion proper. In the accompanying figure A B T t is a parallelogram, and c is a point in A t produced. In the meantime we will



PARALLEL MOTION.



PARALLEL MOTION FOR RICHARD'S INDICATOR.

leave the links C T and B D out of account and consider the parallelogram only. Join B c and we have two similar triangles, B A c and P t c.

$$\therefore \quad \frac{P\,t}{B\,A} = \frac{t\,c}{A\,c}. \qquad \therefore P\,t = B\,A\,\frac{t\,c}{A\,c} = \text{a constant.}$$

That is, in every position of the parallelogram the point, P, remains in one fixed position in the link, T t. Moreover, the ratio

$\frac{B\,c}{P\,c}$ is constant, and, therefore, whatever path P traces out, B will trace out a similar one. This is the principle of the *pantograph*, which is used for enlarging or reducing drawings. Now, we have just seen how we may make P move in an approximate straight line by the link, C T. B will, therefore, also move in an approximate straight line. We might have guided B instead of P with a radius rod, but this would have necessitated longer and heavier links and would have occupied more space.

In applying this motion to his engine, Watt made A *t c* the beam, and attached the piston-rod to B and the air pump-rod to P. The lengths A *c*, *t c* were, therefore, proportional to the strokes of the piston and pump bucket respectively. Sometimes a third link was added so as to get a second parallelogram and a second point moving parallel to P, and this was used to drive the feed-pump.

The right-hand figure shows the parallel motion of Richard's steam engine indicator.* The student will at once see that it is a modification of Watt's parallel motion. In this case the piston-rod, P R, is attached by the link, E F, to the bar, C D, between D and the centre, C. The motion of *p*, to which the pencil is attached, is, therefore, a *magnified* copy of the piston's motion.*

* See Lecture XVI. of the author's *Elementary Manual of Steam and the Steam Engine* for a description of this indicator.

Lecture VI.—Questions.

1. Describe Hooke's joint for connecting two axes whose directions meet in a point. Investigate a method of setting out in a diagram the angles described by the axes in the same given time. (Hons. S. and A. Exam., 1895.)

2. Sketch and describe the double Hooke's joint, and explain why it is used in certain cases in preference to the single joint.

3. The rods of a double-barrelled pump are attached to a double-cranked shaft, at the end of which is a wheel with 30 teeth. The wheel gears with a pinion of 8 teeth driven by a winch handle. Find the number of single strokes performed by each pump rod while the winch handle makes 15 revolutions, and sketch the arrangement. *Ans.* 8.

4. In printing machines the table is sometimes made to reciprocate by running a pinion between two racks, whereof one is fixed and the other is attached to the table. The pinion may be actuated by a crank and connecting-rod, but in that case the reciprocation of the table is not uniform, how may a uniform reciprocation be obtained?

5. What is an epicyclic train, and where are such trains chiefly employed? Investigate a formula for ascertaining the relative velocities of the first and last wheels of such a train. In an epicyclic train, where the first wheel has 20 teeth and is fixed, the second and third wheels are on one axis and have 30 and 40 teeth respectively, and the last wheel has 50 teeth, find the number of rotations of the last wheel for 30 rotations of the arm. In which direction does the last wheel rotate relatively to the arm?

Ans. 14; in the same direction.

6. In a rope-making machine, the reels containing the strands are carried round in a circular path, but no twisting or untwisting of the strands occurs during the operation. Sketch and describe the epicyclic train, or other device, by which you would accomplish this. If a little extra twist be required to be put on the strands, how may this be done? Explain your answer fully.

7. Prove the formula which gives the velocity of rotation of the last wheel of an epicyclic train in terms of the velocities of the first wheel and the arm, and arrange an epicyclic train in which the last wheel and the arm shall rotate with equal velocities in opposite directions.

8. A train of three spur wheels is carried by a revolving arm, the first is a dead wheel of 60 teeth, the second has 30 teeth, and the third has 45 teeth. Prove the formula for determining the number of revolutions of the second and third wheels for each revolution of the arm, and ascertain the actual numbers in this example. *Ans.* $3; -\frac{1}{2}$.

9. Investigate the kinematic properties of an epicyclic train formed by a combination of three equal bevel wheels in gear. Describe, with sketches, the manner in which this combination has been applied in Houldsworth's differential motion. Mention other useful applications of the combination, pointing out the special results obtained.

10. An epicyclic train supported on a frame consists of (1) a spur wheel, A, having 40 teeth; (2) a disc, B, having the same axis as A, and carrying at equal intervals three pinions of 16 teeth, each of which gears with A; (3) an annular wheel, C, of 72 teeth coaxial with A and B, and gearing with the three pinions. If A be made a dead wheel and C be the driver, find the velocity ratio of B to C, both as regards magnitude and direction, proving

any formula which you employ. If B be locked to the frame, and A be the driver, find the same as regards C and A. How would you alter the gearing on B, so that C and A may rotate in the same direction while the disc B remains locked to the frame? How may the driving gear of a bicycle be arranged so that the vehicle may travel more slowly up-hill, while the pedal axis runs at the same rate as on level ground? *Ans.* $\frac{9}{14}$; $-\frac{5}{9}$.

11. Explain the manner in which Watt used the so-called Sun and Planet Wheels as a substitute for a crank and connecting-rod, and account for the result which he obtained.

12. What are elliptical wheels, and for what purpose are they used? What peculiar property of the ellipse has to be taken into account in designing them, and how are they arranged in practice? Give a sketch. How are these wheels applied to the driving of cycles?

13. Sketch a cam for giving a bar a uniform reciprocating motion, and explain how you find the form of its periphery.

14. Set out a form of cam which, when acting on a bar by uniform rotation, will cause the backward and forward motion of the bar to have an interval of rest between each. Describe some other method of obtaining an intermittent motion of this kind.

15. Describe, by the aid of the necessary sketches, how the circular motion of the driving pulley is converted into the reciprocating motion of the punch in an ordinary machine for punching holes in metal plates. Calculate the approximate maximum pressure in pounds at the end of a punch in cutting a hole 1 inch in diameter through a steel plate $\frac{5}{8}$ inch thick, the resistance of the plate to shearing being taken as 50,000 lbs. per square inch of section. *Ans.* 98,175 lbs.

16. Sketch and describe what form of cam you would use when it is required to drive the bar both ways, (1) at right angles to the axis of the cam, and (2) parallel to it.

17. Sketch a pawl and ratchet wheel as used for preventing the recoil of the gear.

18. Sketch and describe some form of pawl which will drive a ratchet wheel during both the forward and backward strokes.

19. Sketch a ratchet feed motion, such as is suitable for a planing machine, and explain the manner in which the amount of feed is regulated.

20. It is sometimes useful to advance a ratchet wheel at every *alternate* forward stroke of the driver, instead of at every stroke, as is commonly the case; describe and sketch a mechanical contrivance which will give such a movement.

21. Describe, with the necessary sketches, some form of silent feed arrangement commonly used instead of a ratchet wheel, for advancing the timber in sawing machines. Explain the principle of the friction grip upon which such a contrivance depends. Within what limit as to deviation of the line of pressure from the common normal is a friction grip possible, and why?

22. Sketch and describe a vertical sawmill, showing how the silent feed is applied.

23. Sketch and describe an arrangement for counting the number of strokes or revolutions of an engine.

24. Explain the principle of Watt's approximate straight line motion, commonly called a parallel motion. By what combination of linkwork is an exact straight line motion obtained? Prove the geometrical proposition upon which the result depends.

25. Show how you would design a cam to secure the following motions:—A slow forward uniform motion, a period of rest, and a quick return.

(C. & G., 1900, O., Sec. A.)

26. Sketch a Watt parallel motion as used:—(*a*) In a Richard's steam engine indicator; (*b*) In a compound beam engine. Is this motion exact? (C. & G., 1900, O., Sec. A.)

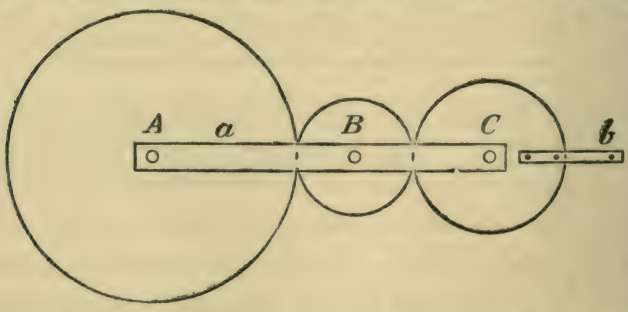
27. A plank, 10 feet long, is rolled over a roller which can roll on the ground. Through what distance will the plank have moved before it rolls off the roller? Give reasons for your answer. (C. & G., 1902, O., Sec. A.)

28. Describe Watt's parallel motion, and show how it is applied, in conjunction with the pantograph, in beam engines. (C. & G., 1902, O., Sec. A.)

29. In a planing machine the table is driven by a rack and pinion. For the cutting stroke the pulley shaft is connected with the rack through the following gearing:—A pinion A rigidly connected to one of the pulleys has 24 teeth, this gears with a wheel B, which has 64 teeth; on the same axis as B is a pinion C of 18 teeth, which gears with a wheel D of 72 teeth, and on the axis carrying the wheel D is the pinion E which has 15 teeth and drives the rack. The pitch of the teeth of the rack is $1\frac{1}{2}$ inches. On the quick return stroke D is driven direct by a pinion F having 18 teeth, and rigidly connected to another pulley on the pulley shaft. The stroke of the table is 6 feet. Find—(*a*) The number of revolutions per minute of the pulleys, if the cutting speed is not to exceed 25 feet per minute. (*b*) The time taken for one complete reciprocation of the table. (*c*) The average force exerted by the tool during one cutting stroke, if the horse-power passing to the planing machine through the belt during the cutting stroke is 3, and if the efficiency of the mechanism is 37 per cent. (B. of E., S. 2 and 3, 1905.)

30. Draw the profile of a cam to do the following work:—It has to lift vertically with uniform velocity a moving bar, the length of the travel of the bar being 6 inches; it then has to allow the bar to descend again with uniform velocity, but at one-half the speed of the ascent. The two movements occupy one revolution of the uniformly rotating cam. The diameter of the roller working on the cam is $\frac{1}{2}$ inch, and the least thickness of metal round the cam centre must be 2 inches. The line of stroke of the moving bar passes through the cam centre. (B. of E., S. 2, 1906.)

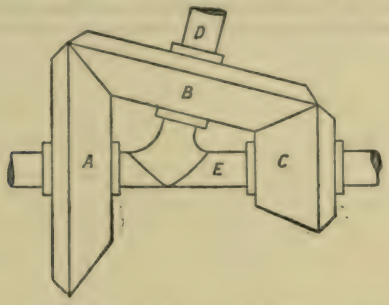
31. In the epicyclic train shown in the sketch, the wheel A, which has 30 teeth, is fixed. The rotating arm *a*, which rotates about the centre A,



carries a wheel B, which gears with A and has 12 teeth, and also a second wheel C, which gears with B and has 15 teeth. To the wheel C is rigidly fixed an arm *b*. If the speed of the arm is *n* revolutions per minute clockwise, what is the speed of the wheel C about its centre C? Find, for one revolution of the mechanism, (*a*) the path of a point on the arm *b*,

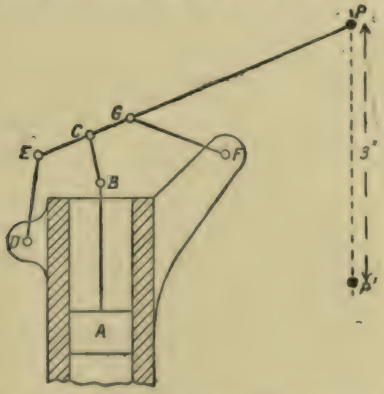
whose distance from the centre C is equal to the distance between the two centres A and C; (*b*) the path of a point on the arm *b*, whose distance from the centre C is one-half that between the two centres A and C. (B. of E., S. 2 and 3, 1906.)

32. In the epicyclic bevel gear shown in the sketch, the wheels A and B have each 40 teeth, and the wheel C has 20 teeth; the shafts D and E are in one solid piece, and rotate together at the rate of 60 revolutions per



minute about the axis of E; each wheel is free to rotate on its own spindle, and the wheel A rotates thirty times per minute in a direction opposite to the rotation of the shaft E. Find the speed and direction of rotation of the wheel C. (B. of E., S. 2, 1907.)

33. The mechanism connecting the pencil and piston of a steam engine indicator is shown in skeleton form in the sketch. The sketch is drawn



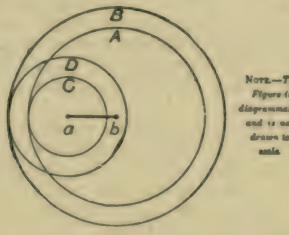
to scale, but the point G is not in its correct position on the bar EP. Determine the correct position of the point G in order that the pencil P may travel approximately in a vertical line parallel to the path of the piston A. (The travel of the pencil will not be greater than 3½ inches to 4 inches.) How far does the pencil P move for a 1-inch travel of the piston A? (B. of E., S. 3, 1907.)

34. In a new coal-cutting machine there is an epicyclic gearing of the following character:—Two concentric annular wheels A and B, which have 37 and 36 teeth respectively, are mounted loosely on the same shaft; a spur-wheel C, which has 32 teeth, is mounted on an eccentric fixed to the shaft in such a manner that C gears with both A and B. The wheel A forms part of a drum which is prevented from rotating by means of a brake. Power delivered to the shaft is transmitted by means of the gearing to a second drum to which the wheel B is fixed. If the driving pulley on the shaft is 26 inches in diameter, and if the drum is 18 inches in diameter, find the velocity ratio of the mechanism. (B of E., S. 3, 1908.)

Lecture VI.—A.M.Inst.C.E. and B.Sc. Exam. Questions.

1. Describe the "parallel motion" of Watt, and show that it gives an approximately straight-line motion. How is the same device extended to serve as a pantograph? (I.C.E., *Feb.*, 1898.)

2. In a bicycle cyclometer A is a fixed annular rack; B a movable annular rack concentric with A, the centre being at (*b*). C and D are pinions rigidly attached to each other and rotating about a centre (*a*) at the end of an arm (*a*, *b*). C gears with A, and D with B. The number of



teeth in A, B, C, D respectively are 22, 23. 19, and 20. For one revolution of the bicycle wheel the arm (*a*, *b*) turns through 72°. Find the diameter of the wheel in order that B shall make one revolution for each mile run. (I.C.E., *Oct.*, 1900.)

3. Describe Watt's "sun and planet" motion. If the diameters of the gearing wheels were in another ratio than equality, how would the number of revolutions be affected? (I.C.E., *Feb.*, 1901.)

4. An epicyclic train of wheels is constructed as follows:—A fixed annular wheel, A, and a smaller concentric rotating wheel, B, are connected by a compound wheel, $A_1 B_1$, the portion A_1 gearing with the wheel A, and B_1 with B. The compound wheel revolves on a stud which is carried around on an arm which revolves about the centre of A and B. A has 130 teeth, B 20, and B_1 80, the pitch of the teeth of A and A_1 being twice the pitch of B and B_1. How many revolutions will B make for one turn of the arm? (I.C.E., *Oct.*, 1902.)

5. What is an epicyclic train? Two wheels, A and B, are so mounted as to form an epicyclic train, and A is a dead wheel having 80 teeth, while B has 20 teeth. Find the number of turns made by B for each rotation of the arm. (I.C.E., *Feb.*, 1903.)

6. Set out a form of cam which is to impart vertical motion to a bar so that the latter shall be raised uniformly during the first half revolution, remain at rest during the next one-sixth, and descend uniformly during the remainder of the revolution. (I.C.E., *Feb.*, 1903.)

7. A plank, 10 feet long, is rolled over a roller which can roll on the ground. Through what distance will the plank have moved before it escapes from contact with the highest point of the roller? Give reasons for your answer. (I.C.E., *Oct.*, 1903.)

8. Describe Watt's parallel motion, and show how it is applied in conjunction with the pantograph, to the mechanism of a beam engine. (I.C.E., *Feb.*, 1904.)

9. An epicycle train of wheels is constructed and arranged as follows:—A fixed annular wheel A, having 120 teeth, and a smaller concentric rotating spur-wheel B, having 20 teeth, are connected by a compound wheel A′B′, the portion A′, with 40 teeth, gearing with the wheel A, and the portion B′ with B. The compound wheel is mounted on a stud carried on an arm which revolves about the common centre of A and B. Supposing the pitch of the teeth of A and A′ is $1\frac{1}{2}$ times that of B and B′, how many revolutions will B make for one turn of the arm? (I.C.E., *Oct.*, 1904.)

10. Describe Hooke's joint for connecting two shafts which are not parallel, and show that the two shafts have not an equal velocity ratio at all points of the rotation. Describe Oldham's coupling—that is, a coupling in which there is a disc fitted with two keys at right angles to each other for connecting two parallel shafts, and show that the two shafts have equal velocity at all points of the rotation. (I.C.E., *Oct.*, 1905.)

11. Sketch and describe any form of quick-return motion, constructed by means of belts and pulleys, and suitable for a planing machine. (I.C.E., *Feb.*, 1907.)

12. A spur-wheel S is keyed on a sleeve which turns freely on a shaft A. Design a compound train of gearing, the first pinion of which is keyed to the shaft A, and the last wheel of which is S, so that when the shaft A makes 240 revolutions per minute, the sleeve will make 15 revolutions per minute, using only one additional shaft, parallel to A and 50 inches from it. The pitch of the gearing is to be about 2 inches. (I.C.E., *Feb.*, 1907.)

13. Define a "cam." Sketch a cam suitable for actuating the exhaust valve of a gas engine. What precautions would you take in this case to minimise noise? Describe two other examples of the use of cams, illustrating your remarks by means of hand-sketches. (I.C.E., *Oct.*, 1907.)

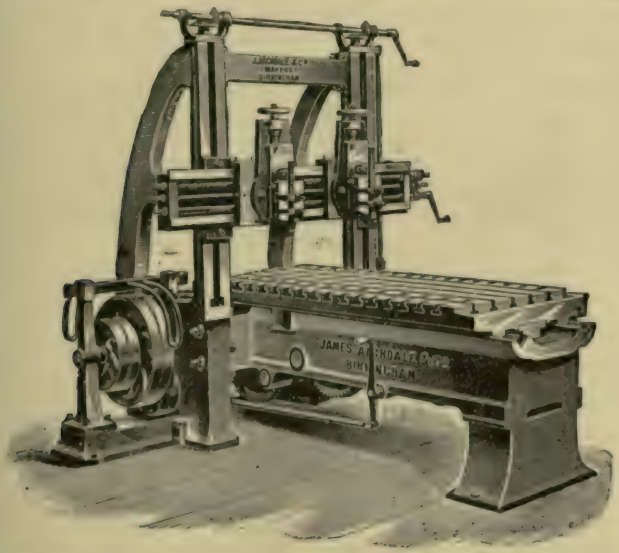
14. Describe a method of finding the angular velocity ratio of the two shafts of a single Hooke's joint, and prove that your construction is correct. When the axis of the two shafts are at 45°, find the maximum and minimum velocity ratios. Give an example of some practical application of this joint. (B.Sc., London, *July*, 1908.)

15. In a compound wheel train *a* 3 inches in diameter gearing with *b* 10 inches in diameter and *c* on the same axle as *b* 3 inches in diameter gearing with *d* 12 inches in diameter, show how to find all the virtual centres. How could this train be replaced by a link mechanism so as to give the same relative velocity between two of the links as between wheels *a* and *d*? (I.C.E., *Oct.*, 1908.)

LECTURE VII.

CONTENTS.—Reversing Motions—Planing Machine—Reversing by Friction Cones and Bevel Wheels — Whitworth's Reversing Gear — Quick Return Reversing Motion — Whitworth's Quick Return Motion — Whitworth's Slotting Machine — Common Quick Return — Horizontal Shaping Machine — Quick Return with Elliptic Wheels — Vertical Slotting Machine — Speed Reducing Gear — Questions.

Reversing Motions—Planing Machine.—The figure below will serve to show the manner in which the quick return and slower forward motions are applied to a modern planing

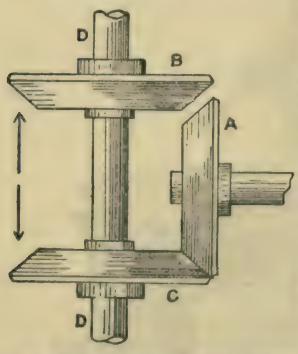


HORIZONTAL PLANING MACHINE WITH QUICK RETURN BELT GEARING BY MESSRS. J. ARCHDALE & CO., BIRMINGHAM.

machine. Here, the smaller fixed and loose pulleys are situated to the extreme left, whilst the larger ones are placed alongside of them. The pulleys are connected by underneath toothed gearing with the rack of the moving table upon which the material to be planed is bolted. The under side of this table has two straight, truly planed and scraped V-shaped surfaces, which accurately fit corresponding V grooves on the upper surface of the strong heavy bed plate. Two vertical standards, with planed and scraped surfaces on their front faces, serve to guide the cross-piece which carries two tool-holders. An up and down motion is given to this cross-piece by means of the uppermost handle, spindle, two pairs of bevel pinions and vertical screws actuating nuts connected to the back of it. A horizontal motion is given to the tool-holders by the lower right-hand handle and horizontal screws. The tool-holders themselves are adjustable up and down by handles and screws as shown. They may also be so set as to cut at any angle, or a horizontal tool-holder can be fixed upon one or other of the upright standards to plane vertical surfaces. All the above-mentioned motions may be actuated from either side of the machine.

We shall now illustrate and describe several other forms which are frequently used in connection with machinery of different kinds.

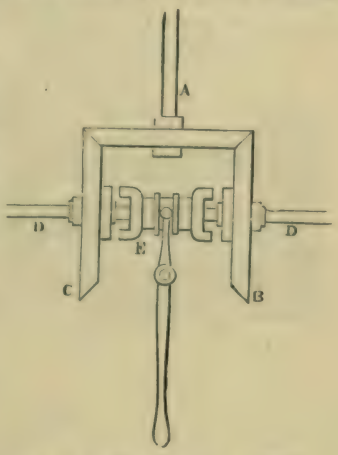
Reversing by Friction Cones and Bevel Wheels.—In the first of the two following figures, the two cones B and C are fixed to the shaft D D, which can be moved up or down so that the cones B or C may be alternately brought into contact with the other cone A, which is fixed to its shaft. Suppose A to be the driver; then, when it is in contact with B, the shaft D D will be turned in one direction, and when in contact with C it will be rotated in the opposite way. The spindle or shaft D D may, however, be driven by a belt or toothed gearing, and consequently when A is in contact with B, it will be turned in one direction and when in contact with C it will be rotated in the opposite way. This device is frequently used in connection with governors for engines, in order to lengthen or



FRICTION CONE REVERSING GEAR.

shorten the rod to the throttle valve or cut-off gear and thus enable the governor balls to regain their normal position, without altering the quantity of steam being admitted.

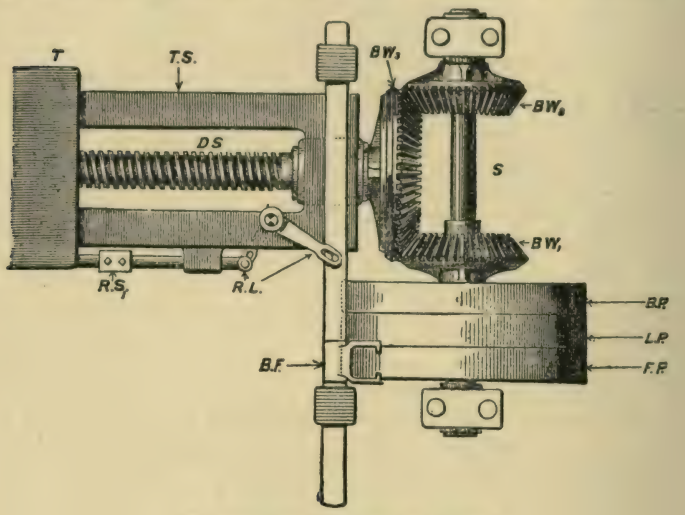
If we require to transmit more force than the friction between the cones will effect without slipping, then we must substitute toothed bevel wheels for plain cones. In such a case, if the speed and stress were considerable the sudden engagement of the wheels on one side or the other would be apt to damage the teeth; hence, it is usual with steam cranes, winches, windlasses, screwing gear, &c., to have B and C free to rotate upon their shaft D D, and always in gear with A, as shown by the second figure. The reversal in this case is effected by means of the clutch E, which can be slid along a feather on the shaft by the lever, so as to engage B or C and thus make it turn with the shaft. It will be easily seen that the direction in which A will rotate depends upon whether B or C is locked to the shaft D D.



BEVEL WHEEL AND CLUTCH REVERSING GEAR.

Whitworth's Reversing Gear.—Another modification of this gear is that made by Sir Joseph Whitworth & Co. for planing machines. In this case, the reversal is effected by shifting the driving belt by the fork B F, from the forward pulley F P to the backward one B P. The latter is cast in one piece along with or keyed to the boss of the bevel wheel $B W_1$, which runs loose on the shaft S; whilst, the former and also $B W_2$ are rigidly connected to this shaft. A loose pulley L P is placed between the forward and backward ones in order to facilitate the shifting of the belt from the one to the other and to carry the belt when the machine is not at work. The table T upon which is placed the casting or other material to be operated upon, has a strong nut fixed upon its under side, and is moved along the bed or slide T S by the driving screw D S. This screw is keyed to the bevel wheel $B W_3$ and is consequently driven in a forward or backward direction according as the belt is on F P or B P. In order to save the time that would otherwise be wasted if the cutting tool

was so fixed as to cut in one direction only, Sir Joseph Whitworth designed a cylindrical revolving tool-holder, which automatically turns the tool half round at the end of each stroke of the table. The reversal of the motion of the table is automatically effected by its pushing the reversing stops $R S_1$ (and $R S_2$ not shown in the figure) as it nears the end of a stroke. These stops are fixed in

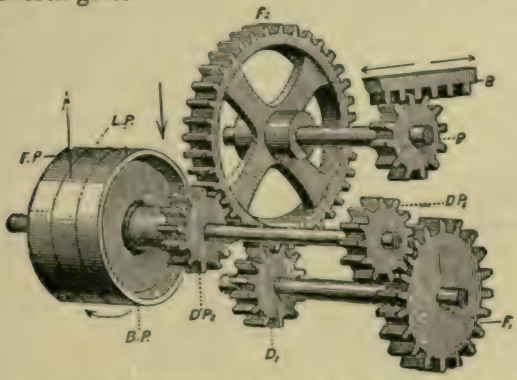


Whitworth Reversing Gear.

such positions upon a rod connected to the reversing levers R L, as to shift the belt fork at the proper time and thus give the required length of stroke.

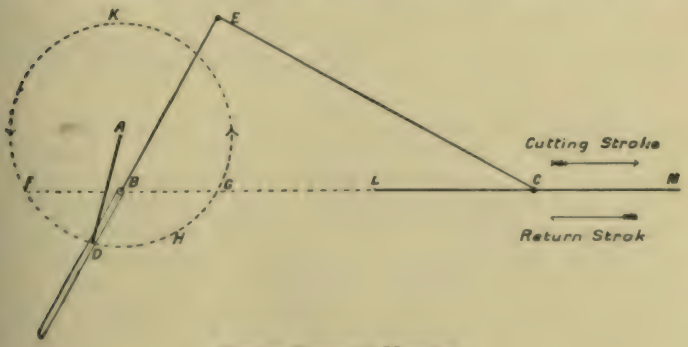
Quick Return Reversing Motion.—Another form of reversing motion, used for planing machines which only cut one way, is made up of a train of spur wheels and a rack R. As in the previous case, there are three belt pulleys, each of the outer ones being connected to a separate pinion. One of these pinions, $D P_2$, drives the rack on the planing table through a spur wheel F_2 and pinion P fixed on an intermediate shaft; the other, $D P_1$, which is connected with F P, transmits its motion through another pair of wheels F_1 and D_1. This will cause the rack and table to move in the opposite direction, and as these wheels are made of unequal sizes, the motion is also slower than when driving through $D P_2$. This slower motion is used for the cutting stroke

and the quicker one for the return stroke. The reversal is effected by shifting the belt from F P to B P, or *vice versâ*, as in the Whitworth gear.



QUICK RETURN REVERSING GEAR FOR PLANING MACHINES.

Whitworth's Quick Return Motion.—In a shaping or slotting machine the table carrying the work is fixed and the tool moves over it, cutting in one direction only. In such a case, the tool usually obtains a reciprocating motion from a compound crank, so arranged as to give a quicker return stroke in order to

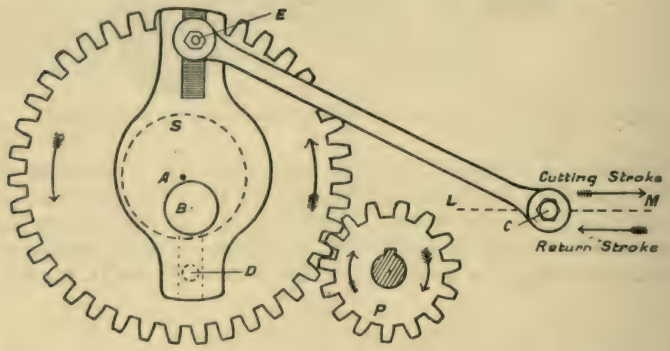


QUICK RETURN MOTION.

save time. Looking at the above diagrammatic figure, A and B are two fixed points and C is a point connected to the tool-holder, so guided as to move along L M at right angles to A B. A crank

D B E, centred at B, has its outer end joined to C, by the connecting-rod E C. A second crank A D rotates round A and drives the first by having a pin at D which moves in a slot B D. Now, when D is at the position G, C will be at the extreme left of its stroke; and when D is at F, C will be at the other end of its stroke. Hence, if A D rotates uniformly in the direction of the arrows; C will make its cutting stroke from left to right while D is moving round G K F; but during its return stroke D only requires to move round F H G. The return stroke will therefore occupy less time than the cutting stroke in the same ratio as the arc F H G is less than the arc G K F.

Sir Joseph Whitworth applied this principle to shaping machines in the manner shown by the next figure, which has the

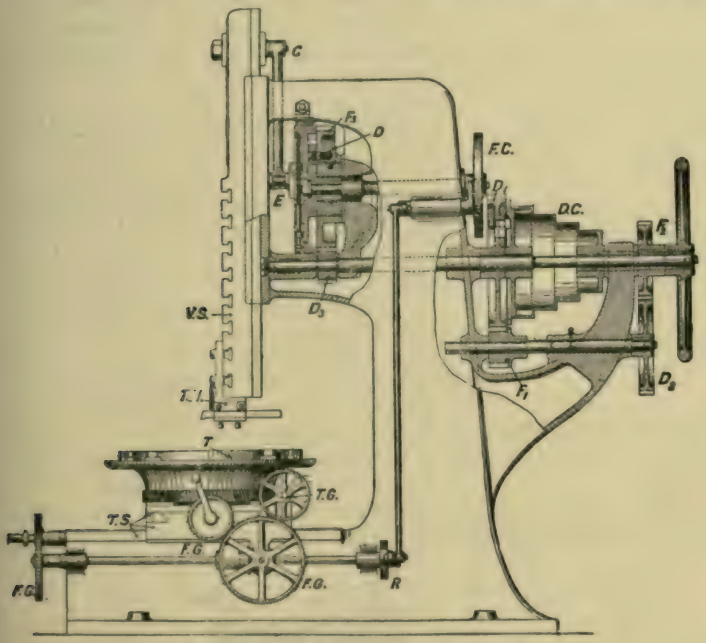


WHITWORTH'S QUICK RETURN MOTION FOR SHAPING MACHINES.

same lettering for corresponding parts as the previous one. Here, the crank A D is obtained by putting a pin D on a toothed wheel which rotates freely on a *fixed* shaft S, whose centre is at A; and the crank D B E is supported by a pin in a hole bored at B in the end of this shaft. In the back of this crank-piece there is a slot B D in which D can slide, and in the front another slot B E in which E can be clamped in any position so as to adjust the length of B E and thus give the required travel to the tool. The large wheel is driven uniformly by a pinion P connected with the belt pulley.*

* Students who desire further information on machine tools such as drilling and milling machines or on measuring appliances should refer to Professor Shelley's "Workshop Appliances," Professor Goodeve's "Elements of Mechanism," and Lineham's "Text-book of Mechanical Engineering," &c.

Whitworth's Slotting Machine.* — The following side elevation of Sir Joseph Whitworth & Co.'s slotting machine will serve to show how their quick return motion is practically applied. The driving stepped cone D C receives its motion from a corresponding overhead cone fixed to the workshop

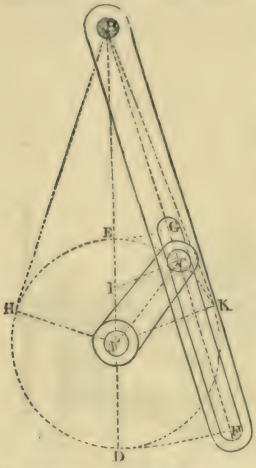


WHITWORTH'S SLOTTING MACHINE.

shafting. Its motion is in turn communicated to the compound crank E B D (just described and illustrated by the two preceding figures) through the toothed drivers and followers D_1, F_1; D_2, F_2; D_3, F_3. The quick up and slow down motion of the vertical slide bar V S, with its tool-holder T H, is obtained from the compound crank through the connecting-rod C E. The table T, upon which the metal to be slotted is bolted, may be shifted by hand levers or automatically moved to and fro, cross-

* The above figure is reduced from a lithographed drawing which appears in Mr. Lineham's book on "Mechanical Engineering," to which students may refer for further views and details.

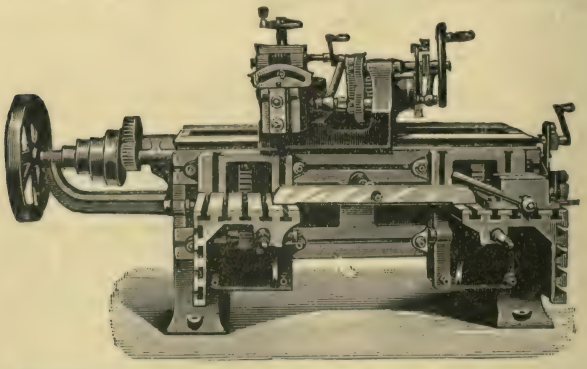
wise and turned at pleasure, by the feed gearing F G, actuated by a ratchet at R. This ratchet has a reversible click of the kind illustrated in the previous lecture and it is driven by a rod from the feed cam F C. The whole of the moving parts are supported by a strong heavy bed plate, cast in one piece with the upright framing so as to prevent vibration in the material being operated upon or chattering of the cutting tool.



COMMON QUICK RETURN FOR SLOTTING MACHINE.

Common Quick Return.—Another form of quick return very often used for shaping and slotting machines is based on the mechanism of the oscillating steam-engine. A crank A C rotates uniformly and imparts motion to a slotted arm B F. This arm will have its extreme positions at B H and B K. It will therefore make its forward and back swings while C is moving round H D K and K E H respectively, and hence it has the quick return motion desired for the tool. The tool-holder is connected by a rod to such a point in B F as will give the required travel.

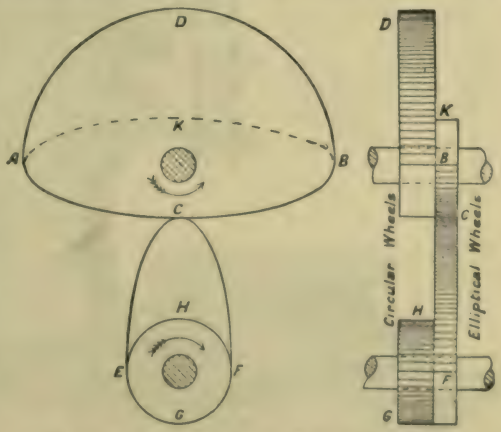
Horizontal Shaping Machine.—The following illustration shows a shaping machine with this gear for driving the cutting



SHAPING MACHINE WITH QUICK RETURN BY MESSRS. SELIG SONNENTHAL & CO., LONDON.

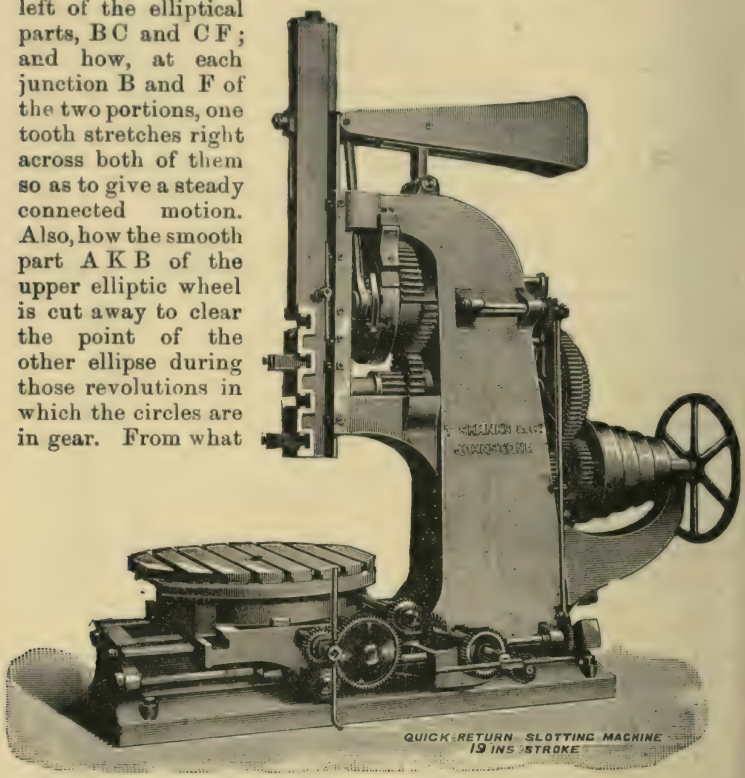
tool. The work is fixed in a vice mounted on a table at the front of the machine, and this table can be moved up and down by a rack or screw, and traversed along the bed in either direction, backwards and forwards by a screw. The tool-holder moves inwards and outwards over this vice and is provided with a screw for moving the tool vertically and a worm for adjusting its inclination. The gear for driving the tool is actuated by a sliding pinion on a shaft which lies along the whole length of the back of the machine and is driven by a stepped cone pulley. The feed of the tool-holder towards the left or right, is effected by a nut and screw underneath. This screw is worked by a ratchet seen to the right, in a similar way to that described for the previous machine.

Quick Return with Elliptic Wheels.—Vertical Slotting Machine.—We illustrate another slotting machine which has a different method of obtaining a quick return. In this machine, the moving bar is actuated by a simple crank, but this crank is driven at different speeds for the cutting and return strokes by the following device. One portion of the circumference of the wheel on the crank-piece is part of an ellipse A C B, and the other, B D A, is circular. The wheel for driving this has an elliptical part E C F, and a complete circle E H F G. Suppose B and F to be in contact. Then, as the lower wheel moves round, the two ellipses keep in contact until A and E come together. The two shafts thus make about half a revolution in the same time. The circular wheels now come into gear, and the lower wheel must make one and a half turns while the upper rotates through its second half. This will bring B and F again into contact and the whole process repeats itself. The quicker motion with the elliptical wheels is used for the upward stroke and the slower motion from the



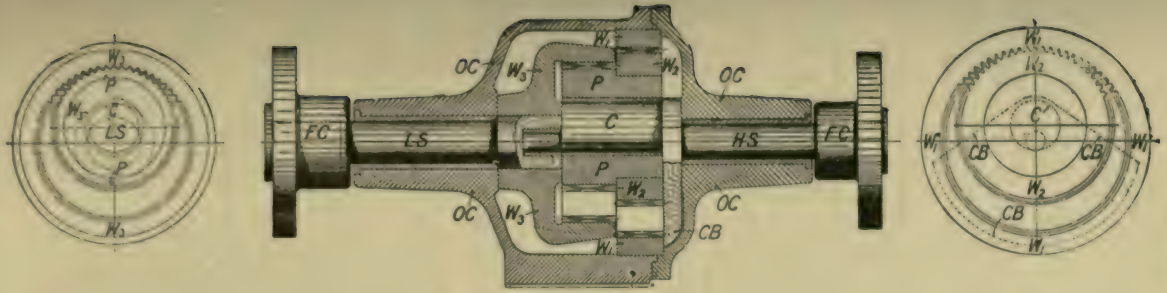
ELLIPTIC WHEELS FOR QUICK RETURN.

circular wheels for the downward or cutting stroke. By looking at the side elevation, and at the view of the complete machine, it will be seen that the circular parts of the wheels D B and H G are placed to the left of the elliptical parts, B C and C F; and how, at each junction B and F of the two portions, one tooth stretches right across both of them so as to give a steady connected motion. Also, how the smooth part A K B of the upper elliptic wheel is cut away to clear the point of the other ellipse during those revolutions in which the circles are in gear. From what



VERTICAL SLOTTING MACHINE BY MESSRS. T. SHANKS & CO., JOHNSTONE.

has already been said about the other machines the student will easily understand the working of this one. He should, however, notice that the moving bar has a counterbalance to relieve the gearing from the weight of the sliding bar, also to reduce the driving force required for the upstroke and to prevent any sudden drop of the tool on the work which may be placed upon the table.



ROSS' SPEED REDUCING GEAR.

Speed Reducing Gear.—The author is indebted to R. G. Ross & Son, Greenhead Works, Glasgow, for the drawing from which the above illustration of their speed reducing gear was made. The high speed shaft, H S, has a crank, C, with a counter-balance, C B, attached. Inside the casings, O C, is a fixed wheel, W_1, gearing with another wheel, W_2, which, together with the pinion, P, works free on the crank, C. The internal wheel, W_3, is keyed on the low speed shaft, L S. When the high speed shaft, H S, is revolving, the fixed wheel, W_1, being a fixture, retards the motion of the wheel, W_2. Hence, a reduction in speed ensues. A flange coupling, F C, is keyed on both the high and the low speed shafts to connect them directly to the motor and the driven machine respectively. This arrangement of reducing gear can also be used for a counting machine, by putting a disc or pointer on the end of the low speed shaft.

NOTE.—Students interested in Mechanisms may consult with advantage *The Engineer's Sketch-Book of Mechanical Movements*, by Thomas Walter Barber.

LECTURE VII.—QUESTIONS.

1. Illustrate and explain the form of reversing gear you would employ for a steam winch in which the engine shaft always runs one way.

2. Show a method of applying a self-acting motion for reversing the motion of the table of a planing machine when a screw is employed for driving it.

3. Describe, with a sketch and index of parts, some form of quick return gearing suitable for a planing machine, the movement being obtained by a combination of pulleys and spur wheels.

4. Sketch and describe an arrangement for driving the table of a planing machine by means of a screw, so that the table may travel 50 per cent. faster in the return than in the forward or cutting stroke. Why is a square threaded screw employed in such a machine?

5. Sketch and describe an arrangement of mechanism for reversing the table in a screw driven planing machine. In what way can a quick return of the table be obtained in such a machine?

6. Sketch and describe a good form of slow forward and quick return for a shaping machine.

7. Describe, with sketches, a planing, a slotting, or a shaping machine, showing clearly how the cutting and feeding motions are effected.

8. Sketch and describe the mechanism for feed motions:—(1) In a machine where there is a reciprocating movement, as in a planing machine; (2) where there is a continuous movement, as in a machine for boring cylinders.

9. Sketch and describe a vertical slotting machine with quick return elliptical gear. Give a separate diagram and explanation of the elliptical gear.

10. Sketch and describe the arrangement of mechanism by which the tool of a planing machine is traversed across the slide of the machine at each stroke of the table. (B. of E., 1900.)

11. Describe, with sketches, the mechanism for giving an automatic feed to the cutting tool of a lathe or shaping machine, and how it is put in or out of action, and the amount of feed varied. (B. of E., 1902.)

12. Describe, with sketches, two quick return motions for driving the table of a planing-machine—one in which the quick return is obtained by belting, and the other in which it is obtained by ordinary gearing.
(C. & G., 1903, O., Sec. A.)

13. The mechanism of the ordinary direct-acting engine is used as a quick return motion for a small shaping machine by simply placing the crank shaft centre below the line of stroke, instead of in the line of stroke produced. The crank radius is $1\frac{3}{4}$ ins., the connecting rod is 7 ins. long, and the centre of the crank shaft is $3\frac{1}{2}$ ins. below the line of stroke. Find, graphically, the stroke and the time ratio of the return and cutting strokes—the crank shaft being supposed to rotate uniformly.

Ans. Length of stroke = $4\frac{1}{8}$ inches; Time ratio = 1 to 1·2.
(C. & G., 1903, O., Sec. A.)

14. Describe, with the aid of sketches, the mechanism known as the crank and slotted lever used to give a quick return motion to the tool of a shaping machine; and if the crank radius is half the distance between the fixed centres, find the ratio of the times occupied in performing the cutting and return strokes. The crank shaft may be supposed to rotate uniformly, and the obliquity of the connecting-rod to the line of stroke of the tool-box neglected. (C. & G., 1904, O., Sec. A.)

LECTURE VIII.

CONTENTS.—Frictional Resistances and Efficiencies of Machines in General—Example I.—Application to the Steam Engine—Efficiency of a Reversible Machine—Example II.—Movable Hydraulic Cranes—Movable Electric Crane—Details of Hoisting Brake and Levers for Working the 3-Ton Electric Crane—Abstract of Report, Tables and Curves on Comparative Trials for Efficiency of 3-Ton Hydraulic and Electric Cranes—Relative Cost of Hydraulic and Electric Power—Crane Tests—Explanation of Efficiency Curves—Electric Cranes for Manchester Ship Canal—Questions.

Frictional Resistances and Efficiencies of Machines in General.—In Lecture VII., Vol. I., we showed how to calculate the work lost in friction in the cases of plane and cylindrical surfaces. As these are the principal kinds of rubbing surfaces in machinery, we might, if we knew the exact pressures between the various surfaces, calculate the total frictional losses and thus find the efficiency of the machine. But there are other losses of work during its transmission, such as the bending of ropes, belts, chains, &c., which it is almost impossible to calculate with exactness. Even if we could calculate all these various losses, the task would be a most tedious and unprofitable one. Hence, in finding the efficiency of any machine, we have recourse to direct experiment on the machine as a whole, the results of which furnish us with data from which we can determine the exact efficiency. In general, however, a machine will not have the same efficiency when working under different loads, owing to the fact that the frictional and other resistances are not proportional to the efforts and loads. To make this clearer, suppose we take the case of a steam engine. Here the friction between the piston and its cylinder, the piston-rod, valve rods, &c., and their stuffing boxes, as well as the friction at the journals due to the weights of the flywheel and shaft, are, severally, constant in amount, whether the engine be developing full power or running light. Hence, in all machines there is a certain proportion of the frictional and other resistances constant, whatever be the magnitude of the effort and the load. This (as just explained in the case of the steam engine) is due to forces between the parts of the machine itself, such as the weights and inertia of the moving parts, or the resistances offered to the bending and stretching of parts, and which have little or no connection with the effort exerted or the load overcome. From these facts we see that the lost work will be proportionally less, and the useful work proportionally more the greater the total work expended. In other words, *the efficiency of a machine will, in general, be higher the greater the load.* This statement, however, is not true for hydraulic and other machines where fluid resistances occur, or where the speed of the machinery is *very* great. As will

be seen later on, fluid resistances increase very rapidly with the speed. In high-speed machinery the effects of the inertia of the moving parts introduce other serious losses which must not be ignored when calculating their efficiency.

The "Principle of Work," when applied to a machine, has already been written in the form :—

Total work expended = Useful work done + Lost work.

Or, $W_T = W_U + W_L$ [see eqn. (I.), Lect. IV., Vol. I.]

The last term, W_L, is made up of two distinct parts—one part depending on W_T and W_U, and a second part which is constant, and, therefore, independent of W_T and W_U. Hence, we may put :—

$$W_L = \mu_1 W_T + \mu_2 W_U + C.$$

Where μ_1 μ_2 are numerical coefficients, and $\mu_1 W_T$, $\mu_2 W_U$ are the frictional resistance due to the effort and load applied; while C represents an amount of lost work which is constant for the same machine.

$$\therefore \qquad W_T = W_U + \mu_1 W_T + \mu_2 W_U + C.$$

$$\text{Or,} \qquad (1 - \mu_1) W_T = (1 + \mu_2) W_U + C. \quad . \quad . \quad . \quad . \quad . \quad (I)$$

This is a *general* equation for the "Principle of Work" as applied to machines. In most machines the coefficient, μ_1, which depends on the effort, Q, must necessarily be very small, and may sometimes be neglected.

Dividing both sides of the equation (I) by $(1 - \mu_1)$ we get :—

$$W_T = \frac{1 + \mu_2}{1 - \mu_1} W_U + \frac{C}{1 - \mu_1},$$

$$\text{,,} \quad = \left(1 + \frac{\mu_1 + \mu_2}{1 - \mu_1}\right) W_U + F.$$

$$\text{Or,} \qquad W_T = (1 + f) W_U + F. \quad . \quad . \quad . \quad . \quad (II)$$

Where $f = \left(\frac{\mu_1 + \mu_2}{1 - \mu_1}\right)$ and $F = \left(\frac{C}{1 - \mu_1}\right)$ are new constants derived from the old ones, as shown.

For some purposes, it is more convenient to write equation (II) in the following forms :—

$$W_T = k W_U + F. \quad . \quad . \quad . \quad . \quad . \quad . \quad (III)$$

$$\text{Or,} \qquad Q x = k W y + F. \quad . \quad . \quad . \quad . \quad . \quad . \quad (IV)$$

Where (as in Lecture IV., Vol. I.) x and y are respectively the displacements of the effort Q and the load W, in a given time.

Suppose the machine to run light. Then $W = 0$, and Q_0 is the effort required to drive the machine.

From equation (IV), we get :—

$$Q_0 x = F.$$

That is, F *represents the work done in driving the machine unloaded.*

Dividing both sides of this equation by x, we get :—

$$Q_0 = \frac{F}{x}.$$

This is the effort required to drive the machine light.

Substituting the above value for F, in equation (IV), we get :—

$$Q x = k\ W y + Q_0 x.$$

Or,
$$Q = k\ W \frac{y}{x} + Q_0$$

$\therefore$
$$Q = k\ W \frac{v}{V} + Q_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (V)$$

This is a *general* equation connecting the effort Q and the load W for any machine. Since the velocity ratio, $\frac{V}{v}$, is constant for the same machine, we might write the last equation in this convenient form :—

$$Q = K\ W + Q_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (VI)$$

Where, $K = k \Big/ \frac{V}{v}$, and can be found by experiment for any machine.

EXAMPLE I.—In an ordinary block and tackle having three sheaves in the upper and two in the lower block, it is found by experiment that a force of 11 lbs. is required to lift a weight of 40 lbs., and a force of 24½ lbs. to lift a weight of 100 lbs. Find a general expression for the relation between Q and W in this arrangement, and the weight which could be raised by a force of 56 lbs. Find, also, the efficiency of the machine in all three cases, and the actual mechanical advantage.

ANSWER.—The general relation between Q and W must be of the form :—

$$Q = K\ W + Q_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

From the results of the first experiment, we get :—

$$11 = 40\,K + Q_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

And from the results of the second experiment, we get :—

$$24\tfrac{1}{2} = 100\,K + Q_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

(3)–(2) $$13\tfrac{1}{2} = 60\,K,$$

$$\therefore \quad K = \frac{27}{120} = \frac{9}{40}.$$

Substituting this value of K in equation (2), we get :—

$$11 = \frac{9}{40} \times 40 + Q_0,$$

$$\therefore \quad Q_0 = 2 \text{ lbs.}$$

Or, the effort required to drive the machine light is 2 lbs.

Substituting the values of K and Q_0 in equation (1), we get :—

$$Q = \frac{9}{40}\,W + 2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which is the general formula required.

To find the weight which could be lifted by an effort of 56 lbs., we substitute this value in equation (4), and get :—

$$56 = \frac{9}{40}\,W + 2,$$

$$\therefore \quad W = \frac{54 \times 40}{9} = 240 \text{ lbs.}$$

The efficiency of the machine in any case is found from the usual formula, viz. :—

$$\textit{Efficiency} = \frac{\text{Useful work done}}{\text{Total work expended}} = \frac{W\,y}{Q\,x} = \frac{W}{Q}\,\frac{v}{V}.$$

Since there are five ropes supporting the weight, W, in this system of block and tackle, it is clear that :—

$$\frac{v}{V} = \frac{1}{5}.$$

$$\therefore \quad \text{Efficiency} = \frac{1}{5}\,\frac{W}{Q}.$$

When raising the weight of 40 lbs., we get :—

$$\text{Efficiency} = \frac{1}{5} \times \frac{40}{11} = \cdot 727, \text{ or } 72 \cdot 7 \text{ per cent.} \quad (1)$$

When raising the weight of 100 lbs., we get :—

$$\text{Efficiency} = \frac{1}{5} \times \frac{100}{24 \cdot 5} = \cdot 816, \text{ or } 81 \cdot 6 \text{ per cent.} \quad (2)$$

When raising the weight of 240 lbs., we get :—

$$\text{Efficiency} = \frac{1}{5} \times \frac{240}{56} = \cdot 857, \text{ or } 85 \cdot 7 \text{ per cent.} \quad (3)$$

The student should notice how the efficiency increases as the load, W, is increased.

$$\left.\begin{matrix}\textit{The actual advantage} \\ \textit{when raising } 40 \textit{ lbs.}\end{matrix}\right\} = \frac{W}{Q} = \frac{40}{11} = 3 \cdot 63.$$

By multiplying the numbers expressing the efficiencies by 5 (the number of ropes attached to the lower block), we get the actual mechanical advantage in each case.

Application to the Steam Engine.—Since the above reasoning is applicable to *all* machines, when the frictional resistances are not greatly influenced by speed, &c., we may here show its application to the steam engine.

Let p_m = Mean pressure on piston in lbs. per square inch.
" p_u = Mean pressure per square inch (being part of p_m) required to overcome the useful load, W.
" p_f = Mean pressure per square inch required to drive the engine when unloaded, or simply the "friction pressure" per square inch.

Since, $\frac{V}{v}$ or the velocity ratio does not enter into this case, and all the pressures are considered as acting on the same piston, we get from equation (V) :—

$$p_m = k\, p_u + p_f.$$

By experiment it is found that the constant p_f, called the "Friction Pressure," has a value between 1 and $1\frac{1}{2}$ lbs. per square inch, in ordinary land engines, and about 2 lbs. per square inch in marine engines. The value of k is about 1·15, but varies with both size and speed of engine. In large or high-speed engines, k is often less than 1·15, though it can never be less than 1.

Efficiency of a Reversible Machine.—The student will have noticed, from the Table of Efficiencies in Lecture II., Vol. I., the great difference in the efficiencies of such machines as the ordinary block and tackle and the Weston's pulley block. In the examples worked out at the end of Lecture IV., Vol. I., it was proved that the efficiency of the former machine may be as high as 75 per cent., while the efficiency of the latter never reaches 50 per cent., and seldom exceeds 40 per cent. He also knows that when the efficiency of any machine is less than 50 per cent. it will not reverse, even if the hauling force or effort be withdrawn. Hence the difference in the working of the two machines just mentioned. The "block and tackle" is, under ordinary conditions, a *reversible* machine (*i.e.*, the load at the lower block is capable of overcoming a smaller load at the hauling part of the rope), while the Weston's pulley block will not reverse even when the hauling force is entirely withdrawn. To lower the load with a Weston's block a force has to be applied to the opposite part of the hauling chain from that at which the effort had to be applied when raising the load.

The screw, wedge, and worm-wheel arrangements are, generally speaking, examples of non-reversible machines. In fact, their usefulness depends to a large extent on this condition.

We can now show that the efficiency of a machine, when working reversed, is not the same as when working in the usual way. Further, if the efficiency of any machine be less than ·5 or 50 per cent., it is not reversible.

We have seen that in any direct working machine the "Principle of Work" takes the form :—

$$(1 - \mu_1)\, W_T = (1 + \mu_2)\, W_U + C. \text{ [equation (I)]}$$

Or,
$$(1 - \mu_1)\, Q\, x = (1 + \mu_2)\, W\, y + C \quad . \quad . \quad . \quad . \quad (1)$$

where the coefficients, μ_1 and μ_2, have the meanings already assigned to them, and C represents a quantity of work absorbed in the machine, but which is independent of both Q and W.

Now, suppose we gradually diminish the effort, Q, until the machine reverses. When this takes place, let the new value of Q be denoted by w, so that the new load is w, and the original load, W, becomes the new effort. The above relation being still approximately true, we only require to substitute the new values for the new effort and load. At the same time it must be observed that the coefficients, μ_1 and μ_2, are taken along with their proper terms, $w\,x$ and $W\,y$; *i.e.*, $\mu_1\, w\, x$ is the lost work due to new load, w, while $\mu_2\, W\, y$ is lost work due to new effort or original load, W.

Then :—

$$(1 - \mu_2)\, W\, y = (1 + \mu_1)\, w\, x + C. \quad . \quad . \quad . \quad . \quad (2)$$

Now subtracting equation (2) from (1), in order to eliminate C, we get :—

$$(1 - \mu_1)\, Q\, x - (1 - \mu_2)\, W\, y = (1 + \mu_2)\, W\, y - (1 + \mu_1)\, w\, x$$

$$\therefore \quad (1 + \mu_1)\, w\, x = 2\, W\, y - (1 - \mu_1)\, Q\, x.$$

Dividing both sides by $(1 + \mu_1)$ and W y, we get :—

$$\frac{w\, x}{W\, y} = \frac{2}{1 + \mu_1} - \left(\frac{1 - \mu_1}{1 + \mu_1}\right) \frac{Q\, x}{W\, y}.$$

$$\textit{Efficiency when reversed} = \frac{\text{Useful work done in raising } w}{\text{Total work expended by W}}.$$

$$\text{,,} \quad = \frac{w\, x}{W\, y},$$

$$\text{,,} \quad = \frac{2}{1 + \mu_1} - \left(\frac{1 - \mu_1}{1 + \mu_1}\right) \frac{Q\, x}{W\, y}.$$

But $\dfrac{W\, y}{Q\, x}$ = the *original efficiency* of the machine, or efficiency when working in the usual manner.

$$\therefore \left.\begin{matrix}\text{Efficiency of Machine} \\ \text{when Reversed}\end{matrix}\right\} = \frac{2}{1 + \mu_1} - \left(\frac{1 - \mu_1}{1 + \mu_1}\right) \times \frac{1}{\eta} \quad . \quad \text{(VII)}$$

where η denotes the original efficiency of the machine.

It is clear that the machine will not reverse unless the above efficiency be greater than 0—

i.e., unless . . $\dfrac{2}{1 + \mu_1} - \left(\dfrac{1 - \mu_1}{1 + \mu_1}\right) \times \dfrac{1}{\eta} > 0.$

i.e., unless . . . . . $\dfrac{2}{1 + \mu_1} > \left(\dfrac{1 - \mu_1}{1 + \mu_1}\right) \times \dfrac{1}{\eta}.$

i.e., unless . . . . . $\eta > \frac{1}{2}(1 - \mu_1).$

Consequently, the machine will not reverse until the original efficiency, η, be greater than $\frac{1}{2}(1 - \mu_1)$, and, if it be less than this, it will not reverse even if the original hauling force, Q, be entirely withdrawn. For, if $\eta = \frac{1}{2}(1 - \mu_1)$, the efficiency of the machine when reversed would vanish (as may be seen by substituting this value in equation (VII)). If $\eta < \frac{1}{2}(1 - \mu_1)$, the efficiency would be negative, which is absurd.

We have already stated that in most machines the friction, μ_1, is *very small*, and may be neglected. Hence we get the important statement that

A machine will not reverse even when the hauling force is entirely withdrawn, if the efficiency is less than $\frac{1}{2}$ or 50 per cent.

EXAMPLE II.—Apply the "Principle of Work" to calculate the relation between P and W in the screw lifting jack, fitted with screw and worm-wheel gear. The handle which works the jack has a radius of 14 inches, pitch of screw 1 inch, number of teeth on worm-wheel 20, and the worm is double threaded; find the force which must be applied at the end of the handle in order to raise a weight of 4 tons, friction being neglected. If the actual force required to raise this weight be 40 lbs., what is the efficiency of the apparatus?

ANSWER. — Let L = length of handle, p = pitch of screw, N = number of teeth on worm-wheel, n = number of threads on worm.

(1) Suppose the handle to make one complete turn. Then, since there are n threads on the worm, and N teeth on the worm-wheel, it is clear, that for one turn of the handle, the worm-wheel, which forms the nut of the screw, will have made $\frac{n}{N}$ part of a complete turn. Hence the weight, W, will have been raised through a height $\frac{n}{N} \times p$.

$\therefore$ By the Principle of Work, we get :—

$$P \times \text{its displacement} = W \times \text{its displacement.}$$

$$\therefore \quad P \times 2\pi L = W \times \frac{n}{N} \times p,$$

$$\therefore \quad \frac{P}{W} = \frac{np}{2\pi L N}.$$

(2) In the example $p = 1''$, $L = 14''$, $n = 2$, $N = 20$, $W = 4 \times 2{,}240$ lbs.

$$\therefore \quad P = 4 \times 2{,}240 \times \frac{2 \times 1}{2 \times \frac{22}{7} \times 14 \times 20} \text{ lbs.},$$

$$\text{,,} = 10{\cdot}18 \text{ lbs.}$$

(3) $$\textit{Efficiency} = \frac{P}{Q} = \frac{10{\cdot}18}{40} = {\cdot}2545, \text{ or } 25{\cdot}45 \text{ per cent.}$$

Movable Hydraulic Cranes.—The following extracts from the "General Specification" by Mr. George H. Baxter, Chief Mechanical Engineer to the Clyde Navigation Trustees, are herewith reproduced along with the accompanying reduced drawings, in order to enable the student to form an idea of the main requirements fulfilled by these double-power 5 and 3-ton hydraulic cranes at the Prince's Dock, Glasgow. The cranes were made by A. Chaplin & Co., Mechanical Engineers, Govan, and have been constantly in use since 1898. Interesting comparative tests were made in March, 1903, between one of these cranes working with its 3-ton lifting ram, and a 3-ton electrical crane, which are described immediately after their specifications. These tests afford useful information regarding the two systems of power supply :—

General Description of Cranes.—"Each crane to be mounted on a steel travelling carriage, with arch for passage of locomotives, having a rail gauge of 14 feet between centres of double rails, which are to be provided and laid on quay and cope of quay wall by the Trustees. Jib to be capable of revolving through at least one and a-quarter turns. The cranes to be double-powered, to lift maximum loads of 5 tons and 3 tons. All the lifts to be by single chain, and the working pressure to be 750 lbs. per square inch. The cranes to have a factor of safety throughout of not less than 8 to 1, excepting the hoisting chains."

Carriages.—"The carriages to be supported by four wheels of cast iron, bushed with gun-metal ⅝-inch thick, with wrought-steel weldless tyres shrunk on each side of central flange after being turned and bored. The axles to be turned steel forgings, secured by strong split cotters. Brackets carrying wheels to be so arranged that the wheels can be easily removed. Shackles and steadying screws to be provided at each corner. The outer side of carriage to be kept at least one foot back from edge of cope, and a hand-rail to be run (the whole length of carriage) outside for convenience in shifting mooring ropes."

Range of Lift.—"Each of the cranes to have a total range of lift of 75 feet, being 35 feet above the cope to 40 feet below, and the chain to project 32 feet beyond face of quay wall when the jib is square to it."

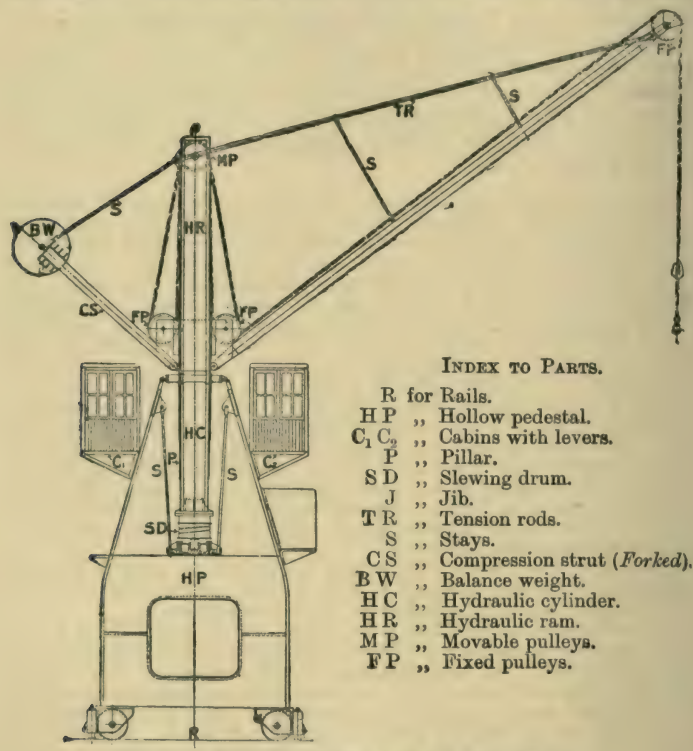
Jib and Counter-balance Weight.—"The jib of each crane to have a rake of not less than 41 feet, and the jib-head pulley to be 60 feet from level of cope of quay wall. Jib stays to be secured independently of pins on which jib-head pulley revolves, so that the latter may be taken down without disturbing the stays. A steel bracket on hinged joint, or other means, to be provided for locking jib when cranes are not in use. Wrought-iron ladder to be fitted to each jib to give access for lubricating jib-head pulley. Counter-balance weight to be high enough to clear roof of shed."

Houses for Driver.—"A house to be provided for the driver on each side of upper part of carriage. Handles for working to be conveniently placed, and marked for hoisting, lowering, and slewing."

Platforms, Hand-rails, Ladders, Guards, &c.—"Platforms, hand-rails, gangways, ladders, footsteps, &c., to be fitted. Suitable lock-up doors and other arrangements to be made to give access to all chains, pulleys, bearings, and other parts, for lubrication and repair. Guards for chains and rollers or rubbing pieces to be fitted wherever required. Elm sheathing, 1½ inches thick, to be laid on inside of jib, and secured thereto by bolts and nut

Chains.—"The hoisting chains to be 1 inch diameter and tested in accordance with the standard requirements of the Clyde Navigation Trustees. Each hoisting chain to have a swivel hook of special form and strength, shown by the drawing. A balance ball to weigh at least 2½ cwts., bolted on in halves a few feet above the hook. Also, a wrought-iron or steel clip of ample strength, bolted to standing part of chain, for adjusting height of hook."

Pulleys.—"The standing pulleys to be of the diameter shown in the



INDEX TO PARTS.

R	for	Rails.
H P	,,	Hollow pedestal.
C_1 C_2	,,	Cabins with levers.
P	,,	Pillar.
S D	,,	Slewing drum.
J	,,	Jib.
T R	,,	Tension rods.
S	,,	Stays.
C S	,,	Compression strut (*Forked*).
B W	,,	Balance weight.
H C	,,	Hydraulic cylinder.
H R	,,	Hydraulic ram.
M P	,,	Movable pulleys.
F P	,,	Fixed pulleys.

GENERAL VIEW OF MOVABLE HYDRAULIC CRANE.
(Made by A. Chaplin & Co., Govan, for the Clyde Trust.)

drawings, and to be arranged on either side of ram as in hydraulic cranes at Queen's Dock. Suitable guards for chain to be provided. Pins for pulleys and carriage wheels to be bored longitudinally, with oil cup in one end and oil hole to each pulley and wheel. All pulleys to be bushed with gun-metal, and be fitted with two steady pins tapped through boss."

Valves.—"The working valves for hoisting, lowering, and slewing to be of the conical description, with levers and weights."

Pipes and Cocks.—"All pipes exposed to the full working pressure to be of strong lap-welded hydraulic tubing of the required thickness, with flanges of cast-iron, turned and faced, with recessed joints to standard sizes. Drain cocks of gun-metal to be fitted to all hoisting and slewing cylinders and all pipes at lowest parts, so as to thoroughly drain them. Air cocks to be fitted to all cylinders and pipes as high as practicable."

Travelling Gear.—"Efficient hand travelling gear to be provided on each crane, so that it may be quickly and easily transported along the quay by not more than six men."

Gas Fittings.—"To prevent damage by frost, each crane to be provided with two gas stoves and burners, including wrought-iron pipes, cocks, and other fittings, for hoisting and slewing cylinders, pipes, &c. A suitable gas stove, with all necessary connections, to be supplied in each crane house. Wrought-iron gas piping, with cock and connection for india-rubber tubing, to be fitted on each carriage, for supplying stoves, &c."

Movable Electric Crane.—The following extracts from the "General Specification," by George H. Baxter, Chief Mechanical Engineer to the Clyde Navigation Trustees, are herewith reproduced, along with the accompanying reduced drawings, in order to enable the student to form an idea of the main requirements fulfilled by the 3-ton movable electric crane, as built and erected at Prince's Dock, Glasgow, by Stothert & Pitt, Bath.* After the specification there follows the data of the comparative tests between this crane and the previously described hydraulic one :—

General Description.—"The crane is to be movable, mounted on a high portal carriage to run on a line of double rails, 14 feet apart between centres, laid along the front of the South Quay, Prince's Dock, Glasgow Harbour. The crane is intended to be used for carrying out an extensive series of tests for the purpose of comparing the relative efficiencies of electric and hydraulic cranes of similar capacity under varying conditions and over a prolonged period. Provision is, therefore, to be made for applying and fitting up suitable recording instruments for the measurement of electric energy delivered to the hoisting and slewing motors, and of the mechanical energy developed by the crane."

General Dimensions.—"The following are the general dimensions of the crane :—

Maximum working load,	3 tons.
Total range of lift,	80 feet.
Radius of lifting rope,	41 ,,
Projection of rope beyond face of quay,	32 ,,
Centre of jib-head pulley above cope,	60 ,,
Clear lift above cope of quay,	50 ,,
Extreme radius of any revolving part at back of crane,	9 ,,
Clear height from cope to lowest revolving part of crane,	29 ,,

Speeds.—"The lifting speed with a load of 3 tons to be 150 feet per minute, the speed with smaller loads to be correspondingly greater. The slewing speed measured at the hook to be 300 feet per minute with maximum load."

* The electric motor, volt and ampere metres for this crane were made by Siemens Brothers & Co., Ltd., of Woolwich and London.

Factors of Safety and Stability.—"The carriage, crane framing and jib, gearing and wire rope, to have a factor of safety of 10 to 1. Stability to be maintained with a load of 6 tons suspended in the crane hook."

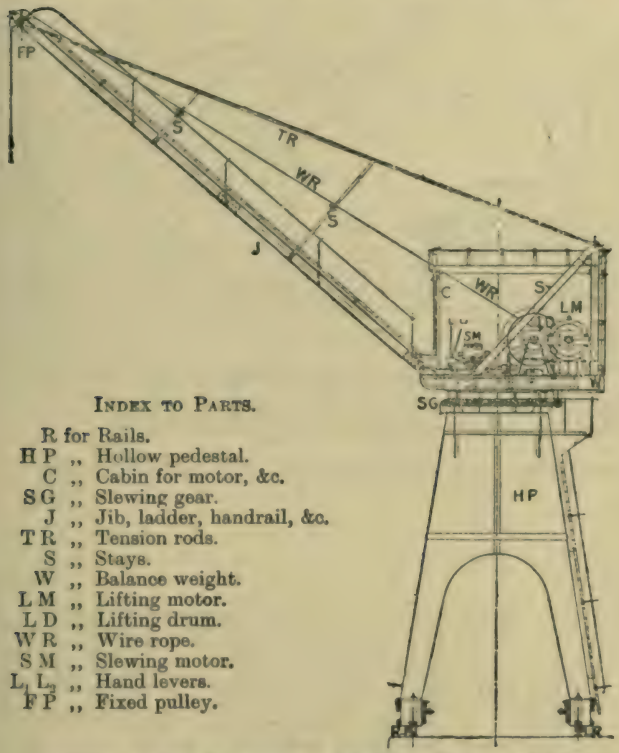
Carriage.—"The carriage is to be constructed with an arch 15 feet high, sufficiently wide for the passage of locomotives. To be made of Siemens steel plates, angles, and bars. The wheel base to be 14 feet. The carriage to be supported by four strong cast-steel wheels turned to 2 feet 6 inches diameter on tread, and having a central flange, 1 inch deep by 1¾ inch wide. The wheel bosses to be bored, faced, and bushed with gun-metal, ⅝ inch thick. The axles to be turned steel forgings, secured by strong split cotters; oil holes to be drilled through centre with short bent lubricating pipes screwed into inner ends. The length and diameter of axle bearings to be specified or shown on plan. Brackets carrying wheels to be of approved design, and so arranged that the wheels can be easily removed if required. Shackles and steadying screws similar to those on hydraulic cranes to be provided at each corner. The side of carriage next dock to be kept at least 1 foot back from the edge of the quay, and a hand-rail to be run the whole length of carriage outside for convenience in shifting mooring ropes. Provision to be made on side of carriage for connecting the motors with the main circuit conductors."

Framing Jib, &c.—"The framing of the revolving part of the crane, jib, and back stays to be made of steel plates and angles, channels, or tee bars carefully fitted and riveted together. The jib stays to be steel forgings or bars drawn or shaped out of the solid without welds, and to be secured by turned pins independently of the pin on which the jib-head pulley revolves, so that the latter may be removed without disturbing the stays. An 18-inch wrought-iron ladder, with hand-rail on each side, to be fitted to jib to give access for lubricating the jib-head pulley. The pulley to be of cast steel, turned in groove to not less than 24 times the diameter of hoisting rope. To be provided with rope, guides, and guards. The pin to run in adjustable gun-metal bushes, and the bearings to have fixed lubricators of approved pattern. Counterbalance weights to be provided in back end of framing to balance the jib and the load."

Centre Post, Roller Path, and Rollers.—"A strong centre post of forged ingot steel, with hole drilled throughout its length for the conductors to pass through, to be turned all over and fitted to strong cast-iron bed plate, or sockets secured to upper part of carriage by turned and fitted bolts. If a roller path is fitted, it is to be a solid ring of Siemens steel accurately turned and faced flat, and secured to carriage by turned and fitted bolts. The rollers are to be of cast steel, truly turned, bored, and keyed to fit strong steel axles, having large bearing surfaces working in gun-metal bushes, adjustable by bolts and nuts, and to be easily removable for repairs."

Plating and Riveting.—"All plate edges are to be planed, all butts to be accurately fitted, and ends of bars neatly trimmed. Rivet holes in framing, jib, &c., are to be drilled in place, and all holes to be made fair without drifting. All rivets to be snap-headed outside."

Hoisting Winch.—"To be single-geared, driven by a separate motor. The bed-plate to be of cast iron, planed to receive the side cheeks and hoisting motor. One of the cheeks to be of box form, planed, bolted together in halves, jointed, and made oil tight; the lower half to form an oil-bath for main spur wheel and the pinion on armature shaft. The barrel to be of cast iron, 24 times the diameter of the hoisting rope, and to be turned with a continuous spiral groove, long enough to take the whole rope required without riding. The maximum load is to be taken on a



INDEX TO PARTS.

R for Rails.
H P ,, Hollow pedestal.
C ,, Cabin for motor, &c.
S G ,, Slewing gear.
J ,, Jib, ladder, handrail, &c.
T R ,, Tension rods.
S ,, Stays.
W ,, Balance weight.
L M ,, Lifting motor.
L D ,, Lifting drum.
W R ,, Wire rope.
S M ,, Slewing motor.
L_1 L_2 ,, Hand levers.
F P ,, Fixed pulley.

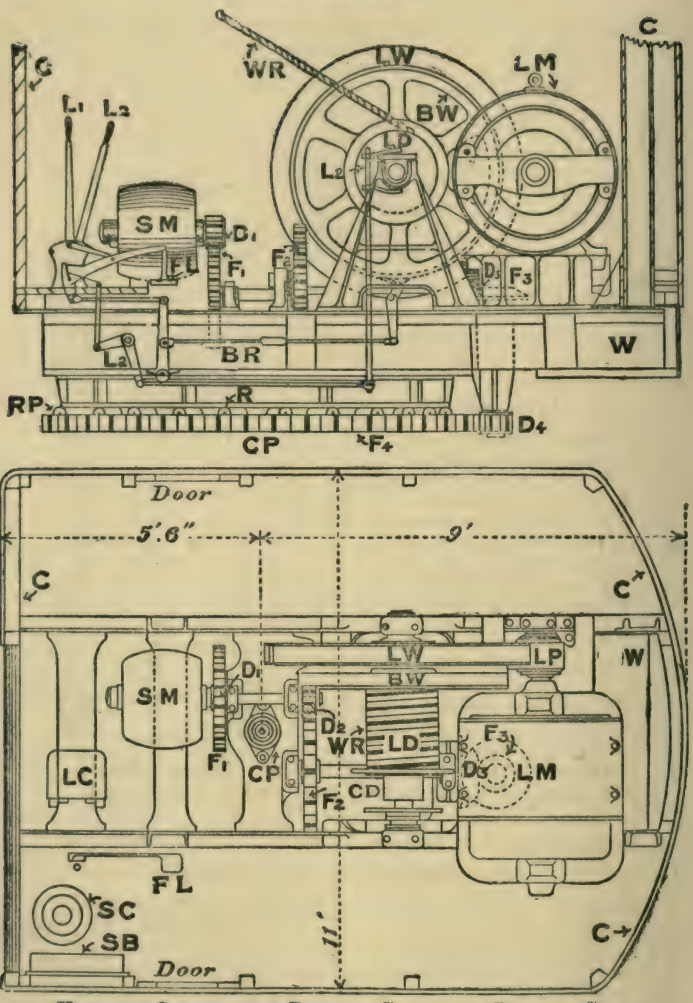
GENERAL VIEW OF A 3-TON MOVABLE ELECTRIC CRANE.
(Made by Stothert & Pitt, Bath, for the Clyde Trust.)

INDEX TO PARTS OF LIFTING, LOWERING, AND SLEWING GEAR.

(*See Section and Plan on opposite page.*)

C for Cabin.
S B ,, Switch board.
L_2 ,, Lever for clutch, C D.
C D ,, Coil-clutch and disc.
L C ,, Lifting controller.
L M ,, ,, motor.
L P ,, ,, ,, pinion.
L W ,, ,, ,, wheel.
L D ,, ,, ,, drum.
W R ,, Wire rope.
F L ,, Foot-brake lever.
B R ,, Brake rod.

B W for Brake wheel.
L_1 ,, Lever for slewing brake.
S C ,, Slewing controller.
S M ,, Slewing motor.
D_1 D_2 D_3 D_4 ,, Driving pinions.
F_1 F_2 F_3 F_4 ,, Follower wheels.
R P ,, Roller path.
R ,, Rollers.
C P ,, Central pivot.
W ,, Balance weight.



VERTICAL SECTION AND PLAN OF CABIN AND DRIVING GEAR
For the 3-Ton Electric Crane belonging to the Clyde Trust.

single rope. The main spur wheel is to be of cast steel, the pinion of forged steel, both to have wide teeth of suitable pitch, accurate form, and machine cut. The ratio of gearing and diameter of barrel and jib-head pulley to be shown on contractor's plan."

Hoisting Brake.—"An automatic brake to be provided. This brake to be worked also by a hand or foot lever, and to effectually hold the maximum load at any point. It must be so designed, that when the current is switched on to the hoisting motor, the brake is automatically released the moment hoisting begins. And, in switching off, or in the event of failure of current, the brake would be put in gear automatically, and without shock." (*See the pages describing construction and action of this brake at the end of this Specification.*)

Hoisting Rope.—"To be made of the highest quality of plough steel wire, specially flexible and free from twisting. The rope to be provided with an overhauling weight in halves, and an eye with solid wrought-iron thimble at end, bored to fit the shackle-pin; also a 3-feet length of $\frac{3}{4}$-inch short-link crane chain, and swivel hook of approved design. The ultimate tensile strength of the wire (which is to be tested and approved of before the rope is made) is not to exceed 110 tons per square inch."

Slewing Gear.—"To be worked by separate motor, having a wrought-steel worm keyed on end of armature shaft, gearing into a phosphor-bronze worm wheel on vertical shaft, with pinion at lower end in gear with the slewing wheel, which is to be secured to top of carriage with turned and fitted bolts. The worm to have large thrust bearings of gun-metal, and to run in an oil-bath with dustproof cover. The vertical shaft to be of large diameter to ensure stiffness, and to have collars forged on at ends. The bearings are to be of gun-metal, adjustable by means of bolts and nuts. All teeth to be of suitable pitch, accurate form, and machine cut. A brake worked by foot lever to be provided to control the speed at any point."

Travelling Gear.—"The crane to be provided with efficient travelling gear fitted to each carriage wheel, worked by hand, and so connected by means of clutches that it may be disengaged to admit of the crane being moved along the quay by a hydraulic capstan. Spur wheels on axles of carriage wheels to be 6 inches clear of the ground."

Electric Motors.—"Both motors are to be of the drum armature enclosed type. They are to be series-wound, and constructed to work on a continuous current circuit at an E.M.F. of 230 volts, but capable of working at 260 volts if required. The hoisting motor to be 50-brake H.P., and slewing motor 10-brake H.P. Both are to be designed to give every facility for examination and repairs. The armatures to be of the strongest form of construction, well insulated throughout, with provision for thorough ventilation. The rise of temperature in the armature must not exceed 60° F. above the surrounding atmosphere. The commutators are to be made of pure annealed copper bars, insulated with mica, and to have large wearing surfaces, with self-adjusting carbon brushes. The armature shafts to be of forged steel, working in long bearings lined with white metal, bored out perfectly smooth and true, and provided with efficient automatic lubricating apparatus of the ring type. Chambers to be formed at the ends of each bearing to hold waste oil, with gauge-cocks, drain-cocks, and pipes led to an oil reservoir."

Controlling Switches.—"These are to be of the most improved type, with graduated resistances suitable for cranes. One controlling switch is to be provided for each motor, and of such dimensions as shall prevent undue heating when the crane is working continuously for six hours. The switch contacts must be large and carefully constructed, so as to prevent sparking. These are to be enclosed in an iron casing, with covers and doors for examination and repairs. The controlling switches to be so constructed that the motors may be started, regulated, stopped, reversed, or ran at

varying speeds, with varying loads in the simplest and most reliable manner. The regulating handles are to be conveniently arranged in such a position that the driver may see the load during the whole time it is suspended in the crane, and the handles should be so connected up, that they move in the direction it is desired to move the load."

Switchboard, Switches, and Cut-outs, &c.—"A double-pole switch of the quick break lever type and two single-pole cut-outs for the hoisting and slewing motors, together with separate switches and cut-outs for the electric lighting and heating circuits, to be all arranged on a suitable switchboard of slate or marble, framed in channel iron $\frac{3}{16}$ of an inch thick lined with teak. The switches and cut-outs to be properly enclosed and protected, so as to prevent risk of fire or shock in any way, and to be conveniently placed within easy reach of the driver. Sweating sockets to be provided, to which the cables and wiring are to be connected. The cut-out terminals shall be so arranged that the fuses of flat tinfoil may be easily and securely fitted by the craneman. The securing nuts are to be of bright mild steel, and the fuse break not less than 5 inches. Each switch and instrument shall be capable of conveying the maximum current continuously, without sensible rise of temperature. The covered flexible connections and cables shall be of such area, that the current density shall not exceed 800, the gun-metal parts 400, and the effective contact surfaces of switches 200 amperes per square inch."

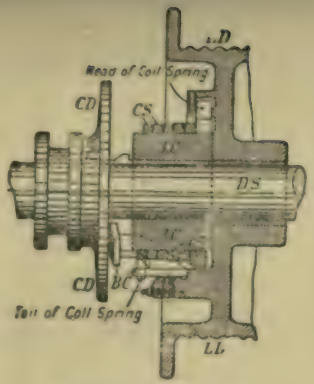
Switchboard Instruments.—"A lamp, an ammeter, a voltmeter, and an automatic circuit breaker (to prevent danger from overloading) to be provided and conveniently arranged on the switchboard, with all necessary connections to the main conductors."

Conductors and Insulation.—"The conductors to be of suitable standard sizes, corresponding to the Cable Makers' Association List. The conductors are to be connected from the terminals of the motors to the controlling switches, from thence to the cut-outs, and therefrom to suitable connections passing through the centre post to sockets at the bottom of crane carriage, and there provided with plug connections to the twin cable. The conductors in the house to be provided with circular sliding contacts and collectors, so as to admit of the crane being slewed in either direction to any extent without difficulty. A portable pair of conductors, with plugs sweated on ends, to be provided to fit sockets on carriage, and also the wall-plugs along the quay. The twin conductor to be of such a length as to extend to a distance of 40 feet on either side of the centre of the crane. The insulation resistance to be not less than 3,000 megohms per mile for the larger, and 4,000 for the smaller conductors."

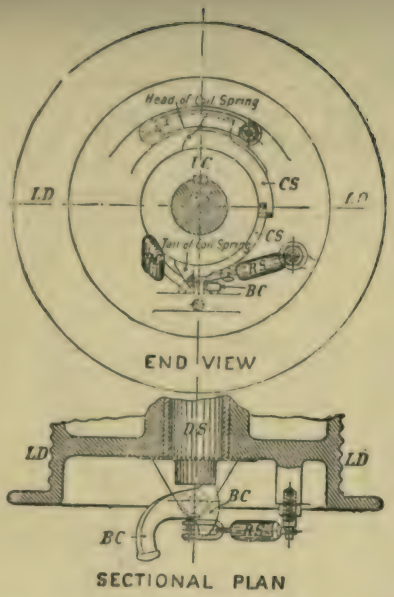
Wall Plugs.—"Wall-plugs to be fitted in the hydraulic supply hydrant-boxes, or elsewhere, along the quay, with suitable terminals for connection to the electric current supply mains. The boxes to be thoroughly water-tight, and every precaution to be taken against damp."

House.—"A substantially-built timber house to be constructed of sufficient size to give easy access to all parts of the machinery, and ample room for working the crane. A suitable electric radiator, or stove, to be provided, and fitted with a graduated resistance switch, to allow of varying degrees of heat being obtained. Two 32 candle-power Ediswan glow lamps, and two plugs for portable lamps, to be fitted up in house, and a cluster of glow lamps half way up the jib."

Details of Hoisting Brake and Levers for Working the 3-Ton Electric Crane.—In order to make the construction and action



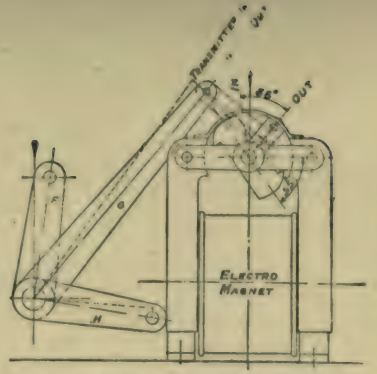
VERTICAL SECTION



SECTIONAL PLAN

DETAILS OF THE COIL CLUTCH FOR 3-TON ELECTRIC CRANE.

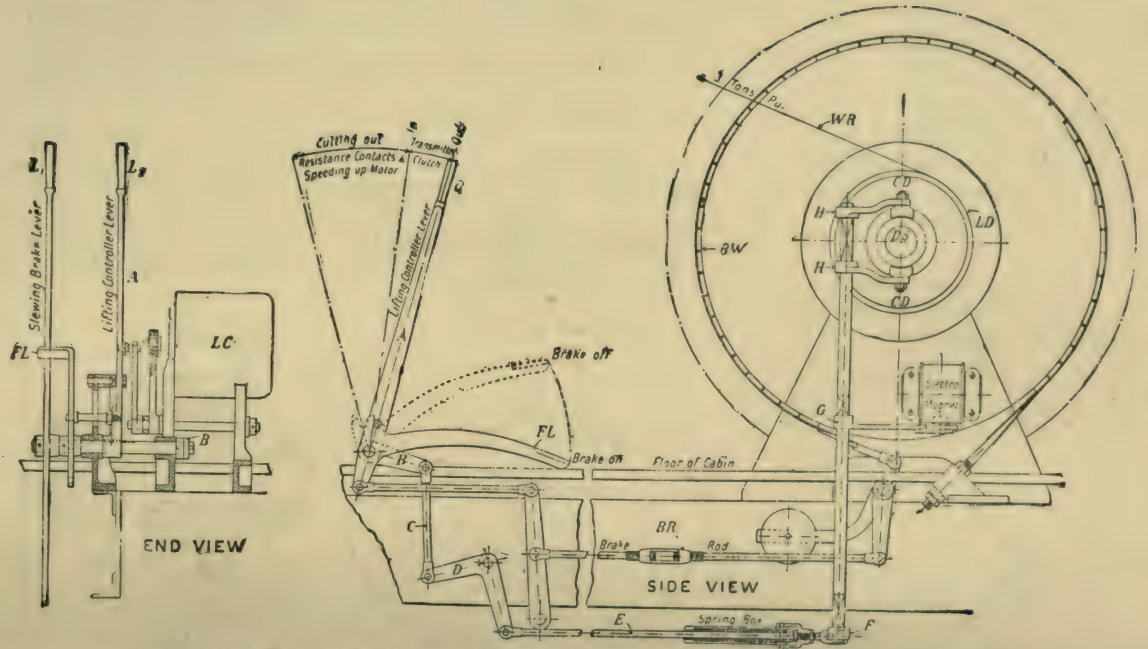
LD for Lifting drum (*see previous and following views*).	CS for Coil spring of soft iron.
DS ,, Drum shaft.	BC ,, Bell crank lever.
IC ,, Iron chilled cylinder.	RS ,, Releasing spring for BC.
	CD ,, Clutch friction disc.



ELECTRO-MAGNET AND LEVERS FOR WORKING COIL CLUTCH.

INDEX TO PARTS OF FIGS
On opposite page and the above one.

L_1 for	Lever to slewing brake.
L_2 ,,	Lever to LC and CD.
LC ,,	Lifting motor controller.
A to H ,,	Levers from L_1 to CD.
DS ,,	Drum shaft.
LD ,,	Lifting drum.
WR ,,	Wire rope.
BW ,,	Brake wheel.
BR ,,	Brake rod.
FL ,,	Foot-brake lever to BR.



LEVERS AND ELECTRO-MAGNET FOR WORKING STOTHERT & PITT'S 3-TON ELECTRIC CRANE.

of these details clear to the student, special views have been herewith reproduced from drawings kindly supplied by the makers.

Coil Clutch.—In the accompanying figures this has been shown by a vertical section, end view, and sectional plan. It will be understood, after comparing the index to parts with these views, that when the clutch friction disc, C D, is pressed forward by the levers, H (shown on the full-page views), it pushes the end of the bell crank lever, B C, so that its inner end pulls upon the tail of the coil spring, C S. This action causes C S to wind close and grip hard upon the iron chilled cylinder, I C. Consequently, we have the lifting drum, L D (which is directly connected to the inner end or head of the coil spring, C S), thereby coupled, through C S, to I C and D S, the drum shaft. The other end of this shaft is keyed to L W, which gears with the lifting motor pinion, L P, and hence with L M, the lifting motor. The load can then be lifted to the desired height.

Braking.—When the clutch friction disc, C D, is disengaged from B C, the latter is brought back to its normal position by the releasing spring, R S. This relieves C S from contact with I C. The lifting drum, L D, with its brake wheel, B W, are thus instantly disconnected from the motor. The load is then, however, held up in position by the brake-strap due to the driver pressing down the treadle or foot lever, F L. The load can now be lowered at any desired speed by simply easing the foot lever, F L, which acts directly on the brake rod, B R, and brake strap of the brake wheel, B W, as shown by the side view on the full-page set of figures. It will thus be seen, that the brake is normally "off," as in a steam crane. The driver lowers a load by pressing his foot more or less on the treadle, thus tightening or slackening the brake-strap in precisely the same way as he would do with a steam crane. The makers admit, that this method of actuating the brake may at first sight appear to be not so automatic or ideal a plan, as that of permitting the weight attached to the brake lever to keep the brake-strap tight on its brake wheel; but, they find it to be much better in actual practice. It will be observed, that if the electric current failed when a load had been raised, and if the driver then stood still and did nothing, the load would undoubtedly drop! However, the current has not been found to fail under these circumstances, and the driver has ample time to press the treadle and check the load.

Lifting.—It will be observed that the first part of the motion of lever L_2 to the left (*i.e.*, from the words "out" to "in"),

puts the clutch, C D, into action, and at the same time it brings the transmitter switch of the lifting controller, L C, into circuit with the lifting motor, L M. The remainder of the movement of lever L_2 in the same direction (*i.e.*, to the left) speeds up the motor, L M, by cutting out resistances in the controller of the motor circuit.

Lowering.—On the other hand, the action of putting on or down the treadle foot-brake lever, F L, for the purpose of stopping the motor before lowering a load, automatically throws back the clutch, C D, and disconnects the motor electrically from its controller. The lifting drum, L D, with its attached brake wheel, B W, are now free to move quite independently of the lifting motor and the drum shaft. The load can then be lowered at the desired speed by the required pressure on the foot lever.

Action of the Electro-Magnet.—In the Clyde Trust 3-ton electric crane, Messrs. Stothert & Pitt have adopted an alternative patent arrangement of electro-magnet for engaging and disengaging the clutch friction disc, C D. When it is used, the levers B, C, D, E, F, and spring box (as shown by the side view) are removed. A separate plan of this electro-magnet, with its levers, G and H, are shown. It will be understood, that the current, in passing from the controller, L C, to the lifting motor, L M, traverses and energises the coils of this electro-magnet.

Abstract of Report, Tables and Curves, on Comparative Trials for Efficiency of the previously-described 3-Ton Hydraulic and Electric Cranes.*—The electric crane was erected on the South Quay, Prince's Dock, and started to work on the 24th March, 1902. It has worked since then with the most satisfactory results, and there has been no hitch or trouble of any kind with the electrical machinery. From that date up to the 28th February, 1903, this crane discharged and loaded minerals and general cargo amounting to 43,954 tons, as compared with 40,833 tons dealt with by the 3-ton hydraulic crane alongside of it, whilst working under precisely similar conditions. Both cranes were tested on the 19th and 20th March, 1903, and the results given in tabulated form, indicate that the efficiency of the electric is greater than that of the hydraulic crane at full load, both in working ore and general cargo. The efficiency of the electric crane is also very much greater at the lighter loads, while the cost of producing and distributing electric power to the cranes is less than that of hydraulic power. The curves of performance of these cranes under tests are shown on diagrams 1 and 2. By comparing the corresponding curves, C, on these two diagrams—which show the cost of power used in discharging 1,000 tons of general cargo at varying loads—the difference in favour of the electric crane may be appreciated. For example, in working with loads of

1 ton	2 tons	2½ tons	3 tons

the cost of power used by the hydraulic crane in discharging 1,000 tons of general cargo is

48/3	25/5	20/6	17/2

and by the electric crane

15/11	16/2	16/1½	16/6½

It will be observed, that the costs for the hydraulic crane are nearly in inverse proportion to the loads raised, for the reason that in this type of crane, the same foot-pounds of water are used to lift the empty hook as to lift the maximum load. On the other hand, with the electric crane the cost is practically the same at all loads. This is due to the valuable feature, that the electrical power used, is much more nearly proportional to the power required to raise the load, than is the case with hydraulic cranes. By raising the electrical pressure at the central power-house from 260 volts (its present value) to, say, 500 or 600 volts

* I am indebted to Mr. George H. Baxter, M.I.E.S., for the liberty to make this abstract from his report to the Clyde Trustees.

(as contemplated for the new Clydebank Dock), and by working upon a greatly increased scale, it is probable that the present average cost of 2·558 pence per Unit will be further reduced.*

Relative Cost of Hydraulic and Electric Power.—Since the hydraulic and electric are the only two practicable and available systems of power transmission for dock appliances, such as cranes, capstans, and coal hoists; and, since both systems are equally reliable and convenient, the choice of one or the other depends entirely on which is the more economical, not only in regard to the cost of production and distribution, but also as regards the cost of utilisation.

One system may cost considerably less to produce and distribute power, but in utilisation the gain in point of economy may not be in favour of that system

There are, indeed, so many factors to be taken into account that, for purposes of comparison, and also to determine whether it would cost less or more to purchase or produce power, the cost of each system should be ascertained at three distinct stages, viz. :—(1) The cost of purchase or production. (2) The cost of transmission or distribution. (3) The cost of utilisation.

In order to compare the costs of hydraulic with that of electric power, it is necessary to reduce both systems to a common standard, and to determine in each case the cost of *one horse-power hour* delivered at the appliance where the power is to be utilised. This is the most convenient unit to adopt, and is applicable alike to both systems.

In making up the working costs, the capital expenditure and working costs have been taken from the annual accounts, and embrace all the capital expended, up to the end of 1902, on buildings, foundations, culvert, machinery, and plant for each of the two systems.

The outputs of hydraulic power during the periods under notice, have been found by keeping strictly accurate records of every revolution of the pumping engines. The outputs of electric power have been got from carefully kept daily records of the hours of lighting, pressure, and current used.

The conclusions arrived at are, that electric power costs less to produce, distribute, and utilise than hydraulic power.

Crane Tests.—Tests were conducted under favourable conditions in order to ascertain the following :—

1. The rapidity of working of both types.
2. Their efficiencies.
3. The cost of power used in doing the same amount of work,

* In a large well-regulated power-house the Cost of Electric Power may be as small as 1d. per Board of Trade Unit—*i.e.*, 1d. per 1,000 volt-ampere-hours or watt-hours.

viz., to raise 30 feet, slew 100 degrees, and discharge into waggons 1,000 tons of cargo and 1,000 tons of ore, when working at loads varying from 1 to 3 tons for cargo, and at 2½ and 3 tons for ore.

Particulars of Hydraulic Crane and its Gauges.—The pressure in the hydraulic 7-inch main was maintained at 750 lbs. per square inch at the Power-House. The 3-ton hoisting ram was used. It is 11 inches diameter, and displaces ·66 cubic foot of water per foot of travel, or ·11 cubic foot per foot of hoist, the velocity ratio being 1 to 6. The turning ram is 9 inches diameter, travels 15½ inches, and uses ·57 cubic foot in slewing through 100 degrees. The water used was measured by the displacement of the rams, and 5 per cent. was allowed for slip and leakage.

Pressure gauges were fitted on the main pressure pipe at the hydrant connected to the crane, and to the upper parts of the hoisting and slewing cylinders. The pressures as indicated by these gauges were noted at each lift.

Particulars of Electric Crane and its Meters.—The hoisting gear is single reduction, and is worked by a 50 H.P. Siemens' motor at a pressure of about 240 volts. The pressure at the Power-House was maintained at 260 volts, whilst the current naturally varied with the load. The turning motor is 10 H.P.

The electric crane was equipped with Siemens' volt and ampere meters, and also with Chamberlain & Hookham's watt meters, which were overhauled and found to be accurate before the tests were commenced.

Details of Testing Operations.—In testing for rapidity of working, each crane was provided with two tipping buckets, one for the load and one for the empty bucket. For convenience, the tests were carried out to specific instructions in double cycles, by working the cranes as rapidly as possible for two hours, under each separate load of 1 ton, 2 tons, 2½ tons, and 3 tons, in such a way, as to imitate as nearly as possible actual conditions of working, without being subjected to the delays so frequently experienced in discharging ore or general cargoes.*

* In calculating the weight of cargo discharged per hour, allowance was made for the 9-cwt. empty buckets and the 2-cwt. hoisting chain balls, as follows; (9 − 2) cwts. = ·35 ton. Now, this weight was lifted only 10 feet during the return journey with the empty bucket, and = (·35 × 10) ft.-tons of work. But each ton of paying load is lifted 30 feet = 30 ft.-tons of work. Hence, for every 30 ft.-tons of general cargo work, there would have to be added in this test, 3·5 ft.-tons for the empty bucket minus the chain ball. Or, $\left(·35 \times \frac{10}{30}\right)$ = ·116 ton, as shown in the two following tables

Curve Diagrams 1 and 2.—The principal curves to which attention may be directed are the speed lines, S, which show that both cranes are nearly alike as regards speed at all loads—varying from 260 to 165 feet per minute in the case of the hydraulic crane, and from 239 to 165 feet per minute in the electric crane, at loads of $\frac{1}{2}$ and $3\frac{1}{2}$ tons respectively.

The efficiency line, E, for the hydraulic crane rises in a straight line from $9\frac{1}{2}$ per cent. to 66 per cent. for loads of $\frac{1}{2}$ and $3\frac{1}{2}$ tons respectively. The efficiency line, E, of the electric crane rises in a full, well-rounded curve, showing at

$\frac{1}{2}$ ton	1 ton	2 tons	$2\frac{1}{2}$ tons	3 tons	$3\frac{1}{2}$ tons *

efficiencies of

31%	49·34%	73·14%	77·1%	75·3%	70%

The other lines, C, show the cost of power used in raising 30 feet, slewing 100 degrees, and discharging into waggons 1,000 tons of cargo. It will be seen, that the cost line for the hydraulic crane rises in a steep curve at the light loads, while that for the electric crane is a straight line nearly parallel with the abscissæ, showing how the cost increases in the case of the former when working at light loads, while the cost is practically constant at any load with the electric crane.*

* I have added to these two diagrams E_C lines. In the case of the hydraulic crane this dotted curve, E_C, indicates the percentage efficiency as found by the pressure gauges attached directly to the hydraulic cylinder. This curve, therefore, represents the ratio of the power got out at the crane hook (when lifting loads 30 feet in height), to the power put into the cylinder during that time. It clearly shows the effect of the loss of pressure between the hydrant on the main supply pipe and the ram cylinder. It also shows by comparison the efficiency of the plant between the crane hook and the cylinder ram with that between the same hook and the power put into the mains at 750 lbs. per square inch.

In the case of the electric crane, the dotted curve, E_C, is derived from special sets of ten observations for each load, taken on March 20th, 1903. The times of simply lifting loads 30 feet were accurately noted, as well as the readings on the crane volt and ampere meters during these respective lifts, quite independently of the meters at the power-house. This curve, therefore, represents the best net combined electrical and mechanical efficiency of the crane, whilst merely lifting different loads. It indicates the ratio of the power got out at the crane hook, to the power put into the lifting motor, or the crane efficiency

Note.—Students may refer here to *Engineering* of June 19 and July 3, 1903, for reports upon two papers, "Economical Speeds of Cranes," and "Hydraulic *versus* Electric Cranes for Docks," read at the London Engineering Conference.

TABLE I.—Results of Tests of 3-Ton Hydraulic Crane Worked as if Discharging General Cargo.

Loads (W).	1 Ton.	2 Tons.	2½ Tons.	3 Tons.
Duration of trial, hours,	2	2	2	2
Pressure in mains, lbs. per sq. in.,	750	750	750	750
Pressure on ram—				
Hoisting, . . ,,	240	400	480	560
Lowering, . . ,,	140	300	380	460
Slewing to windward, ,,	500	500	500	500
,, leeward, ,,	350	350	350	350
1. Number of Cycles per Hour, ...	56	54	52	38
Time of lifting W 30 feet (T), . . seconds,	7·5	8·57	9·25	10
Speed of hoisting loads $(S)=\frac{30\times 60}{T}$ ft. per min.,	240	210	194·6	180
H.P. taken out of crane $(L.H.P.)=W\times S\div 33{,}000$,	16·29	28·51	33	36·65
Horse-power, by water $(W.H.P.)=\frac{95\times 750\times S}{33{,}000\times 6}$,	86·36	75·56	70	64·77
2. Mechanical Efficiency (*Hoisting only*), $\%=100\times\frac{L.H.P.}{W.H.P.}$, .	18·87	37·73	47·14	56·6
Cargo discharged per hour (allowing for empty bucket ·35 × ⅛ ton), tons,	(1·116 × 56) 62·5	(2·116 × 54) 114	(2·616 × 52) 136	(3·116 × 38) 118·5
Water used per cycle—				
Hoisting 30 feet, cub. ft.,	3·3	...	...	...
Slewing 100°, ,,	·57	...	...	...
Hoisting 10 feet, ,,	1·1	...	...	...
Slewing 100°, ,,	·57	...	...	...
Slip, 5 per cent., ,,	·28	...	...	...
Total cubic feet, .	5·82	5·82	5·82	5·82
Water used per hour, cub. ft.,	326·5	314·5	303	221·5
Cargo discharged per hour, tons,	62·5	114	136	118·5
Water used per 1,000 tons cargo, . . cub. ft.,	5,224	2,759	2,228	1,869
3. Cost of Hydraulic Power used in discharging 1,000 tons general cargo, at 110·45 pence per 1,000 cubic feet of water at 750 lbs. per square inch (C), . .	48/	25/6	20/6	17/2

TABLE II.—Results of Tests of 3-Ton Electric Crane Worked as if Discharging General Cargo.

Loads (W)	1 Ton.	2 Tons.	2½ Tons.	3 Tons.
Duration of trial, hours,	2	2	2	¼
E.M.F. at motor terminals, volts,	240	245	240	245
Current when hoisting loads, amps.,	95	110	125	140
Current when lifting empty bucket, amps.,	60	60	60	60
Current slewing to windward, amps.,	15	35	35	40
Current slewing to leeward, amps.,	15	15	15	15
1. Number of Cycles per Hour,	60	53	48	43
Work done by hoisting W 30 feet, foot-lbs.,	67,200	134,400	168,000	201,600
Time of lifting W 30 feet (T), seconds,	8·1	9·25	9·85	10·6
Speed of hoisting loads, $S = \frac{30 \times 60}{T}$ ft. per min.,	222·2	194·3	182·7	170
H.P. taken out of crane (L.H.P.) = W × S ÷ 33,000,	15·08	26·42	31	34·62
Horse-power expended, $(E.H.P.) = \frac{\text{volts} \times \text{amps.}}{746}$,	30·56	36·12	40·21	46
2. Combined Mechanical and Electrical Efficiency, $\% = 100 \times \frac{L.H.P.}{E.H.P.}$ (*Hoisting only*),	49·34	73·14	77·1	75·3
Cargo discharged per hour (allowing for empty bucket ·35 × ⅛ ton), tons,	(1·116 × 60) 67	(2·116 × 53) 112	(2·616 × 48) 125·5	(3·116 × 43) 134
Units used per hour at various loads,	5	8·5	9·5	10·4
Units required for 1,000 tons cargo (U),	74·6	75.8	75·7	77·6
3. Cost of Electric Power used in discharging 1,000 tons general cargo, at 2·558 pence per unit (C),	15/11	16/2	16/1½	16/6½

DIAGRAM No 1.

Curves showing performance of 3 Ton Hydraulic Crane at South Quay, Prince's Dock, under loads of 1 Ton, 2 Tons, 2½ Tons and 3 Tons, tested on 19th and 20th March, 1903.

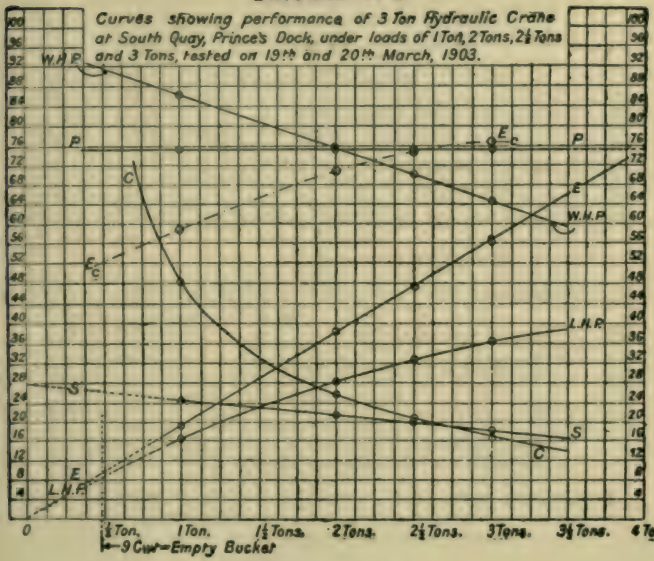
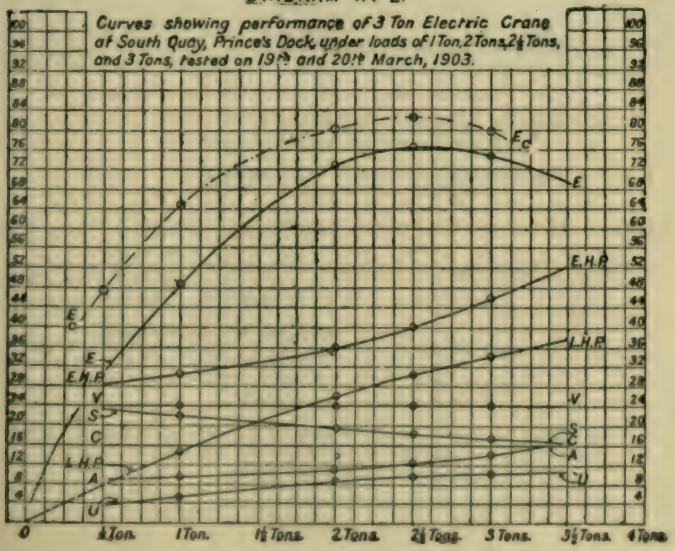


DIAGRAM No 2.

Curves showing performance of 3 Ton Electric Crane at South Quay, Prince's Dock, under loads of 1 Ton, 2 Tons, 2½ Tons, and 3 Tons, tested on 19th and 20th March, 1903.



EXPLANATION OF EFFICIENCY CURVES.

Hydraulic Crane.

P. Pressure in mains during tests.
Each division = 40 lbs. per square inch.

S. Speed of hoisting in feet per minute.
Each division = 40 feet per minute.

W.H.P. Horse-power put into crane by water at 750 lbs. per square inch, hoisting only.
Each division = 4 H.P.

L.H.P. Horse-power taken out of crane in hoisting only.

E. °/₀ Mechanical efficiency = $\frac{\text{L.H.P.}}{\text{W.H.P}} \times 100$ (*hoisting only*).
Each division = 4 per cent.

E_C. °/₀ Efficiency = $\frac{\text{Power got out at crane hook}}{\text{Power put into hydraulic ram}}$, when *merely* lifting different loads 30 feet during special trials.
Each division = 4 per cent.

C. Cost of power used in raising 30 feet, slewing 100°, and discharging into waggons 1,000 tons of cargo.
Each division = 4 shillings.

Electric Crane.

V. Volts while hoisting at speed (**S**).
A. Amperes ,, ,,
Each division = 40 volts and also = 40 amperes.

S. Speed of hoisting in feet per minute.
Each division = 40 feet per minute.

E.H.P. Horse-power delivered to hoisting motor.
Each division = 4 H.P.

L.H.P. Horse-power taken out of crane in hoisting only.

E. °/₀ Electrical and mechanical efficiency = $\frac{\text{L.H.P.}}{\text{E.H.P.}} \times 100$.
Each division = 4 per cent (*when hoisting only*).

E_C. °/₀ Crane efficiency = $\frac{\text{Power got out at crane hook}}{\text{Power put into motor}}$ when *merely* lifting different loads 30 feet during special trials.
Each division = 4 per cent.

C. Cost of power used in raising 30 feet, slewing 100°, and discharging into waggons 1,000 tons of cargo.
Each division = 4 shillings.

U. Units used per hour at various loads.
Each division = 4 units.

Electric Cranes for Manchester Ship Canal.—As one more instance of the rapid progress which the application of electricity to cranes is making, we here show two views of those recently made and fitted by George Russell & Co., of Motherwell, for the Manchester Ship Canal Warehouse Co. There are twenty-two cranes placed on the roof of the warehouse. Of these, twelve are portable cranes on the dock side of the warehouse for lifting 30 cwts., with a radius of 42 feet. The remaining ten are fixed cranes on the land side of the warehouse, three of which lift 30 cwts., and seven 10 cwts., both having a radius of 16 feet.

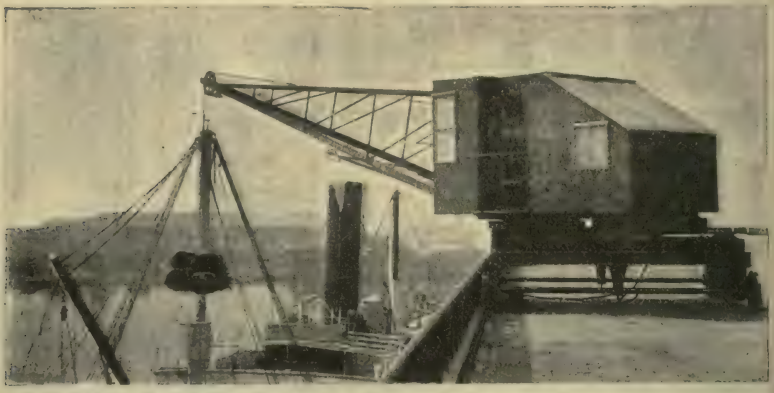
The electrical motors and controllers were supplied by the British Thomson Houston Co., Rugby. The hoisting motors for

FIXED AND MOVABLE ELECTRIC CRANES ON THE MANCHESTER CANAL WAREHOUSE.
(Made by George Russell & Co., Ltd., Motherwell, N.B.)

the 30-cwt. cranes are rated at 28 H.P. when their armatures have a speed of 355 revs. per minute, and are supplied at 500 volts. They are geared direct to the lifting barrels by single reduction machine-cut spur wheels. The barrel of each of these cranes is 24-inches diameter, and the lift is effected by a single wire rope. The speed of lifting was specified to be 200 feet per minute, with a full lift of 75 feet. A strap brake operates upon a brake wheel, keyed upon the barrel shaft. Under normal conditions the brake bears upon its brake wheel through the action of a lever weight, just as in the previously described Clyde Trust Electric Crane. But, in this instance, the weight is raised by an electro-magnet in series with the lifting motor,

so that whenever current is switched on to the motor the weight is lifted and the brake released. In the event of the current failing through any cause the pull of the magnet ceases, and the weight automatically applies its brake to prevent the load from falling. The brake may be also released by a foot tread, so that the load may then be lowered without using any current.*

From a final test of one of the portable cranes it was found that whilst lifting four bales of cotton at a time, having an



MOVABLE ELECTRIC CRANE UNLOADING A STEAMER AT THE MANCHESTER CANAL.

average weight of 3,136 lbs., through 43½ feet, that the time was 11·5 seconds (or 227 feet per minute), current 38·5 amperes at 510 volts; consequently, the efficiency under these circumstances reached 82 per cent.

* For a fuller description of the details of these cranes students may refer to the March and April numbers of the leading British electrical and mechanical journals, such as *The Tramway and Railway World*, April, 1903, to which paper I am indebted for the two views here reproduced.

LECTURE VIII.—QUESTIONS.

1. Explain why the force necessary to drive a machine does not vary in exact proportion with the load.

2. With a pair of three sheaved blocks it is found by experiment that a weight of 40 lbs. can be raised by a force of 10 lbs., and a weight of 200 lbs. by a force of 40 lbs. Find the general relation between P and W, and also the efficiency when raising 100 lbs. *Ans.* $P = \frac{3}{16} W + \frac{5}{2}$; ·784 or 78·4 per cent.

3. With a screw jack it is found that a force of 47·5 lbs. must be applied at the end of the handle to lift 1·5 tons, and a force of 85 lbs. to lift 3 tons. Find what force will be required to raise 2 tons. *Ans.* 60 lbs.

4. If the length of the handle in the above example be 2 feet and the pitch of the screw $\frac{3}{4}$ inch, find the efficiency in each case. *Ans.* 35·3, 39·5, and 37·3 per cent.

5. Find the "*friction pressure*" of a steam engine which requires a mean effective pressure of 24 lbs. per square inch to drive it at full load, 20 lbs. being taken up in overcoming the load. At three-quarters load a mean effective pressure of 18·5 lbs. is required, of which 15 lbs. is similarly taken up. *Ans.* 2 lbs. per square inch.

6. It is found that a force of 2 lbs. must be applied to the handle of a crane in order to wind up the rope when no weight is attached and of 80 lbs. when lifting a weight of 10 cwts. If the velocity ratio be 20, find the efficiency in this last case and also when the force at the handles is lessened so as just to allow the weight to descend. *Ans.* 70 per cent.; 57 per cent.

7. A machine is concealed from sight except that there are two vertical ropes; when one of these is pulled downwards the other rises. If the falling of a weight, A, on one, causes a weight, B, on the other to be steadily lifted, first when A is 100 lbs. and B is 3,250 lbs., second when A is 50 lbs. and B is 1,170 lbs., what is the probable value of A when B is 2,000 lbs.? How would you find the efficiency of this lifting machine in these three cases? What new measurement must be made?

Ans. 69·95 lbs. (B. of E. Adv., 1900.)

8. A machine is concealed from sight, except that there are two vertical ropes; when one of these is pulled downwards the other rises. If the falling of a weight, A, on one, causes a weight, B, on the other to be steadily lifted, first when A is 12 lbs. and B is 700 lbs., second when A is 7·6 lbs. and B is 300 lbs. What is A likely to be when B is 520 lbs.? If B rises 1 inch when A falls 70 inches: what is the efficiency of this lifting machine in each of the three cases? (B. of E. Adv., 1901.)

9. In a lifting machine an effort of 26·6 lbs. just raises a load of 2,260 lbs., what is the mechanical advantage? If the efficiency is 0·755, what is the velocity ratio? If on this same machine an effort of 11·8 lbs. raises a load of 580 lbs., what is now the efficiency? (B. of E. Adv. & H., Part I., 1902.)

10. If the diameter of the 3-ton lifting ram of the hydraulic crane described in this lecture is 11 inches, and its stroke be 5 feet when the chain hook rises 30 feet, and if the inside diameter of the water service pipe from the power-house to the crane dock hydrant opposite this crane is 7 inches; find what length and weight of a column of water in the service pipe is used at each complete 5-foot stroke of the lifting ram.

11. Referring to the previous question, if a 3-ton load be lifted 30 feet high in 10 seconds, and if the lengths of 7-inch service piping between the crane hydrants and the power-house accumulator be 500 and 1,000 feet respectively for two different cranes, what will be the loss in pressure in lbs. per square inch between the power-house and each of these two hydrants, when accumulator applies a steady pressure of 750 lbs. per square inch at the power-house, and if the coefficient of friction between the water and its service pipe (or the "friction factor") is ·02 at the velocity under these circumstances? Also, calculate (*a*) the ft.-lbs. of work; (*b*) the watt-hours of work lost through the friction in the pipe at each lift of 3 tons; (*c*) the ft.-lbs. per second; (*d*) the watts or the lost power in the water service pipe during the lift.*

12. Referring to Table I. of tests and the curves for the 3-ton hydraulic crane in this Lecture, how do you account for the differences in the loss of the pressures per square inch between the water in the mains and the pressures on the ram whilst hoisting different loads? What would be the calculated corresponding loss of pressure in each case between the power-house and the crane hydrant if the distance of the main supply pipe was 1,000 feet long and the times of lifting 1, 2, 2½, and 3 tons as stated in Table I.?

13. Check by calculations the several values for W.H.P., L.H.P., E.H.P., E., and C., in the case of the hydraulic and electric cranes described in this Lecture, and draw their curves upon squared paper neatly to a scale at least double the size of that given in the text.

14. Referring to Tables I. and II., &c., in this Lecture, calculate and compare the loss of power in ft.-lbs. per second (or in watts) for the hydraulic and the electric mains whilst lifting 1, 2, 2½, and 3 tons, and show clearly which system of distribution of power is more efficient. How would you propose to render the distribution of power from the power-house to the electric crane more economical?

15. In a crane an effort of 122 lbs. just raises a load of 3,265 lbs. What is the mechanical advantage? If the efficiency be 60 per cent., what is the velocity ratio? Would this crane overhaul? If the efficiency had been 40 per cent., would the crane overhaul? Give clear reasons for your answer. (B. of E., Adv. and H., Part I., 1903.)

16. Electric current is supplied to a certain motor plant at 220 volts, and 150 amperes are taken. What H.P. does this represent? How much would it cost if used for an average of 6·5 hours per day for a whole year of 313 days? The power is supplied at 2·24d. per H.P. hour (*i.e.*, 3d. per B.T.U.). (B. of E., Adv., 1903.)

17. An electrically-driven overhead crane raises a weight of 5 tons at the rate of 90 feet per minute. What is the horse-power? Convert this into watts. The motor drives the lifting machinery, whose efficiency is 70 per cent. What current must be supplied to the motor if the voltage is 220, and the efficiency of the motor is 87 per cent.? If the current is supplied by a company which charges at the rate of 2½d. per Board of Trade unit (1 kilowatt hour) for power purposes, what is the cost of lifting in pence per foot-ton? (B. of E., S. 2, 1905.)

LECTURE VIII.—A.M.INST.C.E. AND B.SC. EXAM. QUESTIONS.

1. What is meant by the efficiency of a machine? How would you measure the efficiency (1) of a screw-jack, (2) of a steam pump, (3) of a turbine? (I.C.E., *Oct.*, 1897.)

2. An hydraulic company charges 15d. per 1,000 gallons of water at 820 lbs. per square inch; what is the cost per H.P. hour? (I.C.E., *Oct.*, 1898.)

3. The saddle of a lathe weighs 5 cwts., and it is moved along the lathe bed by a rack and pinion arrangement. What force applied at the end of a handle 10 inches long will be just capable of moving the saddle, supposing the pinion to have twelve teeth of 1¼ inch pitch, the coefficient of friction between the saddle and lathe bed to be 0·13, and if 35 per cent. of the work done in working the rack and pinion is expended in overcoming friction. (I.C.E., *Feb.*, 1902.)

4. An experiment on a windlass to determine the effort required to lift different loads, gave the following results:—

Load (lbs.)	0	28	56	84	112	140	168	196	224
Effort (lbs.)	1·5	2·75	4	5·2	6·4	7·6	8·8	9·9	11

The velocity ratio of the machine was 52. Plot the results on squared paper, showing the effort curve on a load base, and draw also the friction and efficiency curves, finding at least five points in each. Find the effort for a load of 210 lbs., and find the mechanical advantage and mechanical efficiency of this load. (I.C.E., *Feb.*, 1902.)

5. Explain why the loss by friction in an engine is neither constant nor a constant fraction of the power developed. In an experiment with an engine, when running at 400 revolutions per minute, the B.H.P. was 21 and the I.H.P. 25, and at 200 revolutions per minute the B.H.P. was 4¾ and the I.H.P. 6. Estimate the probable efficiency at 500 revolutions per minute, the I.H.P. being 44. (I.C.E., *Oct.*, 1904.)

6. In the equation $P = (1+e)\,R\,r + P_o$, P is the urging force applied to a machine, R is the working resistance, r is the velocity ratio of the working point to the urged point, e is a friction coefficient, and P_o is the urging force required to keep the machine in motion when there is no working resistance. Derive the equation from fundamental considerations, and show how it is modified to suit the case in which P is zero and R just reverses the machine. By means of a crane, in which the velocity ratio of fall to lift is 25 to 1, a weight of ½ ton can be raised by the application of a force of 67 lbs., and a weight of 4 cwts. is just able to run down and reverse the crane. Find the force necessary to lift 1 ton, and the efficiency when that load is being raised. (I.C.E., *Feb.*, 1905.)

7. Define the terms "efficiency" and "mechanical advantage" as applied to machines. In a jib crane a force of 40 lbs. is required to raise 1 ton and 105 lbs. to lift 5 tons. The velocity ratio is 220. Find the force necessary to lift 3½ tons, and state the efficiency for that load. (I.C.E., *Feb.*, 1907.)

8. Explain what is meant by—the brake efficiency of an engine; the transmission efficiency of shafting and countershafting; and the mechanical efficiency of a machine tool. What are the principal causes of loss in each case? A steam engine has to be provided for driving a number of machine tools under the following conditions:—Average total power to be delivered at the machine tools, 100 H.P. Average mechanical efficiency of machine tools, 75 per cent. Transmission efficiency of shafting and countershafting, 65 per cent. Cost of 1 lb. of steam per annum, 4s. 15 per cent.

per annum to be allowed for depreciation, interest, and repairs. Which of the two following propositions would be the best to adopt?—(*a*) Steam consumption, 14 lbs. per I.H.P. per hour; brake efficiency, 86 per cent.; cost of engine, complete, £2,000. (*b*) Steam consumption, 16 lbs. per I.H.P. per hour; brake efficiency, 93 per cent.; cost of engine, complete, £1,100. (I.C.E., *Oct.*, 1907.)

9. A steam engine is direct-connected to a dynamo. At full load the electrical output is 1,000 H.P. The dynamo losses (at full load) consist of two parts, one of 20 H.P. which is constant—*i.e.*, is independent of the load—and the other 40 H.P. which varies with the load. In the case of the engine, the loss at full brake-load is 100 H.P., and of this loss 90 H.P. is constant and 10 H.P. varies with the load. Explain how you would find the indicated H.P., at full load and at half-load of the dynamo. What is the combined efficiency in both cases? (I.C.E., *Feb.*, 1908.)

10. In a screw-jack the couple in lb.-feet required to raise a load of W lb. is 0·029 W, the pitch of the screw being $\frac{3}{4}$ inch, and the mean diameter of the screw thread 3 inches. Find a similar expression for the couple required to lower the load. Find also the efficiency of the screw-jack.

(B.Sc., Lond., *July*, 1909.)

TOOTH, FRICTION, BELT, ROPE, CHAIN AND MISCELLANEOUS GEARING, WITH THEIR APPLICATIONS TO MACHINES.—SHAPES AND STRENGTHS OF TEETH.—AUTOMATIC TOOTH-CUTTING MACHINES.—VELOCITY-RATIO AND POWER TRANSMITTED BY GEARING.

LECTURE IX.

CONTENTS.—Definition of Gearing—Train of Wheels—Pitch Surface—Pitch Circle—Definitions of Pitch Surface, Pitch Line or Pitch Circle, Pitch Point—Sizes of Spur and Bevel Wheels—Velocity-Ratio of Two Wheels in Gear—Angular Velocity-Ratio—Definition of Angular Velocity—Velocity-Ratio of a Train of Wheels—Definition of Value of Train—Example I.—Intermediate or Idle Wheel—Marlborough Wheel—Change Wheels for Screw Cutting Lathes—Example II.—Force-Ratio and Power Transmitted by Gearing—Examples III. and IV.—Cable Grappling, Picking-up and Paying-out Gear—Details of Jamieson's Cable Gear—Changing Speed by Friction Wheels—Changing Speed by Toothed Wheels—Changing Speed by Bands and Pulleys—Changing Speed with Electro-Motor Connections—Mechanical Speed Gear combined with Variable Speed Electric Motors—Electrically-Driven Lathe—Electric Overhead Travelling Crane—Questions

Definition of Gearing.—The term *gearing* is applied generally to any arrangement of wheel-work or link-work, for transmitting motion and power from one place to another. Engineers, however, restrict the term to denote any combination of *wheels* used for the transmission of motion and power from one shaft to another.

When the wheels are so arranged that they are capable of communicating motion from one to the other, they are said to be *in gear*; otherwise they are said to be *out of gear*.

Train of Wheels.—When a number of wheels are employed in transmitting motion and power from one place to another, it is usual to so arrange the wheels that each shaft, except the first and last, shall carry two wheels of different sizes; the smaller of these is made to gear with the larger one on the shaft next in order. Such an arrangement is termed a **Train of Wheels.**

In any train of wheels, that wheel which causes motion is termed the **Driver**, and that which receives the motion is called the **Follower.** Usually, however, the terms driver and follower are applied to *any contiguous* pair of wheels in the train.

The connection between a driver and its follower may be made in either of three ways :—

I. By *rolling contact* at their surfaces, as in *toothless wheels* or *friction gearing.*

II. By *sliding contact* of their surfaces, as in *toothed gearing.*

III. By *belts*, *ropes*, or *chains.*

The first two methods are adopted when the shafts to be con-

nected are close together. In such cases the wheels are in actual contact with each other. The third method is adopted when the shafts are so far apart that it becomes impracticable or inconvenient to use friction or toothed wheels. However, circumstances other than the mere distance apart of the shafts often determine which method of connection should be adopted. The advantages and disadvantages attending each method will appear in what follows.

In this Lecture we shall deal with the velocity- and force-ratios communicated by wheels in gear, particularly with friction and toothed gearing, leaving their theory of construction and design to subsequent Lectures. We begin by defining a few of the general terms employed.

Pitch Surface—Pitch Circle.—Motion may be communicated from one shaft to another by rolling contact between the surfaces of bodies rigidly fixed to the shafts. The shafts to be connected may be (1) parallel, (2) intersecting, or (3) neither parallel nor intersecting. The third case will not come under our notice in this text-book. The velocity-ratio transmitted may require to be *constant* or *variable*. It is only for very special machinery that a variable velocity-ratio is required, and we shall, therefore, not consider it here. We have, therefore, to consider the case of a constant velocity-ratio between two parallel or intersecting shafts. The rigid bodies fixed to the shafts, and through which the motion has to be transmitted, must be cylindrical for parallel shafts, and conical for intersecting shafts. These bodies we shall now call wheels, being **spur** or **bevel**, according as they are cylindrical or conical—*i.e.*, according as they are used to connect parallel or intersecting shafts.

The surfaces of the wheels, which, by their rolling contact, communicate the required velocity-ratio, are called **Pitch Surfaces**. In ordinary friction gearing these pitch surfaces have a real physical existence, but in toothed gearing they have no such existence. However, for constructive and other purposes, it is necessary to imagine such surfaces as also existing in their case. Hence, we have the following:—

DEFINITION.—**The Pitch Surface of a Toothed Wheel is an ideal surface (intermediate between the crests of the teeth and the bottoms of the spaces), which, by rolling contact with the pitch surface of another wheel would communicate the same motion that the toothed wheels communicate.**

DEFINITION.—**The Pitch Line or Pitch Circle is a section of the pitch surface perpendicular to it and to the axis of the shaft.**

In the case of cylindrical pitch surfaces, the surface of section is a *plane* perpendicular to the axis of the shaft; while for conical pitch surfaces it is a *sphere* having its centre at the apex of the cone.

Definition.—**The Pitch Point of a pair of wheels in gear is the point of contact of their pitch lines or circles.**

In bevel gearing only frusta of the conical surfaces are employed. Equal bevel wheels having the angle at the apex of their pitch cones equal to a right angle are called **Mitre Wheels.** Mitre wheels are used when two shafts at right angles to each other have to rotate with equal speeds.

Sizes of Spur and Bevel Wheels.—The size of a spur wheel is measured by the diameter of its pitch circle. With bevel wheels, however, the pitch circle is of variable diameter. For some purposes, the size of the bevel wheel is measured at the larger end of the conical frustum, while for other purposes it is measured at a pitch circle half way between the larger and smaller ends of the frustum. For mere convenience in stating the relative sizes of bevel wheels, the first of these methods is adopted; but for calculations relating to power transmitted, &c., we require to adopt the second method. In applying the following results of this Lecture to bevel gearing, it must, therefore, be remembered that the diameter, pitch circle velocity, &c., refer to the pitch circle *half way* between the larger and smaller ends of the frustum.

Velocity-Ratio of Two Wheels in Gear.—When toothed gearing has properly formed teeth, and the wheels are accurately in gear, then the velocity-ratio transmitted is kinematically identical with that obtained by the rolling of their pitch surfaces in contact without slipping. Hence, in what follows, we shall consider the velocity-ratio of the pitch circles only.

Let A and B be the centres of two wheels in gear.
„ D_1, D_2 = Diameters of pitch circles A and B respectively.
„ N_1, N_2 = Number of revolutions of wheels A and B „ in unit time.
„ n_1, n_2 = Number of teeth on „ „ „

Then, since no slipping takes place, we must have :—

Circumferential Velocity of A = *Circumferential Velocity of* B.

$$\therefore \quad \pi D_1 N_1 = \pi D_2 N_2 .$$

Or,
$$\frac{N_2}{N_1} = \frac{D_1}{D_2}$$

Also,
$$\frac{n_1}{n_2} = \frac{D_1}{D_2}$$

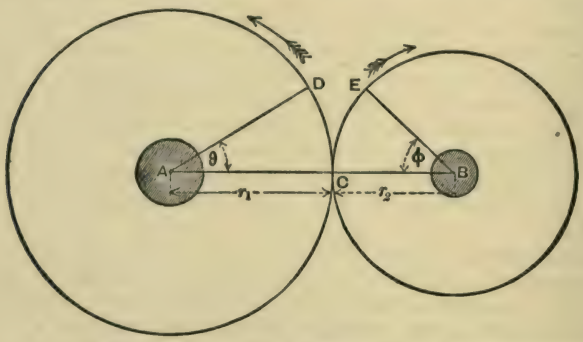
$$\therefore \quad \frac{N_2}{N_1} = \frac{D_1}{D_2} = \frac{n_1}{n_2} . \quad . \quad . \quad . \quad . \quad . \quad . \quad (I)$$

Consequently, *the speeds of two wheels in gear, are inversely as their diameters or number of teeth.*

Angular Velocity-Ratio.—Sometimes we require to know the *angular* velocity-ratio of two wheels in gear in terms of their diameters or number of teeth. This we now proceed to determine, but, in the first place, we give the following :—

DEFINITION.—**The Angular Velocity of a rotating body is the circular measure of the angle described in unit time by any line in that body.**

Let ω_1, ω_2 = Angular Velocity of wheels A and B respectively.



VELOCITY-RATIO OF TWO CIRCULAR DISCS.

During a small interval of time let the radii, A C, B C, be displaced through the angles θ and φ into the positions A D, B E respectively.

Then, $$\frac{\omega_1}{\omega_2} = \frac{\theta}{\varphi}.$$

But, $$\text{Arc C D} = \text{Arc C E}.$$

i.e., $$r_1 \theta = r_2 \varphi.$$

Or, $$\frac{r_1}{r_2} = \frac{\varphi}{\theta}.$$

$\therefore$ $$\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = \frac{D_2}{D_1}.$$

Hence, $$\frac{\omega_1}{\omega_2} = \frac{N_1}{N_2} = \frac{D_2}{D_1} = \frac{n_2}{n_1}. \quad . \quad . \quad . \quad . \quad . \quad \text{(II)}$$

Consequently, *the angular velocities of two wheels in gear, are inversely as their diameters or number of teeth.*

It may be useful here to state the relation between the angular velocity of a wheel and the linear velocity of a point on its pitch circle.

Let V = Linear velocity of a point on the pitch circle.
„ ω = Angular velocity of wheel.
„ r = Radius of pitch circle.

Since ω is the angle described in unit time by any radius, then :—

Arc described in unit time by a point at distance, r, from axis, $\Big\}$ $= \omega r$.

But the length of this arc represents the velocity, V.

Hence, $$V = \omega r.$$

Again, since $$V = 2 \pi r N.$$

$\therefore$ $$2 \pi r N = \omega r.$$

Or, $$\omega = 2 \pi N.$$

Velocity-Ratio of a Train of Wheels.— In questions relating to trains of wheels there is a certain advantage in denoting the radii, diameters, or number of teeth on the various wheels by single letters, such as A, B, C, . . . , which letters also may indicate the wheels themselves.*

DEFINITION.—**The ratio of the number of revolutions of the last wheel in a train to the number of revolutions of the first wheel in the same time is called the Value of the Train.**

Thus, denoting the value or velocity-ratio of the train by e, we get :—

$$e = \frac{\text{Revolutions of last wheel in any time}}{\text{Revolutions of first wheel in the same time}}.$$

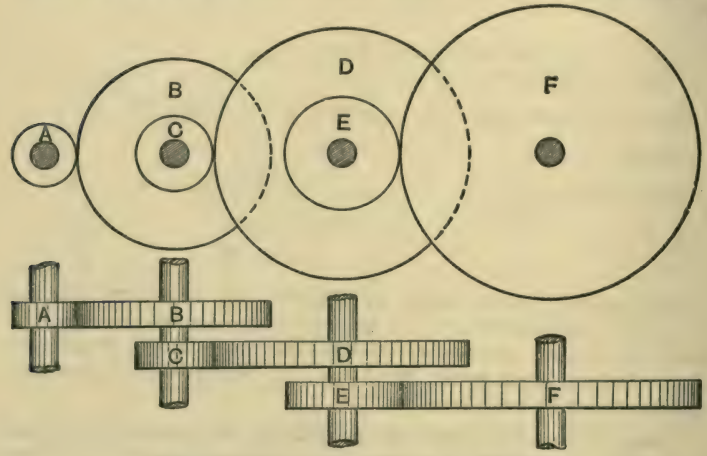
Let N_1, N_l denote the number of revolutions made by *first* and *last* wheels in a train in a given time ; then :—

$$e = \frac{N_l}{N_1} \text{ ; or, } N_l = e N_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad (III)$$

* In the *Elementary Manual on Applied Mechanics*, we denoted the drivers of a train of wheels by D_1, D_2, D_3, &c., and the followers by F_1, F_2, F_3, &c. (see Lecture XII. of the same). Students may adopt either method of notation.

The figure shows a train of wheels, A, B, C, D, E, and F, whereof A may be called the first wheel, or driver, and F the last wheel, or follower. On each of the intermediate shafts, B C, D E, there are two wheels of different sizes, the smaller wheel of each shaft gearing with the larger one on the following shaft. From what has been said at the beginning of this Lecture, we may consider each of the pairs of wheels, A, B; C, D; and E, F, as driver and follower with respect to each other. The first and last shafts, A and F, each carry one wheel only.

Let N_1, N_2, N_3, N_4 denote the speeds or number of revolutions



VELOCITY-RATIO OF A TRAIN OF WHEELS.

per minute of the first, second, third, and fourth shafts respectively :—

Then, $$\frac{N_2}{N_1} = \frac{A}{B}; \quad \frac{N_3}{N_2} = \frac{C}{D}; \quad \frac{N_4}{N_3} = \frac{E}{F}.$$

Multiplying together the corresponding sides of these three equations, we get :—

$$\frac{N_2}{N_1} \times \frac{N_3}{N_2} \times \frac{N_4}{N_3} = \frac{A}{B} \times \frac{C}{D} \times \frac{E}{F},$$

$$\therefore \quad \frac{N_4}{N_1} = \frac{A \times C \times E}{B \times D \times F}.$$

i.e., The velocity-ratio $e = \dfrac{A \times C \times E}{B \times D \times F}$. . . . . (IV)

This is true, however many wheels are in the train, and thus we have the following rule :—

$$\text{Value of a Train} = \frac{\left\{\begin{array}{l}\text{Product of radii, diameters, or number}\\ \text{of teeth of all the drivers}\end{array}\right.}{\left\{\begin{array}{l}\text{Product of radii, diameters, or number}\\ \text{of teeth of all the followers}\end{array}\right.}.$$

Example I.—The table of a planing machine has to be moved backward and forward at the rate of 12 feet per minute by a rack and pinion arrangement underneath it. The gearing consists of three shafts; the first carrying the pinion which gears with the rack; the last carrying the pulley on which a belt works. The pinion gearing with the rack has 12 teeth of $1\frac{1}{4}$ inches pitch. On the same shaft as this pinion is a wheel of 40 teeth. This wheel gears with a pinion of 15 teeth on the second shaft. A wheel of 30 teeth on this shaft gears with a pinion of 12 teeth on the last shaft. Find the number of revolutions of the last shaft per minute.

Answer.—Let A, B, C, D denote the numbers of teeth on the wheels gearing together, where :—

$$A = 40,\ B = 15,\ C = 30,\ D = 12,$$

Then,
$$e = \frac{A \times C}{B \times D} = \frac{40 \times 30}{15 \times 12} = \frac{20}{3}.$$

Since the pitch of the teeth on rack and pinion is $1\frac{1}{4}$ inch, and the speed of the rack is 12 feet per minute :—

$$\therefore \left.\begin{array}{l}\textit{Revolutions of first}\\ \textit{shaft per minute}\end{array}\right\} = \frac{\textit{Speed of table}}{\textit{Circumference of pinion}},$$

$$\text{,, ,,} = \frac{12 \times 12\ \textit{inches per minute}}{12 \times 1\frac{1}{4}\ \textit{inches}},$$

$$\text{,, ,,} = \frac{48}{5} = 9\tfrac{3}{5}.$$

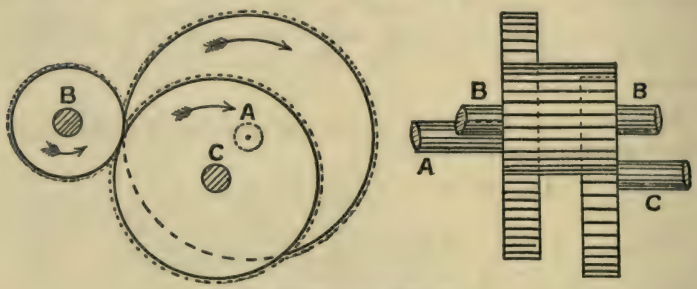
$$\therefore \left.\begin{array}{l}\text{Revolutions of last}\\ \text{shaft per minute}\end{array}\right\} = e\ N_A = \frac{20}{3} \times 9\tfrac{3}{5} = 64.$$

Intermediate or Idle Wheel.—Sometimes a wheel carried on a separate axle is interposed between two other wheels, or introduced into a train of wheels, for the purpose of *changing the relative directions of rotations* of the first and last wheel. Such an intermediate wheel is called an *idle* wheel, because it does not affect the

numerical value of the train, but only its *sign*. That an idle wheel has the effect just stated may easily be proved by considering the velocity-ratio of a pair of wheels connected through an intermediate one.

Instances of the use of idle wheels are very common in gearing, but the two following will serve as examples :—

I. **Marlborough Wheel.**—When two parallel shafts, A and C, are so close together that they cannot be conveniently connected in the ordinary way, a broad wheel, B, called a *Marlborough wheel*, may be introduced as shown. This wheel has the effect of causing the shafts, A and C, to rotate in the same direction, but in no way does it affect the velocity-ratio which would be obtained by the direct gearing of the wheels A and C. There are other methods whereby the same object could be attained as with a



MARLBOROUGH WHEEL.

Marlborough wheel arrangement; but with these we are not at present concerned.

II. **Change Wheels for Screw Cutting Lathes.***—In screw cutting lathes, a train of wheels, called change wheels, is interposed between the back end of the lathe spindle and the leading screw, for the purpose of transferring motion to the saddle, and determining that the cutting tool shall be moved through a definite pitch for each rotation of the cylinder to be turned or screwed. Every turn of the leading screw moves the saddle and cutting tool through a distance equal to its pitch, and, consequently, if the bar to be screwed turn at the same rate as the leading screw, the pitch of the screw cut upon it will be the same as that of the leading screw. If it move faster than the leading screw, the pitch will be less; and if slower, the pitch will be correspondingly greater. It therefore follows as a matter of

* See the author's *Manual on Applied Mechanics*, Lecture XVI., for description, &c., of a screw cutting lathe.

course, that if we fit wheels on the lathe spindle and on the leading screw of the same diameter, or having the same number of teeth, the screw being cut will have the same pitch as the leading screw. If we fix a small pinion, or one with few teeth, on the lathe spindle and a wheel of large diameter, or many teeth, on the leading screw, the pitch of the screw to be cut will be small, compared with that of the leading screw. The leading screw is generally right-handed, in which case the screw to be cut will be right-handed or left-handed, according as its direction of rotation is the same as, or different from, that of the leading screw. In the former case, there must be at least one intermediate axis between the lathe mandril and the leading screw. If the wheels on the lathe mandril and on the end of the leading screw are of the proper size for the necessary velocity-ratio, then the intermediate axis must carry an idle wheel. Sometimes the wheels required to give the proper velocity-ratio and relative direction of rotation cannot be correctly adjusted without the interposition of more wheels in the train, when it may be necessary to introduce one or more idle wheels.

Let p_c = Pitch of screw to be cut, in inches.
„ p_l = Pitch of parent or leading screw, in inches.
„ e = Effect or value of train of change wheels.

Then,

$$\frac{\textit{Pitch of screw to be cut}}{\textit{Pitch of leading screw}} = \frac{\textit{Speed of leading screw}}{\textit{Speed of lathe mandril}}.$$

$$\therefore \quad \frac{p_c}{p_l} = e. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (V)$$

The problem, then, consists in finding a train of wheels which shall have its value, $e = \frac{p_c}{p_l}$.

Example II.—The leading screw of a lathe is $\frac{1}{2}$ inch pitch and right-handed. The set of change wheels belonging to the lathe consists of the following:—20, 25, 30, 35, 40, 45, 50, 60, 70, 80, 90, 100, 110, and 120 teeth respectively. Devise suitable trains to cut (1) a right-handed screw of 8 threads to the inch, and (2) a left-handed screw of 12 threads to the inch.

Answer.—(1) The number of threads on the screw to be cut being 8 per inch, its pitch is, therefore, $p_c = \frac{1}{8}$ inch.

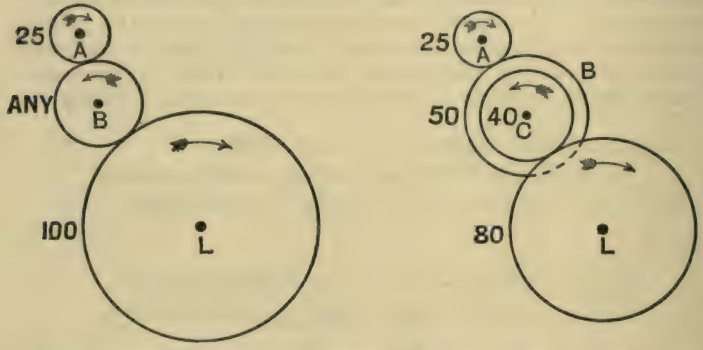
Hence, $$e = \frac{p_c}{p_l} = \frac{\frac{1}{8}}{\frac{1}{2}} = \frac{1}{4}.$$

Both screws being right-handed, they must rotate in the same direction. Hence the train of change wheels must have an *odd* number of axes. Let there be *three* axes in the train.

(*a*) The sizes of the first and last wheels in the train may be such that the required velocity-ratio could be obtained by those wheels in direct gear. In this case the intermediate axis must carry an idle wheel of *any* convenient size, such as B, in the left-hand figure.

$$\therefore \quad \frac{A}{L} = e = \tfrac{1}{4} = \frac{20}{80}; \text{ or, } = \frac{25}{100}; \text{ or, } = \frac{30}{120}.$$

(*b*) The sizes of the first and last wheels need not be such as



CHANGE WHEELS FOR CUTTING A RIGHT-HANDED SCREW.

give the required velocity-ratio by their direct gear. In this case the intermediate axis must carry two wheels of different sizes as in the right-hand figure.

The following train will answer the purpose :—

$$\frac{A \times C}{B \times L} = e = \tfrac{1}{4} = \frac{25 \times 40}{50 \times 80}.$$

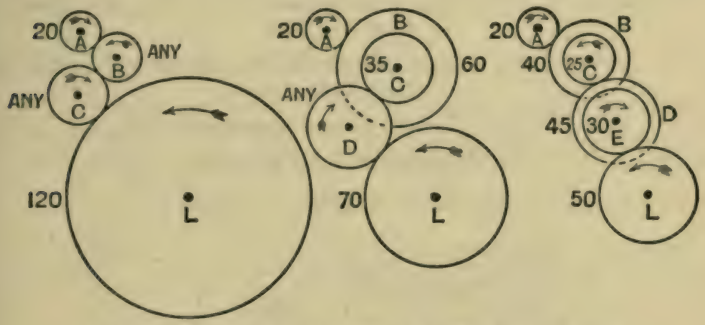
(2) The screw to be cut being left-handed, the leading screw must rotate in the opposite direction from that of the lathe mandril; hence the train must consist of an *even* number of axes. Let there be *four* axes in the train.

(*a*) Let A and L be of such sizes that they would transmit the proper velocity-ratio if geared directly. Then the two intermediate axes must each carry an idle wheel as in the left-hand figure.

Here, $$e = \frac{p_c}{p_l} = \frac{\frac{1}{12}}{\frac{1}{2}} = \frac{1}{6}.$$

$$\therefore \quad \frac{A}{L} = e = \frac{1}{6} = \frac{20}{120}.$$

(*b*) The wheels A and L, not being of the above sizes, we may arrange the train so that *one* of the intermediate axes carries an idle wheel, or that there are no idle wheels in the train. These



CHANGE WHEELS FOR CUTTING A LEFT-HANDED SCREW.

arrangements are shown by the middle and right-hand figures. The trains may be as follows :—

With one idle wheel, $$\frac{A \times C}{B \times L} = e = \frac{1}{6} = \frac{20 \times 35}{60 \times 70}.$$

Or, With no idle wheel, $$\frac{A \times C \times E}{B \times D \times L} = e = \frac{1}{6} = \frac{20 \times 25 \times 30}{40 \times 45 \times 50}.$$

Force-Ratio and Power Transmitted by Gearing.—For certain purposes, as in hoisting and similar machinery, a small effort, P, moving through a comparatively great distance, may be utilised in overcoming a much greater effort or resistance, W, through a much smaller distance. For other purposes, where speed is the ultimate desideratum, the converse of the above would be adopted; *i.e.*, a large effort, P, moving slowly, overcomes a smaller resistance, W, moving rapidly. In any case, it is evident from the *Principle of Work*, that what is lost or gained in speed is gained or lost in the resistance overcome. To modify the effort during its transmission to the working point, suitable mechanism has

to be employed. Several forms of mechanism for this purpose have been considered in previous Lectures; consequently, we proceed at once to express the force-ratio obtained by a train of wheels.

Let the effort, P, be applied by hand or by an engine to the end of a lever or crank rigidly fixed to the *first* axle or shaft at A, while the resistance to be overcome is applied at the circumference of a drum or pulley keyed to the *last* axle or shaft, F, in the following train of wheels :—

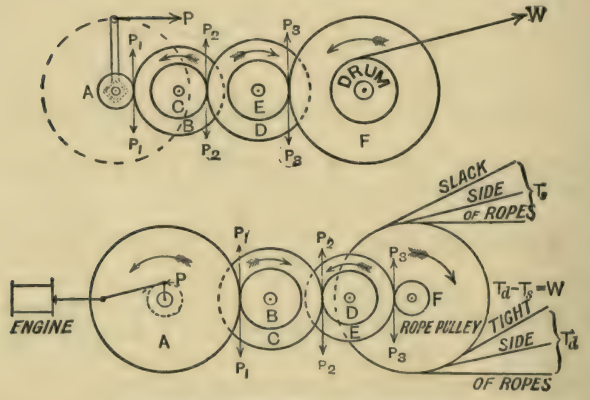
Let P_1, P_2, P_3 = Tangential pressures at points of contact of wheels in gear,

„ r_a, r_b, &c. = Radii of wheels denoted by the capitals of these suffix letters,

„ R = Length of lever handle or crank,

„ r = Radius of drum or pulley,

„ e = Value of train.



POWER TRANSMITTED BY GEARING.

Then, by the *Principle of Moments*, we get :—

$$P \times R = P_1 \times r_a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$P_1 \times r_b = P_2 \times r_c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$P_2 \times r_d = P_3 \times r_e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$P_3 \times r_f = W \times r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Multiplying together the corresponding members of these equations, and cancelling the terms P_1, P_2, and P_3, we get :—

$$P \times R \times r_b \times r_d \times r_f = W \times r \times r_a \times r_c \times r_e .$$

$$\therefore \quad \frac{P}{W} = \frac{r}{R}\left(\frac{r_a \times r_c \times r_e}{r_b \times r_d \times r_f}\right)$$

$$\text{But,} \quad e = \frac{r_a \times r_c \times r_e}{r_b \times r_d \times r_f} \quad . \quad .$$

$$\therefore \quad \frac{P}{W} = \frac{r}{R} \times e . \quad . \quad . \quad .$$

. . (VI)

The results expressed in equation (VI) may be more easily arrived at in the following way :—

By the *Principle of Work* (neglecting friction, &c.), we get :—

$$P \times \textit{its displacement} = W \times \textit{its displacement.}$$

$$\text{Or,} \quad \frac{P}{W} = \frac{\text{W's displacement in a given time}}{\text{P's displacement in the same time}}.$$

$$\text{Hence,} \quad \frac{P}{W} = \frac{\text{Velocity of W}}{\text{Velocity of P}} = \frac{2\pi r \times N_2}{2\pi R \times N_1}.$$

$$\text{Or,} \quad \frac{P}{W} = \frac{r}{R} \times e .$$

Equations (1), (2), (3), and (4) may be used in finding the tangential resistances P_1, P_2, P_3.

Neglecting friction, the power transmitted at any stage of the transmission is constant. Thus, the power transmitted from the second to the third shaft is the same as that done at the driving or working ends.

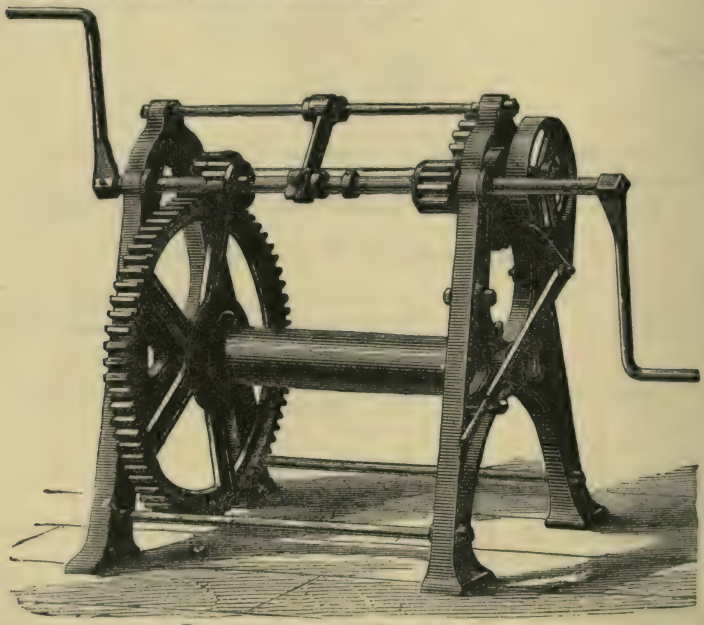
Let H.P. = Horse-power transmitted,

„ P = Tangential pressure in lbs. at pitch surface of any given wheel in the train,

„ V = Velocity in feet per minute at the pitch surface of the given wheel.

$$\text{Then, clearly,} \quad \text{H.P.} = \frac{P\,V}{33{,}000} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(VII)}$$

EXAMPLE III.—The handles of a double-purchase lifting crab are 16 inches long. The pinions have 14 and 20 teeth respectively, and the wheels gearing with these have 84 and 100 teeth respectively. The diameter of the barrel is 12 inches, thickness of rope 1 inch. Find the effort, P, necessary to raise a load of 30 cwts., friction being neglected. If the diameter of the pinion on the first motion shaft be 6 inches, find the pressure between the teeth of each pair of wheels in gear.



DOUBLE-PURCHASE WINCH OR CRAB.

ANSWER.—The accompanying figure shows a general view of a double-purchase lifting crab. For sketches and description of similar lifting machinery we must refer the student to the author's *Manual on Applied Mechanics*, Lecture XIII.

Here R = 16 inches, r = radius of barrel + $\frac{1}{2}$ thickness of rope = $6\frac{1}{2}$ inches.

$$\text{Value of train} = e = \frac{14 \times 20}{84 \times 100} = \frac{1}{30}.$$

$$\therefore \quad \frac{P}{W} = \frac{r}{R} \times e.$$

Or, $$\frac{P}{30 \times 112} = \frac{6\frac{1}{2}''}{16''} \times \frac{1}{30}.$$

i.e., $$P = 45{\cdot}5 \text{ lbs.}$$

Since there are two handles, the effort exerted on each will be:—

$$\tfrac{1}{2} P = 22{\cdot}75 \text{ lbs.}$$

Next, to find the pressure between the teeth of the wheels.

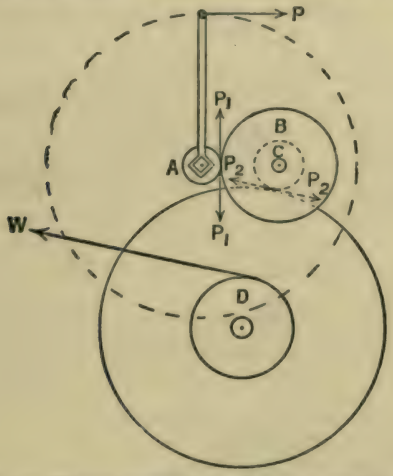


DIAGRAM OF WHEEL-WORK OF A DOUBLE-PURCHASE CRAB TO ILLUSTRATE EXAMPLE III.

Let P_1, P_2 = Pressures between the first and second pinions and their followers respectively.

Then, $P_1 \times$ radius of pinion A $= P \times R$.

$\therefore$ $$P_1 \times 3 = 45{\cdot}5 \times 16.$$

$\therefore$ $$P_1 = 243 \text{ lbs.}$$

Also, $P_2 \times$ radius of pinion C $= P_1 \times$ radius of wheel B.

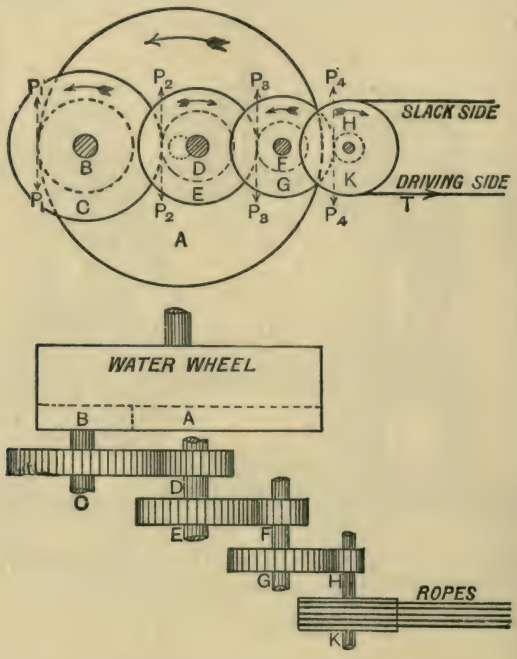
$\therefore$ $$P_2 = P_1 \times \frac{\text{radius of wheel B}}{\text{radius of pinion C}}.$$

Or, $$P_2 = P_1 \times \frac{\text{number of teeth on B}}{\text{number of teeth on C}}.$$

i.e., $$P_2 = 243 \times \frac{84}{20} = 1{,}019 \text{ lbs.}$$

From these results we notice that the wheels should be made stronger as they approach the barrel shaft.

EXAMPLE IV. — A water-wheel making two revolutions per minute is provided with an internal toothed wheel of 24 feet diameter at the pitch circle. This wheel gears with a train of wheels, thus :—Pinions of 8, 6, 4, and 2 feet diameter, and wheels of 12, 10, and 8 feet diameter. The last shaft in the train carries



TO ILLUSTRATE EXAMPLE IV.

POWER TRANSMITTED FROM A WATER-WHEEL BY TOOTHED-GEARING AND ROPES.

a rope pulley of 8 feet diameter. If 40 horse-power be transmitted, find the total driving tension in the ropes and the pressure between the teeth of each pair of wheels in the toothed gearing.

ANSWER.—The arrangement is shown by the accompanying sketch.

The value of the train is :—

$$e = \frac{24 \times 12 \times 10 \times 8}{8 \times 6 \times 4 \times 2} = 60.$$

$$\therefore \quad \left.\begin{matrix}\textit{Circumferential speed} \\ \textit{of rope pulley}\end{matrix}\right\} = V = \pi D N_l = \pi D N_1 e$$

$$\text{,,} \qquad = \frac{22}{7} \times 8 \times 2 \times 60 \text{ ft. per minute.}$$

Let T = Driving tension in ropes in lbs.

Then, $$\text{H. P.} = \frac{T V}{33{,}000},$$

$$\therefore \quad T = \frac{40 \times 33{,}000}{\frac{22}{7} \times 8 \times 2 \times 60} = 437{\cdot}5 \text{ lbs.}$$

Let P_1, P_2, &c., denote the tangential pressures at the pitch circles of the various pairs of wheels in gear.

Let r_a, r_b, &c., denote the radii of wheels, A, B, &c., respectively.

Then, since the power developed at pitch circle of wheel A, is equal to the power transmitted by ropes on pulley K, we get :—

$P_1 \times$ *Circumferential speed of* A $= T \times$ *Circumferential speed of* K.

$$\therefore \quad P_1 \times 2\pi r_a N_1 = T \times 2\pi r_k N_l .$$

$$\therefore \quad P_1 = T \times \frac{r_k}{r_a} \times e.$$

Or, $$P_1 = 437{\cdot}5 \times \frac{4}{12} \times 60 = 8{,}750 \text{ lbs.}$$

Next, $$P_2 \times r_c = P_1 \times r_b .$$

$$\therefore \quad P_2 = 8{,}750 \times \frac{4}{6} = 5{,}833{\cdot}\dot{3} \text{ lbs.}$$

But, $$P_3 \times r_e = P_2 \times r_d .$$

$$\therefore \quad P_3 = 5{,}833{\cdot}\dot{3} \times \frac{3}{5} = 3{,}500 \text{ lbs.}$$

Lastly, $$P_4 \times r_g = P_3 \times r_f .$$

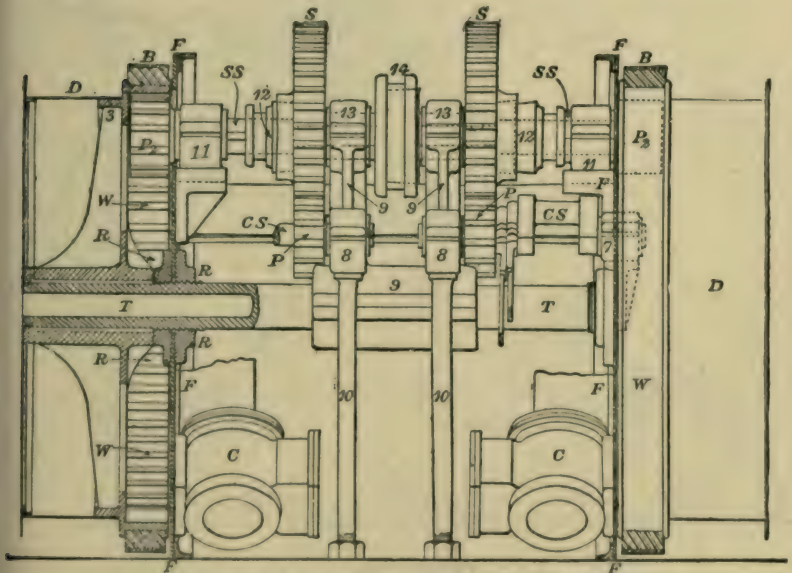
$$\therefore \quad P_4 = 3{,}500 \times \frac{2}{4} = 1{,}750 \text{ lbs.}$$

Cable Grappling, Picking-up and Paying-out Gear.—As another practical example of power transmitted by toothed gearing, we herewith give illustrations and description of one form of picking-up and paying-out gear, devised by the author for use on board submarine cable-repairing steamers. If the student refers back to the first folding page in Lecture VIII., Vol. I., he will observe the position of the "grappling and forward paying-out gear." He will also be able to trace the cable or grapnel rope's line of action—as the case may be—during its passage upwards from the sea over the bow sheaves and the forward-guide pulley, then under the dynamometer pulley and over its after-guide one to the drum of the picking-up or grappling gear, from which it goes to the forward deck or to the cable tank as indicated. Most of the smaller and newer cable repairing steamers, possess only one set of such gear. It serves the several purposes of paying out and picking up the grappling rope, deep buoy ropes, and the comparatively short lengths of cable, which have to be dealt with during these interesting and necessary operations. On the larger steamers which have to lay down long lengths of submarine cable, paying-out gear is also fitted on the after part of the ship, as it has been found more convenient in many ways to deal with the submersion of lengths of, say, over 20 or 30 nauts from the stern of the vessel; whereas, during repairs to a cable, the work of grappling, picking-up and paying-out, may be entirely confined to the forward part of the ship, and hence only one set of gear—such as we are about to describe—is necessary.

The stresses which have to be endured by grappling and picking-up gear, are often very sudden and severe. These are indicated by the dynamometer, as explained near the end of Lecture VIII., Vol. I. For example, the author, whilst engaged repairing a submarine cable off the Portuguese coast, which was laid at a depth of over 11,000 feet, observed, with 2,200 fathoms (13,200 feet) of grapnel rope out, that the mean working stress was 55 cwts. (6,160 lbs.). After hooking the cable, and whilst lifting the same 700 fathoms from the bottom, the stress rose at times to 90 cwts. (10,080 lbs.). Again, whilst grappling in shallow waters of 30 to 100 fathoms on a rocky bottom, the grapnel—if of the strong, long-pronged, unrelieving type—often caught on the rocks, and the stresses jumped very suddenly to the breaking point of the rope or the grapnel toes. The author has frequently experienced under such circumstances sudden jerks on the grapnel rope amounting to 15,000 lbs. and more, as well as found parts of the cable gear give way.

Seeing that such stresses act through the grapnel rope at the

circumference of the picking-up drum—which is seldom less than 6 feet in diameter—it is of the greatest importance to have the large main spur-wheel rigidly fixed to the drum, with its tooth pitch line of nearly the same radius. Also, to have as little "backlash" as possible between the teeth of the various wheels in gear. In the year 1902 the teeth of one of the pinions in the (old) gear of the cable ship "Newington" was completely stripped whilst working on rocky bottom in the West Indies. Such accidents cause great annoyance, delay, and expense.*

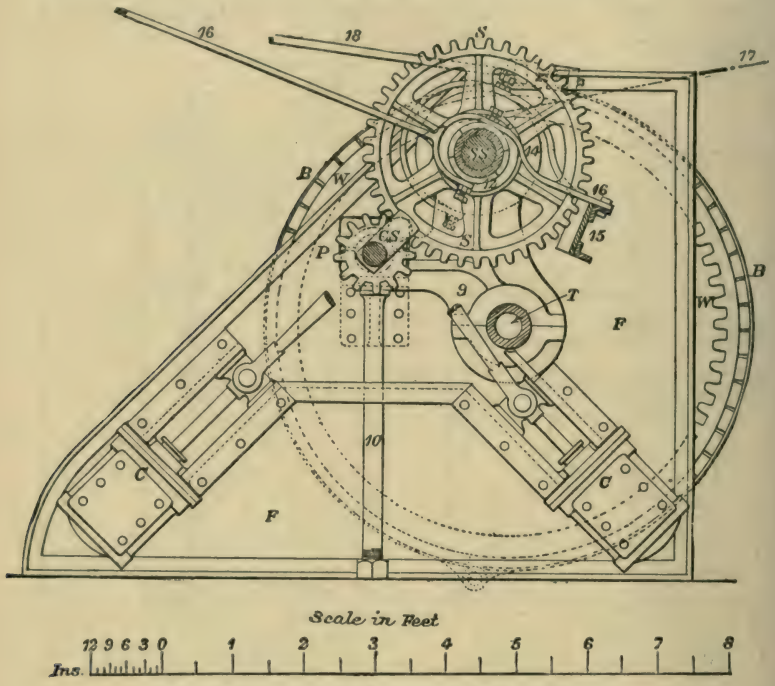


SIDE ELEVATION OF JAMIESON'S CABLE GEAR.
(With Engines for Picking-up and Paying-out at the same time.)

Details of Jamieson's Cable Gear.—Two steel plate frames, F, F, strengthened and stiffened by angle irons all round their edges, are bolted between the main and upper decks of the ship, in

* Strong steel helical teeth of the form described at the end of Lecture XIV. are sometimes used in modern up-to-date cable gears. Machine-cut gear is seldom employed. To prevent excessive stresses due to the grapnel toes getting jammed in rocks, the author devised and patented a self-relieving form of grapnel, as described in Munro and Jamieson's *Pocket-Book of Electrical Rules and Tables*, which was used with good effect on rocky bottoms. It cleared rocks, but caught and held the cable.

the forward position shown by the first folding plate in Lecture VIII., Vol. I., at the place indicated by the words engines and drum. A strong central hollow steel shaft or tube, T, is fixed to these frames by strong, screwed, round flanged nuts, R R, on each side of the frames as shown in section at the left-hand side of the side elevation. The tube, T, is in two parts connected together, and supported at its centre by a strong bracket and



End Elevation of Jamieson's Cable Gear.

bearing, 9, so that each half can revolve quite independently of the other half. On the two outer overhanging ends of T, there are fitted steel plate drums, D. Each drum, D, is rigidly connected to its own large annular spur-wheel, W, gearing with a pinion, P_2, as illustrated and described at the end of Lecture XII. The outer rims of the wheels, W, W, are accurately turned to receive the brake strap with its working faced blocks, B. The drums, D, D, with wheels, W, W, are there-

fore free to turn on the outside of the tubes, T, T, which extend beyond the frames, F, F, quite independently; or, they may be coupled together by the clutch, 14, to the secondary shafts, SS, so as to act together. This independent and collective action of the two drums, is of the greatest importance in cable work; since, when a cable is brought to the bows and cut, one end may be pulled on board and connected to the electrical testing room, whilst the other end is being slacked or eased out, and *vice versâ*, to lesson the stresses on either the port or the starboard cable.

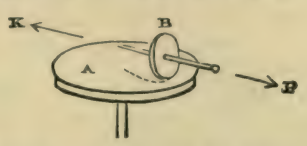
The power for turning the drums is obtained from two independent diagonal steam engines, with their cylinders as shown at C, C. The piston-rods act in the usual way through their connecting-rods and crank-shafts, CS, pinions, P, spur-wheels, S, and clutches, 12, attached to the second shafts, SS, pinions, P_2, and large spur-wheels, W, with drums, D. Bearings 7 and 11 carry the outer ends of the crank shafts, CS, and the second shaft, SS; whilst the inner ends of the crank-shafts are supported by bearings, 8, and pillars, 10, secured to the main deck. The inner ends of the shafts, SS, are supported by the bearings, 13, and bracket, 9. The lower part of this bracket engages with the tubular shafts, T, T, and its arms with the crank-shafts, CS, by the bearings, 8, 8, already mentioned. The whole of the bracket, 9, is supported by rigid connections to the decks (not shown). On the end elevation may be seen the lever, 16, for working one of the clutches, 12; whilst the centre line of the three or four turns of cable or rope on the drum, D, is indicated by line 17. The brake tightening or slackening (right- and left-handed) screw-shaft is marked 18. All starting, stopping, reversing, clutch, brake lever handles and wheels are led to one handy platform on the upper deck, where the engine cableman can easily manipulate them with a minimum of trouble, whilst keeping his eye upon the cable engineer as he stands on the bow-baulks and directs the various ticklish operations, of steaming slowly ahead or astern, picking-up or paying-out, easing or holding fast, &c., as the case may be.

By merely disengaging clutches, 12, any required length of cable may be paid out over the bows, with any desired stress, for any depth of water, by means of either the port or starboard drum, D, and its own brake without working its particular engine; whilst the other drum and engine may be kept in readiness for picking up the buoyed end to which the freshly laid cable is to be joined, when the ship comes up to the mark buoy.

The whole of the operations of making, laying, testing, fault-

finding and repairing submarine cables, forms one of the most interesting and scientific industries, in which Great Britain has hitherto taken the leading part.*

Changing Speeds by Friction Wheels. — A combination of friction-wheels in which the velocity ratio is changed by degrees during the motion, is shown by the accompanying figure. It consists of a circular disc, A, revolving round an



VARIABLE SPEEDS OF ROTATION BY FRICTION WHEELS.

axis perpendicular to its plane, and giving motion to a rolling wheel, B, fixed upon an axis which intersects the axis of the disc, A, at right angles. The angular velocity of the wheel, B, will continually decrease as it is shifted towards the centre of the disc, A. It will be apparent, without any proof, that the wheel, B, will change the direction of its rotation as soon as it has passed over the centre of the circular disc, A. When the circumference of B is at the centre of the driving plate or disc, A, we have the wheel, B, at a standstill. Friction gear is inefficient, and cannot be applied for large powers.

Changing Speeds by Toothed Wheels.—The ordinary method of producing definite changes of the angular velocity ratio of two rotating shafts is by means of change-wheels. There may be several pairs or trains of wheels, suited to a certain series of velocity ratios; and one or other of those pairs or trains of wheels is put into gear according to the comparative speed that is required at the time. The three classes of *Spur Gear*, viz. :— (*a*) spur gear with metal wheels, (*b*) spur gear with metal and raw hide, (*c*) spur gear with mortice-wheels—are all good. The speed, power and permissible noise usually determine which class should be adopted. A limit of about 1,000 feet per minute is allowable for metal spur gears, beyond which the noise becomes unpleasant in ordinary machine shops. At this speed, cut gears with well-formed teeth are necessary. Raw-hide and metal gears can easily be used up to 2,000 feet per minute.

* For a complete list of the present "Cable Laying and Repairing Fleet of the World," see the Sixteenth or later Editions of Munro & Jamieson's *Pocket-Book of Electrical Rules and Tables*, as well as detailed Specification of a Submarine Cable, and the various methods of testing and locating faults in cables.

Changing Speeds by Bands and Pulleys.—Belting is of course applicable to nearly all cases, the slipping being a positive advantage where heavy shocks and reversals of machines take place. The most convenient method of changing the velocity ratio of rotation of a pair of shafts, where absolute precision in the ratio is not required, is by means of "speed-cones." Where a perfectly smooth gradation is necessary, this can be obtained by using conoidal pulleys or reversed cones along with guiding pulleys to keep the belt in the desired position (as shown in Lecture XV. under "Link Chain Belting"). There are two forms of conoidal pulleys, one where the oblique edges of a section are parallel straight lines, and the other where the convexity of one section exactly fits into the concavity of the other. The rotation of the upper cone being uniform, it is evident that the rotation of the lower cone will decrease as the strap is shifted towards the right hand in this instance.

Changing Speeds with Electro-Motor Connections.—It is becoming a very common practice in workshops, to drive the machine tools in the various shops by means of motors. But certain classes of machines require variable speeds of rotation for the wide range of work to be performed by them. For example, in the case of cranes, portable drills, grinders, drilling machines and lathes, a variable speed motor which will increase its torque as the speed decreases, and *vice versâ*, may be connected to these machines. Variable speed motors should be shunt wound and the speed varied as far as possible by a rheostat in the shunt circuit. With all the resistance in, full speed will be got, and with all out the lowest speed. This is as it should be, for at low speeds most torque is wanted, and at high speeds less torque is required to get the same power.* Variable speed is not obtainable by alternating single phase or polyphase motors, hence most of the applications are made by continuous current motors and controllers. The application of electro-motors to all purposes, where belts and shafting are now used, is going on rapidly, and is at the present time a prominent section of electrical engineering.

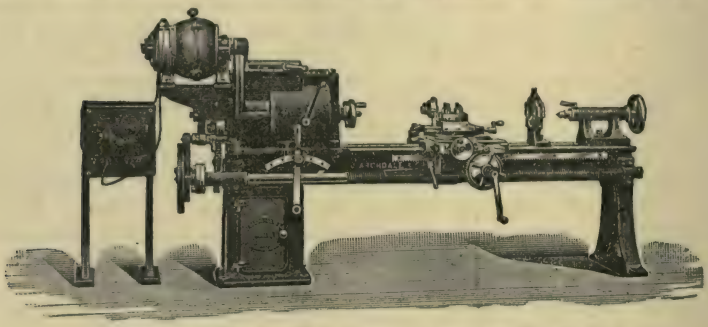
Hitherto, it has been customary and necessary for the most part, to bring the "job" which had to be machined to the tool. Now, however, by the aid of the flexibility of electrical transmission of power, in the case of large heavy "jobs," portable

* For a complete consideration of this method of changing the speed with an electric motor, students are referred to the *Transactions of the Institution of Engineers and Shipbuilders in Scotland*, vol. xlvi., Session 1902-1903, for a Paper by Mr. W. B. Sayers on "Speed Control of Electric Motors when driven from Constant-Pressure Mains."

electric motors driving suitable cutting tools, can be taken to and applied to the "job" in a most satisfactory manner.

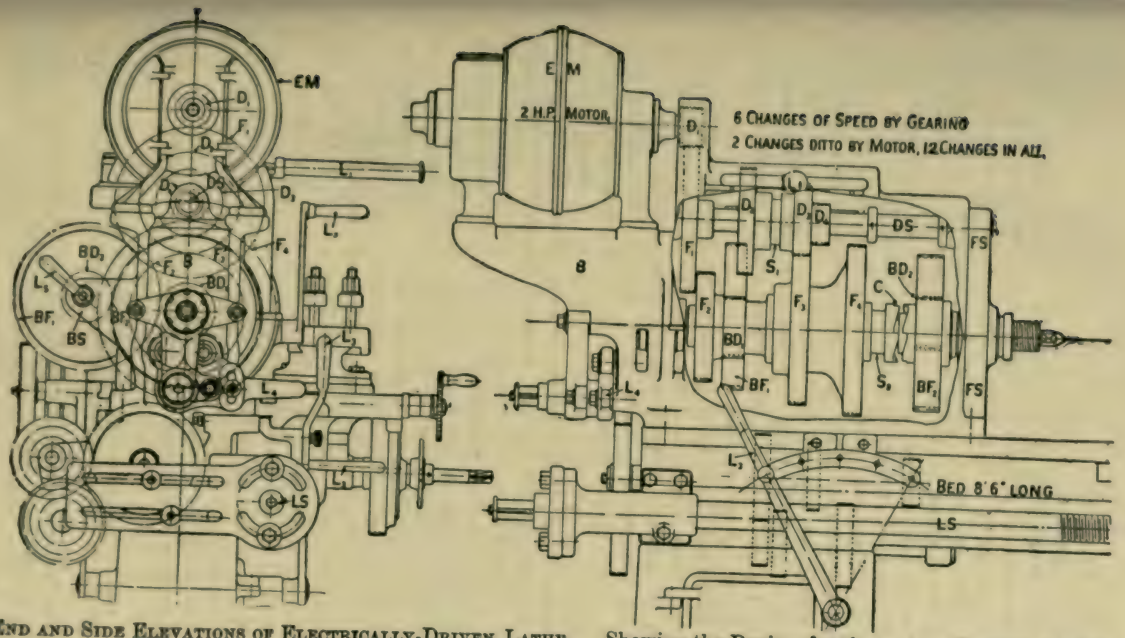
Mechanical Speed Gear combined with Variable Speed Electric Motors.—In order to avoid the necessity for a large motor, the speed variations might be obtained by means of step-cone pulleys, or spur-wheel speed-gear, combined with the means of a variable speed constant pressure motor. To illustrate this method of obtaining the required range of speed variations, three views are given of an electrically-driven lathe as made by Jas. Archdale & Co., Ltd., of Birmingham.

Electrically-Driven Lathe.—The first figure gives a general idea of the lathe, motor, and starting switch. The second figure



General View of Electrically-Driven Lathe.

is arranged to show the method adopted for obtaining the changes in speed. The lathe is driven by a 2-H.P. motor attached to a specially designed headstock. A bracket, B, is cast on one end of the fast headstock, F S, upon which the electro-motor, E M, is supported. A raw-hide pinion, D_1, is fixed to the motor spindle, and gears with the follower wheel, F_1, which is keyed to the driving shaft, D S. On the driving shaft is mounted a sliding sleeve, S_1, with three pinions or drivers, D_2, D_3, and D_4, of varying diameters. These pinions gear respectively with spur-wheels or followers, F_2, F_3, and F_4, on the lathe spindle. The sliding sleeve, S_1, is so moved by the hand lever, L_1, that three changes of speed can be obtained. Another lever, L_2, actuates a clutch, C, by which the back gear may be engaged or disengaged. By placing this lever, L_2, in its mid-position the lathe spindle is brought to rest. In this manner, six changes of speed can be got by the movement of these two levers, L_1 and L_2. In addition to which, the starting switch is arranged to admit of the speed of the electro-motor,



END AND SIDE ELEVATIONS OF ELECTRICALLY-DRIVEN LATHE. Showing the Devices for the various Changes of Speed.

INDEX TO PARTS.

E M for Electric motor.
B ,, Bracket for motor.
F S ,, Fast headstock.
D_1 D_2 D_3 D_4 ,, Driving pinions.
F_1 F_2 F_3 F_4 ,, Follower wheels.

D S for Driving shaft.
S_1 S_2 ,, Sliding sleeves.
C ,, Clutch.
B S ,, Back (gear) shaft.

$B D_1$ $B D_2$ for Back (gear) drivers.
$B F_1$ $B F_2$,, ,, ,, followers.
L_1 L_2 L_3 L_4 L_5 ,, Levers.
L S ,, Leading screw.

E M, being varied between 350 and 1,200 revolutions per minute. The regulating resistance or starting switch may be placed in any convenient position, whilst the motor, E M, is completely enclosed, thus protecting the armature, brushes, and winding from the entrance of chips, cuttings, filings, or dust. A gear-driven leading screw, L S, is fitted for self-acting, sliding, and surfacing. The speeds of L S can be changed by the movement of the lower front lever, L_3. The direction of rotation of the guide screw, L S, can be reversed by moving the handle, L_4, which is situated at the end of the fast headstock, F S. The lever, L_5, is used for putting the back gear shaft, B S, in or out of gear.* The advantages due to the control of the speed of individual machines has not yet been fully appreciated, for not only can better work be done, but there is also an increase in the quantity turned out by a lathe fitted with a separate motor and a shunt regulating resistance.

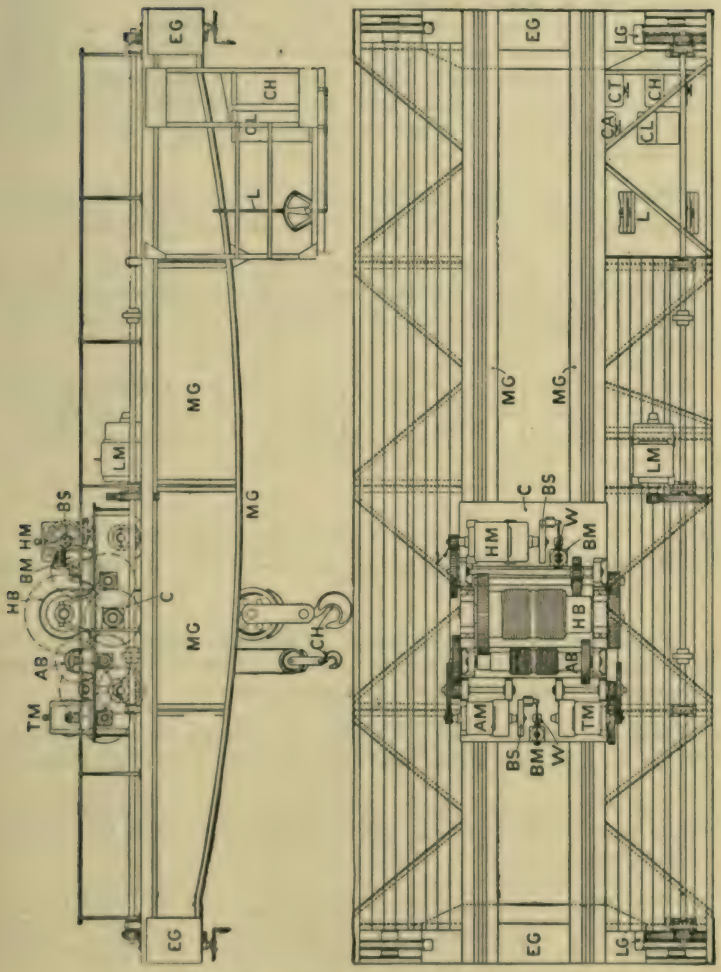
Electric Overhead Travelling Crane.—The accompanying side elevation and plan is an illustration of one of the most recent adaptations of electric transmission to engineering workshop cranes. The two main girders, M G, are of box section, and cover a span of 38 feet 3 inches. Their top flanges are stiffened in a horizontal plane, by angle bracing to a steel channel running parallel to the girders, which gives the requisite stiffness for resisting the lateral stresses, produced by the high acceleration of starting and stopping the travelling motion of the crane. The bracing also serves to support a wide platform on each side for access to the crab, motors, and gearing.

Crane Crab and its Motors.†—The crab, C, is built entirely of mild steel, and it is braced vertically as well as horizontally. Three series wound motors are mounted upon this crab.—(1) The motor, H M, for the main hoist, which is rated for 20 H.P. at 480 revs. per minute.—(2) The motor, A M, for the auxiliary 5-tons hoist is of 8 H.P. at 530 revs. per minute.—(3) The motor, T M, for traversing the crab of 8 H.P. at 530 revs. per minute. Current is conveyed to these motors and to the two brake-magnets, B M, by means of twelve bare trolley wires situated between the two main girders.

Main Hoisting Motor and its Gearing.†—The main hoisting

* Further information regarding the construction and action of a screw-cutting lathe will be found in the author's *Elementary Manual on Applied Mechanics.*

† All the motors are *series wound;* hence, increased speeds are obtainable when handling *light* loads, without any changes in the gear. The motors and controllers were made by the British Schuckert Electric Company, Limited. The pressure of the supply current is only 110 volts.



30-Tons Four-Motor Electric Overhead Crane.

(Designed and made by George Russell & Co., Ltd., Motherwell.)

motor, H M, is geared to the hoisting barrel, H B, through four reductions of spur gearing. These gears, as well as all the other wheels, are machine cut from solid blanks, with the exception of the last large one on the hoisting barrel shaft, which is of cast steel. The numbers of the teeth on the respective driving pinions and the follower wheels, as well as the value of the train, e, are as follows, starting from the first or the motor pinion :—

$$\frac{\text{Drivers}}{\text{Followers}} = \frac{15}{64} \times \frac{14}{57} \times \frac{14}{45} \times \frac{10}{41} = \frac{1}{229} = \frac{\text{Revs. of } \textit{last} \text{ wheel}}{\text{Revs. of } \textit{first} \text{ pinion}} = e.$$

The barrel is 2 feet in diameter (d_1), and since the load is lifted by four parts of a steel rope, which forms a double purchase in *duplicate*, the speed of hoisting with full load is as follows :—

$$\left.\begin{matrix}\text{Revs. per}\\ \text{min. of}\\ \text{motor}\end{matrix}\right\} \times \left\{\begin{matrix}\text{Value of}\\ \text{train}\end{matrix}\right\} \times \left\{\begin{matrix}\odot\text{ce of}\\ \text{drum, } d_1\end{matrix}\right\} \div \left\{\begin{matrix}\text{Velocity ratio}\\ \text{of the}\\ \text{rope purchase.}\end{matrix}\right.$$

$$480 \quad \times \quad \frac{1}{229} \quad \times \quad \pi d_1 \quad \div \quad 2$$

$$\text{Or, Speed of main lifting} = \frac{480}{229} \times \frac{3{\cdot}1416 \times 2}{2} = 6{\cdot}6 \text{ feet per min.}$$

Auxiliary Hoisting Motor and its Gearing.—The motor, A M, for the auxiliary hoist, drives through three reductions of spur gearing, having the following velocity ratio :—

$$\frac{\text{Teeth on drivers}}{\text{Teeth on followers}} = \frac{25}{135} \times \frac{21}{64} \times \frac{14}{45} = \frac{1}{53} = \frac{\text{Revs. of } \textit{last} \text{ wheel}}{\text{Revs. of } \textit{first} \text{ pinion}} = e.$$

The diameter (d_2) of the auxiliary barrel is 1 foot; hence, with the double purchase the hoisting speed with full load is the following :—

$$\left.\begin{matrix}\text{Revs. per}\\ \text{min. of}\\ \text{motor}\end{matrix}\right\} \times \left\{\begin{matrix}\text{Value of}\\ \text{train}\end{matrix}\right\} \times \left\{\begin{matrix}\odot\text{ce of}\\ \text{drum, } d_2\end{matrix}\right\} \div \left\{\begin{matrix}\text{Velocity ratio}\\ \text{of the}\\ \text{rope purchase.}\end{matrix}\right.$$

$$530 \quad \times \quad \frac{1}{53} \quad \times \quad \pi d_2 \quad \div \quad 2$$

$$\text{Or, Speed of auxiliary lifting} = \frac{530}{53} \times \frac{3{\cdot}1416 \times 1}{2} = 15{\cdot}7 \text{ ft. per min.}$$

Traverse Motor and its Gearing.—The crab rail wheels are 2 feet in diameter (d_3), and are driven through three reductions of machine-cut spur gear, which gives a traversing speed when the traverse motor, T M, is fully loaded, of :—

Revs. per min. of motor × Value of train × ⊙ce of rail wheels, d_3.

$$530 \times \frac{25}{135} \times \frac{21}{64} \times \frac{15}{42} \times \pi d_3$$

$$\text{Or, Speed of traversing} = \frac{530}{1} \times \frac{25}{135} \times \frac{21}{64} \times \frac{15}{42} \times \frac{3{\cdot}1416 \times 2}{1} = 73 \text{ ft. per min.}$$

Longitudinal Travel Motor and its Gearing.—The motor, L M, for the longitudinal travelling motion is rated for 20 H.P. at 480 revs. per minute. It is mounted at the centre of one of the girders, and is geared to the long shaft stretching across the crane, by means of which the rail wheels at both ends of the girders are driven.

The rail wheels, d_4, are 2 feet 9 inches in diameter. Consequently, the longitudinal travelling speed when the motor is fully loaded is :—

Revs. per min. of motor × Value of train × ⊙ce of rail wheels, d_4.

$$480 \times \frac{15}{64} \times \frac{18}{68} \times 3{\cdot}1416 \times 2{\cdot}75$$

Or, **Speed of longitudinal travel = 258 feet per min.**

Controllers.—The four controllers, C H, C A, C T, and C L, for the afore-mentioned main and auxiliary hoisting, traverse, and longitudinal motors respectively, are situated at the back of the craneman's cage, and are operated by the four vertical levers, L, which are conveniently placed near the front of the cage. The craneman has, therefore, an unobstructed view, and has no clutch gears or foot levers to attend to.

Lowering Loads.—This is governed entirely by means of the controllers, C H and C A, which, in lowering, short-circuit their motors through their regulating resistances. The motors, H M and A M, being rotated by the falling load, generate electric currents, which develop heat in the regulating resistances, and thus retard the falling load. The speed of lowering is easily controlled by varying the amount of resistance in circuit with the motors.

When the current is switched off, the load is held by a strap

brake, B S, applied by means of a weight, W. This brake is released by an electro-magnet, B M, whenever the motor is switched into circuit for hoisting or for lowering. In the event of a failure of the current through any cause, the electro-magnet, B M, allows the weight, W, to fall, so that the brake strap, B S, is instantly applied.

The travelling controllers, C T and C L, are also provided with rheostatic brakes, so that the crane can be quickly brought to a standstill in any desired position.

The crane hooks, C H, are of forged steel, and are carried upon ball bearings to facilitate turning the load.

LECTURE IX.—QUESTIONS.

1. Define the following terms as applied to gearing:—Pitch surface, pitch circle, and pitch point. Illustrate your answers by reference (1) to a spur wheel, (2) to a bevel wheel, and (3) to a rack.

2. Define angular velocity. Given the angular velocity of a body about a given axis, show how you would find the linear velocity of any point in the body.

3. Define the *pitch circle* of a toothed wheel. Prove that when two wheels, whose axes are parallel, gear together, their angular velocities are inversely as the diameters of their pitch circles. Two parallel shafts are at a distance of $4\frac{1}{2}$ feet, and they are to rotate with velocities as the numbers 7 and 11 respectively. Determine the diameters of the pitch circles of a pair of wheels which would give the required motion. *Ans.* $5\frac{1}{2}$ ft. and $3\frac{1}{2}$ ft.

4. Define the term *train of wheels*, and explain how to find the value of a given train. Arrange trains of wheels for the following values of e, no pinion to have less than 12 teeth, and no wheel to have more than 120:—
$\frac{2,000}{240}$, $\frac{1,200}{490}$, $\frac{880}{35}$, $\frac{35}{880}$, $\frac{490}{1,200}$, $\frac{240}{2,000}$.

5. What are mitre and idle wheels, and for what purposes are they used? Give instances of the use of both.

6. Explain, by aid of a sketch, the use of a Marlborough wheel. A shaft is divided into two parts, the parts being still in line. Sketch an arrangement of wheels whereby one part of the shaft may drive the other at twice the speed of the first.

7. Describe the operation of cutting a screw in a lathe, showing the wheels required, and how they are placed to cut a right-handed screw with 8 threads to the inch in a lathe whose leading screw is of $\frac{1}{4}$ inch pitch.

8. Explain the use of change wheels in a screw-cutting lathe. It is desired to cut a screw of $\frac{3}{8}$ inch pitch in a lathe with a leading screw of 4 threads to the inch, using 4 wheels. If both screws be right-handed, what wheels would you employ?

9. The leading screw in a self-acting lathe has a pitch of $\frac{1}{2}$ inch, show an arrangement of change wheels for cutting a screw of $\frac{3}{8}$ inch pitch.

10. You are required to cut a left-handed screw of 5 threads to the inch in a lathe fitted with a right-handed guide-screw of $\frac{1}{2}$ inch pitch. Show clearly by the aid of sketches the change-wheels which you would employ for the purpose, indicating how they would be respectively carried, and the number of teeth in each wheel.

11. The leading screw of a lathe is $\frac{1}{4}$ inch pitch and right-handed. The set of change wheels belonging to the lathe consists of the following:—20, 25, 30, 35, 40, 45, 50, 60, 70, 80, 90, 100, 110, and 120 teeth. From these devise suitable trains to cut the following screws and draw up a table of your results:—4, $4\frac{1}{2}$, 5, $5\frac{1}{2}$, 6, $6\frac{1}{2}$, 7, 8, 9, 10, 12, 14, and 16 threads per inch.

12. A wheel of 40 teeth is driven by a winch handle 14 inches long, and gears with a rack having teeth of 1 inch pitch; apply the principle of work to find the driving pressure exerted on the rack when a force of 50 lbs. is applied at the end of the winch handle. *Ans.* 110 lbs.

13. Sketch, in side elevation, the wheelwork of an ordinary 5-ton lifting crane. In doing so, it will be sufficient to represent the wheels by their pitch circles. If the weight raised moves through 1 inch when the driving

handle moves through 40 inches, find the weight which could be raised by 60 lbs. applied at the end of the lever handle. *Ans.* 2,400 lbs.

14. In a model of a lifting crab, the circumference of the circle described by the end of the winch handle is 43 inches, and the circumference of the drum which raises the weight is 14·9 inches. The wheelwork gives an advantage of 8 to 1, and it is found by trial that a force of 3·1 lbs. on the winch handle just suffices to raise a weight of 56 lbs. hanging on a cord wound upon the drum. What proportion of the power exerted is lost in this model? *Ans.* 21·76 per cent.

15. Sketch two views of a treble-purchase lifting crab. In doing so it will be sufficient to represent the wheels in side elevation by their pitch circles. Apply the Principle of Work, *or* the Principle of Moments to determine the force-ratio, P : W, and the pressure between the teeth of each pair of wheels in gear. *Ex.* In a treble-purchase lifting crab the handles are 16 inches long; diameter of drum 16 inches, thickness of rope 2½ inches. The wheelwork consists of the following:—Pinions 6, 6, and 8 inches diameter; wheels 24, 30, and 36 inches diameter. Supposing two men to work at each handle, each man exerting a force of 30 lbs., find the weight which could be raised, and the pressure between the teeth of each pair of wheels in gear. Allowing 30 per cent. for friction, find the actual weight which could be raised by the four men. *Ans.* (1) 8·34 tons nearly, 640 lbs., 2,560 lbs., 9,600 lbs.; (2) 5·84 tons nearly.

16. A water-wheel making two and a-half revolutions per minute is provided with an internal toothed wheel of 20 feet diameter at the pitch circle. This wheel gears with a train of wheels, thus:—Pinions of 6, 4, and 2 feet in diameter, and wheels of 10 and 9 feet in diameter respectively. On the last shaft a pulley 10 feet in diameter is keyed, on the rim of which there are four ropes. The horse-power transmitted by the ropes is 20. You are required to find the pull on each rope, the pressure between the teeth of each pair of wheels gearing together, and the revolutions per minute made by the pulley. *Ans.* 56 lbs., 4,200 lbs., 2,520 lbs., 1,120 lbs., 93·75 revolutions.

17. Describe, with the aid of sketches, the construction and use of the quadrant for carrying the change wheels of the screw cutting lathe,* and explain the manner in which change wheels are employed in cutting screw threads of different pitches in the same lathe. What are the sources of inaccuracy in the screw threads so produced? What number of wheels would you employ, how would they be arranged, and how many teeth would each wheel have in order to cut upon a bolt a left-handed screw thread of ⅝ inch pitch, on a lathe whose leading screw is right-handed and has two threads to the inch? In which direction would the saddle of the lathe travel when cutting a left-hand screw thread in the lathe above named? (S. & A. Adv. Exam., 1896.)

18. Make a sketch showing two pieces which move about fixed parallel axes in rubbing contact. Given the angular velocity of one of them, find that of the other, also the normal velocity and the velocity of sliding at the point of contact. (S. & A. Hons. Exam., 1897.)

* 19. Sketch and describe the mechanism by which the slide or saddle of a screw-cutting lathe can be put into gear with the leading screw, so as to traverse the slide automatically along the bed of the lathe, or be thrown out of gear so as to allow of the slide being traversed by hand. Describe how a screw-thread can be cut in a lathe, and determine the number of

* See Lecture XVI. Prof. Jamieson's *Elementary Manual on Applied Mechanics* for full description and figures of a screw cutting lathe.

teeth on the change wheels, and sketch their arrangement when cutting a screw-bolt with 10 threads to the inch on a lathe in which the leading screw is of ½-inch pitch. (S. and A. Adv., 1899.)

20. A hand crane has the following gear wheels:—On the drum axis a wheel of 56 teeth, on the intermediate axis a pinion of 18 teeth and a wheel of 64 teeth, on the handle-bar axis a pinion of 16 teeth: calculate—(*a*) The velocity ratio of this train of wheels. (*b*) The force at the handles to lift 30 cwts., given that the radius of the handle bars is 16 inches, and the effective diameter of the chain drum 9 inches, and that 32 per cent. of the power applied is wasted in friction. (C. & G., 1900, O., Sec. A.)

21. Two shafts are to be as nearly as possible 3 feet apart, and you wish to drive the one by the other through a pair of spur-wheels, the driven shaft to have twice the speed of the driver. The pitch of the teeth of the two wheels is to be 2 inches. What numbers of teeth would you employ in the two wheels, what would be the diameter of the pitch circles, and what would be the exact distance apart of the two shafts?

(C. & G., 1901, O., Sect. A.)

22. In a crane the chain barrel is driven by a 25-H.P. motor, on the spindle of which is keyed a spur-wheel of 14 teeth. This gears with a spur-wheel of 68 teeth keyed to the same spindle as a spur-wheel of 12 teeth. The last wheel gears with a spur-wheel of 50 teeth keyed to the same spindle as a wheel of 25 teeth, and the latter gears with a wheel of 54 teeth keyed to the chain barrel spindle. The chain barrel is 16½ inches in diameter. Find the revolutions per minute of the motor when 20 feet of chain per minute are wound on the chain barrel. If the load lifted, at this speed, be 10 tons, and the motor be developing its full power, find the efficiency. (C. & G., 1902, O., Sec. A.)

23. Design an overhead travelling crane worked by electric motors, the supply being continuous current at 200 volts. Clear span, 30 feet; maximum load, 5 tons. State the lifting speed and the travelling speeds you fix upon. You are not expected to design the motors, but you must specify the power each is to be if you use more than one, or the power of a single one if you prefer to do everything by one motor.

(C. & G., Hons., *Machine Design*, 1903.)

24. Referring to the Cable Gear in this Lecture, make an enlarged scale drawing and describe it, as well as the operations which the gear performs. If the diameter of each piston be 12 inches and its stroke 12 inches; if the ratio of the number of teeth in wheel, S, to pinion, P, be 3 to 1; and that of wheel, W, to pinion, P_2, be 6 to 1, whilst the radius of the drum is 3 feet, what is the maximum stress on the grappling rope which will just balance a pressure of 50 lbs. per square inch on the pistons: (*a*) if the port or starboard gear be worked singly; (*b*) when working in combination as explained in the text? Allow that 30 per cent. of the stress on the rope at sea level is absorbed by friction of the different pulleys and gear before it acts on the pistons and that the brakes are quite free.

25. Referring to the previous question, at what rate could you pick up the grapnel rope, if the maximum calculated stresses in it at sea level were ½ of values in cases (*a*) and (*b*)? Allow the whole gear up to the bows, to have 25 per cent. of the calculated maximum efficiency and torque of the previous case, wherein the mechanical advantage of the total pressure on one piston was taken at its maximum effort.

Lecture IX.—A.M.Inst.C.E. and B.Sc. Exam. Questions.

1. Sketch and describe a usual arrangement of change-wheels for a lathe, showing wheels arranged for cutting a left-handed screw of three threads to the inch when the leading screw has a pitch of ½ inch. (I.C.E., *Oct.*, 1897.)

2. The effective diameter of a worm is 6 inches ; the pitch of the thread of the worm is 2½ inches. The worm is secured on the shaft of an engine of 60 brake H.P. The worm gears with a spur wheel on a second shaft whose axis is at right angles to that of the engine shaft. If the coefficient of friction is 0·16, find the efficiency of the mechanism and the H.P. transmitted by the second shaft. (I.C.E., *Oct.*, 1900.)

3. Why is a lathe headstock often back-geared? If the two wheels have 63 and 63 teeth respectively, and each pinion has 25 teeth, find the reduction in the velocity ratio of the lathe spindle due to the gear. Sketch the arrangement. (I.C.E., *Oct.*, 1901.)

4. Explain the mode in which change wheels are employed in a screw-cutting lathe. The leading screw being ⅛ inch pitch, arrange, in a sketch, the change wheels as required for cutting a screw of 15 threads to the inch, marking the numbers on each wheel. (I.C.E., *Feb.*, 1903.)

5. A turbine making 1,080 revolutions per minute is required to be connected to a shaft by means of two pairs of spur-wheels of such diameters that the shaft may be driven at a speed of 90 revolutions per minute. The pitch of the teeth of the first pair of wheels is 1½ inches and that of the second pair 2 inches. Determine the number of teeth you would adopt on the wheels in order that the diameter of the largest wheel may be as small as possible, the least number of teeth on any wheel being 16. (I.C.E., *Oct.*, 1904.)

6. Sketch the quadrant plate of a lathe and the arrangement of wheels provided for putting the screw-cutting train in or out of gear without moving the quadrant plate, and for altering the train to cut a right- or a left-hand thread without putting an idle wheel into the train on the quadrant plate. Find a train of wheels for cutting eighteen threads per inch, right-hand, the pitch of the leading screw of the lathe being ¼ inch right-hand. (I.C.E., *Oct.*, 1906.)

7. Prove the conditions that must be satisfied by the teeth of wheels in gear so that the wheels may revolve at a constant velocity ratio. Show how these conditions are satisfied by the usua lmethods of designing wheel teeth. (B.Sc., Lond., *July*, 1907.)

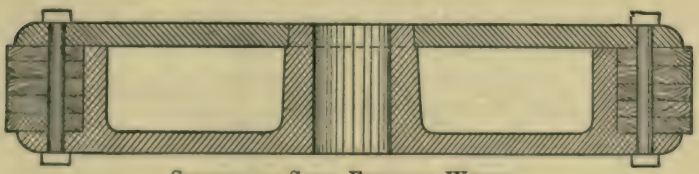
LECTURE X.

CONTENTS.—Friction Gearing—Power Transmitted by Ordinary Friction Gearing—Examples I. and II.—Robertson's Friction or Wedge Gearing—Power Transmitted by Wedge Gearing—Questions.

Friction Gearing.—Friction gearing is that form of gearing wherein the wheels in contact are driven, the one by the other, by reason of the friction between their pitch surfaces. The wheels require to be pressed together in a direction normal to their pitch surfaces at the line of contact, with a force sufficient to give a frictional resistance greater than the tangential resistance to motion. The wheels may be spur or bevel, according as the shafts are parallel or intersecting.

In order to insure sufficient frictional resistance and smooth working, it is usual to face one wheel of the pair with some compressible material, such as wood, leather, india-rubber, compressed paper, &c. When slipping takes place between the wheels, "flats" are soon formed on the face of the *follower* (this being the wheel which lags behind the other), while the face of the driver gets equally worn all round. For this reason the driver, and not the follower, is faced with the softer material.

The usual forms of rims suitable for spur and bevel friction wheels are shown by the accompanying figures.

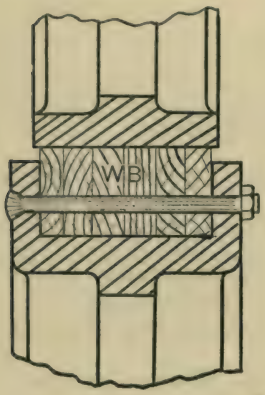


SECTION OF SPUR FRICTION WHEEL.

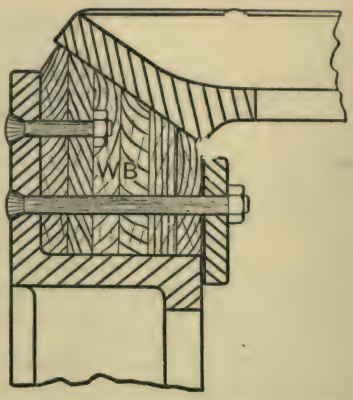
SECTION OF BEVEL FRICTION WHEEL.

The rim of the driver is faced with wood or leather, the different

layers of which are nailed or glued together and then held in position by bolts as shown. When wood is used the grain should lie in a direction tangential to the working surfaces, the wear being



SECTIONAL VIEW OF RIM FOR SPUR FRICTION WHEEL.



SECTIONAL VIEW OF RIM FOR BEVEL FRICTION WHEEL.

then more uniform all over. The rim of the follower is of cast iron turned in a lathe.

Friction gearing of this kind is more employed in America than in this country, being often applied for driving saw-mills, &c.

Power Transmitted by Ordinary Friction Gearing.—We shall now proceed to calculate the necessary pressure to be applied to the wheels in order to transmit a given power.

(1) *By Spur Friction Gearing.*

Let P = Pressure between wheels at pitch line in *pounds.*

„ T = Tangential resistance at pitch line in *pounds.*

„ V = Circumferential velocity of wheels in *feet per minute.*

„ μ = Coefficient of friction for surfaces in contact.

Then, *Work transmitted* = T V ft.-lbs. per minute.

$$\therefore \quad \text{H.P.} = \frac{\text{T V}}{33{,}000} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(I)}$$

If there is to be no slipping, we must have :—

$$T \leqq \mu P.$$

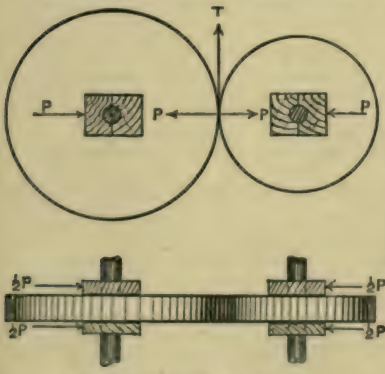
Or,
$$P \geqq \frac{T}{\mu}.$$

But,
$$T = \frac{\text{H.P.} \times 33{,}000}{V}.$$

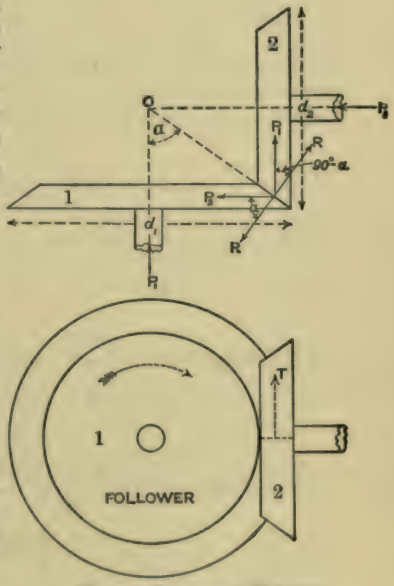
$\therefore$
$$P \geqq \frac{\text{H.P.} \times 33{,}000}{\mu V} \quad . \quad . \quad . \quad . \quad . \quad \text{(II)}$$

Equation (II) gives the least pressure between the wheels in order to transmit a given power.

The pressure, P, should always be greater than $\frac{\text{H.P.} \times 33{,}000}{\mu V}$ so as to provide against contingencies, such as oil or water getting on to the surfaces of the wheels. If P be less than this, then slipping must take place.



Spur Friction Gearing.



Bevel Friction Wheels.

(2) *By Bevel Friction Gearing.*

Let P_1, P_2 = Thrusts along shafts 1 and 2 respectively.
„ d_1, d_2 = Mean diameters of wheels 1 and 2 respectively.
„ R = Normal reaction between pitch cones.
„ T = Tangential resistance at pitch line.
„ 2α = Angle of pitch cone 1.

Then, $90° - \alpha$ = Half-angle of pitch cone 2.

Since the pressures along the axes of the shafts are P_1 and P_2, it follows that these are also the pressures at the pitch line, and act as indicated by the figure.

The normal pressure between the surfaces is :—

$$P_1 = R \cos (90° - \alpha) = R \sin \alpha.$$

Or, $$P_2 = R \cos \alpha.$$

But, $$\tan \alpha = \frac{\frac{1}{2} d_1}{\frac{1}{2} d_2} = \frac{d_1}{d_2}.$$

$$\therefore \quad \sin \alpha = \frac{\tan \alpha}{\sqrt{1 + \tan^2 \alpha}} = \frac{d_1}{\sqrt{d_1^2 + d_2^2}}.$$

And, $$\cos \alpha = \frac{1}{\sqrt{1 + \tan^2 \alpha}} = \frac{d_2}{\sqrt{d_1^2 + d_2^2}}.$$

$$\therefore \quad R = P_1 \frac{\sqrt{d_1^2 + d_2^2}}{d_1}.$$

Or, $$R = P_2 \frac{\sqrt{d_1^2 + d_2^2}}{d_2}.$$

But, $$T \leqq \mu R.$$

Or, $$R \geqq \frac{T}{\mu}.$$

And, as before, $$T = \frac{\text{H.P.} \times 33{,}000}{V}.$$

$$\therefore \quad R \geqq \frac{\text{H.P.} \times 33{,}000}{\mu V}.$$

Consequently, $$P_1 \geqq \frac{\text{H.P. } 33{,}000}{\mu V} \cdot \frac{d_1}{\sqrt{d_1^2 + d_2^2}} \quad \ldots \quad \text{(III)}$$

And, $$P_2 \geqq \frac{\text{H.P. } 33{,}000}{\mu V} \cdot \frac{d_2}{\sqrt{d_1^2 + d_2^2}} \quad \ldots \quad \text{(IV)}$$

Equations (III) and (IV) determine the least values of the axial thrusts in order to insure sufficient frictional resistance for transmitting a given power.

The following values of μ may be taken *:—

For metal on metal, . . .	$\mu = \cdot 15$ to $\cdot 20$.
For wood on metal, . . .	$\mu = \cdot 25$ to $\cdot 30$.
For millboard on metal, . . .	$\mu = \cdot 20$.

* Unwin's *Machine Design*, part I., p. 283 (1892).

Taking the greatest and least of the above values for μ, we see that in spur friction gearing, the smallest value of P must lie between $\frac{T}{\cdot 3}$ and $\frac{T}{\cdot 15}$, *i.e.*, between $3\frac{1}{3}$ T and $6\frac{2}{3}$ T.

In practice, the width of the face of friction wheels is about the same as that of a single leather belt which is required to transmit the same power. The tangential force may be taken at from 15 to 30 lbs. per inch of width when the face of the driver is lined with wood. In the case of a wheel with millboard face, the tangential force transmitted was observed, by Prof. Unwin, to be as great as 80 lbs. per inch of width.*

It will be apparent that friction gearing of the above kind is unsuited for transmitting great power. The constancy of velocity-ratio between the wheels cannot be relied upon. Gearing of this kind is only used (1) when the power to be transmitted is small; (2) when the speed is so high that toothed gearing would be noisy; (3) when the wheels require to be frequently put into or out of gear.†

EXAMPLE I.—In a spur friction gearing the driving wheel is faced with wood, and gears with a metal wheel $3\frac{1}{2}$ feet in diameter. The latter makes 200 revolutions per minute, and transmits 10 H.P. Find the tangential resistance at the circumferences of the wheels, and the necessary thrust to be applied to the bearings of the shafts, taking $\mu = \cdot 25$.

ANSWER.—Using the same notation as in the text, we have, H.P. = 10; $V = \pi d n = \frac{22}{7} \times 3\frac{1}{2} \times 200 = 2{,}200$ ft. per minute.

$$\therefore \quad T = \frac{\text{H.P.} \times 33{,}000}{V} = \frac{10 \times 33{,}000}{2{,}200} = 150 \text{ lbs.}$$

$$\text{Also,} \quad P = \frac{T}{\mu} = \frac{150}{\cdot 25} = 600 \text{ lbs.}$$

EXAMPLE II.—5 H.P. has to be transmitted through a pair of bevel friction wheels. The diameters of the wheels are 2 feet and $1\frac{1}{2}$ feet respectively. The circumferential speed of the wheels is 1,000 feet per minute. Find the normal and tangential pressures at the surface of contact of the two wheels, and the axial thrusts to be applied to each shaft, taking $\mu = \cdot 2$.

ANSWER.—Here $d_1 = 2$ ft., $d_2 = 1\frac{1}{2}$ ft., V = 1,000 ft. per min.

* Unwin's *Machine Design*. † *Ibid.*

Then :—

$$\text{Normal pressure} = R = \frac{\text{H.P.} \times 33{,}000}{\mu V},$$

$$\text{"} \qquad \text{"} = \frac{5 \times 33{,}000}{\cdot 2 \times 1{,}000} = 825 \text{ lbs.}$$

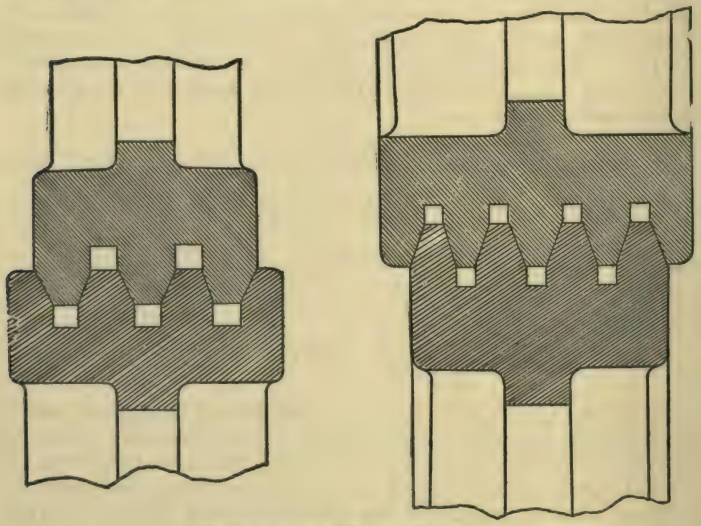
$$\text{Tangenital pressure} = T = \mu R = \cdot 2 \times 825 = 165 \text{ lbs.}$$

$$\text{Also,} \qquad P_1 = R \frac{d_1}{\sqrt{d_1^2 + d_2^2}},$$

$$\therefore \qquad P_1 = \frac{825 \times 2}{\sqrt{2^2 + (1\frac{1}{2})^2}} = 660 \text{ lbs.}$$

$$\text{Similarly,} \qquad P_2 = \frac{825 \times 1\frac{1}{2}}{\sqrt{2^2 + (1\frac{1}{2})^2}} = 495 \text{ lbs.}$$

Robertson's Friction or Wedge Gearing.—One objection to the friction gearing just described is the great pressure which is brought on the bearings, due to the force with which the wheels



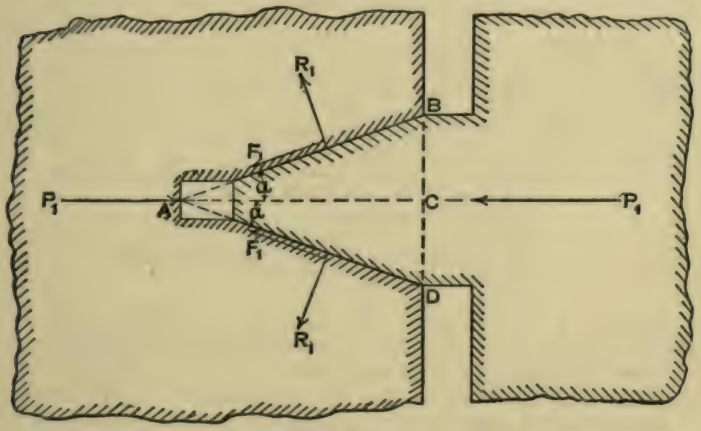
ROBERTSON'S WEDGE GEARING.

require to be pressed together in order to secure sufficient frictional resistance at their surfaces. To overcome this and the previous objections to the ordinary friction gearing, the wheels are now

made of cast iron with parallel wedge-shaped projections round their rims. The projections of the one wheel fit into the wedge-shaped grooves on the other; and by this means the friction is greatly increased.

Sections of the rims of such wheels are shown by the accompanying figures, and need no further explanation.

There is, however, one serious objection to such wheels—viz., the grinding action, and consequent excessive wear, while working. This is due to the sliding contact between the sides of the projections and those of the grooves. On this account the wheels sometimes work with great noise. This difficulty can be overcom



WEDGE GEARING.

to a certain extent by making the depth of the surfaces in contact as small as possible.

The depth of the acting surface (*i.e.*, the distance which the wheels penetrate each other when in gear) is given by the formula :—

$$t = 0{\cdot}025 \sqrt{T}.*$$

Where t is the depth of acting surface, and T the tangential resistance between the wheels.

Power Transmitted by Wedge Gearing.—Consider the action of one of the wedge-shaped projections in its groove. When the wheels rotate, the action is similar to that of a wedge thrust into the groove by a horizontal force.

* Unwin's *Machine Design*, part I.

Let P_1 = Horizontal force on projection considered.
„ R_1 = Total normal reaction on each side of groove.
„ F_1 = Total friction between projection and each side of groove.
„ 2α = Angle of wedge.

Then, $$P_1 = 2R_1 \sin\alpha + 2F_1 \cos\alpha.$$
But, $$F_1 = \mu R_1.$$
$\therefore$ $$P_1 = 2R_1(\sin\alpha + \mu\cos\alpha).$$

Then, for all the wedges, if P denote the total force pushing the wheels together, and R the total reaction, $P = \Sigma P_1$, $R = \Sigma . 2R_1$, and we get :—

$$P = R(\sin\alpha + \mu\cos\alpha).$$

$\therefore$ as before, $$T \lessgtr \mu R.$$

Or, $$R \gtrless \frac{T}{\mu}.$$

But, $$T = \frac{\text{H.P.} \times 33{,}000}{V}.$$

$\therefore$ $$R \gtrless \frac{\text{H.P.} \times 33{,}000}{\mu V}.$$

$\therefore$ $$P \gtrless \frac{\text{H.P.} \times 33{,}000}{V} \cdot \frac{\sin\alpha + \mu\cos\alpha}{\mu} \quad . \quad (V)$$

Equation (V) determines the least pressure with which the wheels must be forced together in order to transmit a given power.

From this equation it is seen that the number of grooves or projections has no effect on the power transmitted. The number of grooves may be anything we please, but generally there are no fewer than two nor more than ten. The pitch of the grooves may vary from $\frac{1}{8}$ inch to $1\frac{3}{4}$ inches.

Usually the groove angle 2α is 40°. Hence, taking $\mu = \cdot 15$, we get :—

$$P \gtrless T \frac{\sin\alpha + \mu\cos\alpha}{\mu},$$

$$\text{„} \gtrless T \frac{\sin 20° + \cdot 15 \cos 20°}{\cdot 15},$$

$$\text{„} \gtrless T \frac{\cdot 342 + \cdot 15 \times \cdot 94}{\cdot 15},$$

$$\text{„} \gtrless 3 \cdot 22\, T.$$

i.e., P must be at least $3\frac{1}{4}$ times T. In practice we may take $P = 4T$.

Lecture X.—Questions.

1. Define friction gearing. State the advantages and disadvantages of such a gearing, and mention under what circumstances it is likely to be employed in preference to other kinds of gearing.

2. Describe, with sketches, the construction of the wheels used for friction gearing. State your reasons why, in ordinary friction gearing, one of the wheels only is faced with a softer material than the other, and say which wheel it is.

3. In a spur friction gearing, the driving wheel is faced with wood and gears with a metal wheel of $2\frac{1}{2}$ feet in diameter. The circumferential speed is 2,000 feet per minute. The force pressing the wheels together is 550 lbs. Taking $\mu = \frac{1}{4}$, find the maximum H.P. which can be transmitted. Sketch the arrangement, showing a method of engaging and disengaging the wheels. *Ans.* 11·1 H.P.

4. The diameters of a pair of bevel friction wheels are $1\frac{1}{2}$ feet and 4 feet respectively. The larger wheel makes 200 revolutions per minute, and transmits 10 H.P. Find the normal and tangential pressures at the pitch surfaces, and the axial thrusts on each shaft, $\mu = \cdot 25$. Prove the formula which you employ. *Ans.* 525 lbs.; 131·25 lbs.; 184 lbs.; 491 lbs.

5. Describe, with sketches, Robertson's wedge gearing, and deduce a formula for the pressure between the wheels for a given H.P.

6. A cone clutch has an angle of 50° and a mean diameter of 10 inches. Find the thrust on the sliding part parallel to the shaft when the clutch transmits 2 horse-power at 120 revolutions per minute, if the coefficient of friction between the two parts of the clutch is 0·35. (B. of E., S. 3, 1906.)

LECTURE XI.

CONTENTS.—Constancy of the Velocity-Ratio of Toothed Gearing—Proportions of Teeth of Wheels—Clearance—Arc of Action—Relation between Length of Arc of Action and Pitch of Teeth—Clock and Watch Wheels—Primary Conditions for Correct Working of Toothed Wheels—Curves which satisfy the above Conditions—Particular Cases—(I.) When the Cycloid is a Straight Line—(II.) When the Epicycloid is the Involute of the Base Circle—(III.) When the Hypocycloid is a Straight Line—(IV.) When the Hypocycloid is a Point—Cycloidal Teeth—Gee's Patent Toothed Gearing—Exact Method of Drawing the Curves for Cycloidal Teeth—Practical Method of Drawing the Curves for Cycloidal Teeth—Application of Preceding Principles to the Case of a Rack and its Pinion—Particular Forms of Teeth as Dependent upon Changes in the Sizes of the Generating Circles Employed—First Particular Case—When the Hypocycloid is a Straight Line—Rack having Teeth with Radial Flanks—Practical Method of Drawing the Involute Curves for the Faces of the Teeth on the Pinion—Second Particular Case—When the Hypocycloid is a Point—Pin Wheels—Pins are always placed on the Follower—Rack and Pinion—Disadvantage of Pin Wheels—Questions.

Constancy of the Velocity-Ratio of Toothed Gearing.*—In nearly every case of the transmission of motion by friction or belt gearing, slipping takes place to a greater or less extent, and hence these methods of transmitting motion are unsuited where an exact or constant velocity-ratio is desired. In such a case it is best to employ toothed gearing. But, to insure a constant velocity-ratio and smooth working with toothed gearing, the teeth of the wheels must be carefully constructed, and of such shapes that, when gearing together, certain geometrical conditions are fulfilled. In this Lecture we shall endeavour to explain the principles according to which all properly constructed teeth of wheels are made and act.

In the first place, we shall give some further definitions and general explanations relating to toothed gearing.

DEFINITION.—**The pitch of the teeth is the distance from the centre of one tooth to the centre of the next tooth, measured along the pitch line.**

* The student may refer to the following books on gearing:—

Practical Treatise on Gearing, by Browne & Sharpe, printed by J. W. Pratt & Son, New York.

Odontics, by Geo. B. Grant, published by the Lexington Gear Works, Lexington, Mass.

Handbook on the Teeth of Gears, by Geo. B. Grant, Boston, Mass.

Bevel-Gears, by John W. Newall & Co, Manchester.

A paper on "Setting out the Curves of Wheel Teeth," by W. J. Last, in *Proc. Inst. Civil Engineers*, vol. lxxxix., p. 335.

Elements of Machine Design, by Prof. Unwin, published by Longmans, Green & Co.

Let d = Diameter of the pitch circle *in inches*.
„ p = Pitch of teeth *in inches*.
„ n = Number of teeth on wheel.

Then, $$n\,p = \pi\,d$$
Or, $$p = \frac{\pi\,d}{n}$$
And, $$d = \frac{n\,p}{\pi}$$ . . . . . (I)

The pitch, as measured in this way, is called the **Circular Pitch**, and is the one chiefly used by engineers. But another and more convenient method of measuring the pitch is sometimes adopted, and is called the **Diametral Pitch.** According to this method the pitch of the teeth is stated as a fraction of the diameter of the pitch circle.

Thus, if the diameter of the pitch circle be 40 inches, and the number of teeth on the circumference be 120, then :—

$$\textit{Diametral Pitch} = \frac{40}{120} = \tfrac{1}{3} \text{ inch.}$$

In this case, the size of the wheel would be spoken of as one of 3 teeth per inch of pitch circle diameter.

Let p_d = Diametral pitch of teeth *in inches*.

Then, using the same notation as in the case of circular pitch:—

$$p_d = \frac{d}{n}$$
Or, $$d = n\,p_d$$ . . . . . (II)

The student will notice that equations (II) are much simpler than equations (I), from the fact, that it is easier to treat of the sub-division of a straight line than that of a circle. It is for this reason, that several engineers, especially American, advocate the adoption of this method as being more convenient for stating the sizes of wheels. In this country the circumferential pitch is the one chiefly used, except in the case of small wheels (such as the change wheels for a lathe) whose sizes are often stated in terms of their number of teeth per inch in diameter.

The relation between the circular and diametral pitches can be stated thus :—

From (I), $$d = \frac{n\,p}{\pi},$$
„ (II), $$d = n\,p_d,$$
$\therefore$ $$p_d = \frac{p}{\pi}.$$
Or, $$p_d : p = 1 : \pi.$$ . . . . . (III)

In this book the term *pitch* must always be understood as meaning the *circular pitch*, unless otherwise expressly stated.

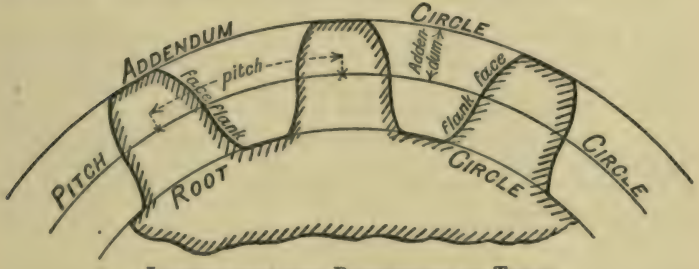
Definitions.—The Face of a tooth is that part of its acting surface which lies between the pitch surface and the crest of the tooth.

The Flank of a tooth is that part of the acting surface which lies between the pitch surface and the bottom of the spaces between the teeth.

The Addendum Circle is that imaginary circle which touches the crests of all the teeth on the wheel.

The Root Circle is that imaginary circle which touches the bottoms of the spaces between the teeth.

The Addendum of a tooth is the length of the tooth projecting beyond the pitch surface; or, it is that part of the tooth lying between the pitch surface and the addendum surface.



Illustrating the Definitions of Terms.

The above terms will be understood from the accompanying figure.

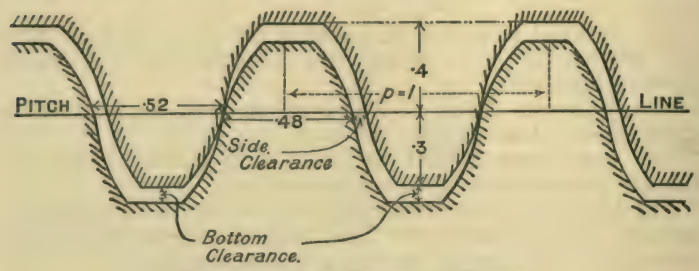
Proportions of Teeth of Wheels.—The proportions for the teeth of wheels vary slightly with different makers, but the following rules, as given by Prof. Unwin, represent good average practice :—

Let p = *Pitch of teeth in inches.*

Then,	*Height of tooth above pitch line,* .	= ·3 p,
	Depth of tooth below pitch line, .	= ·4 p,
	Thickness of tooth (measured along pitch line), ...	= ·48 p,
	Width of space between the teeth, .	= ·52 p,
	Width of face of tooth, . . .	= 2 p to 3 p.

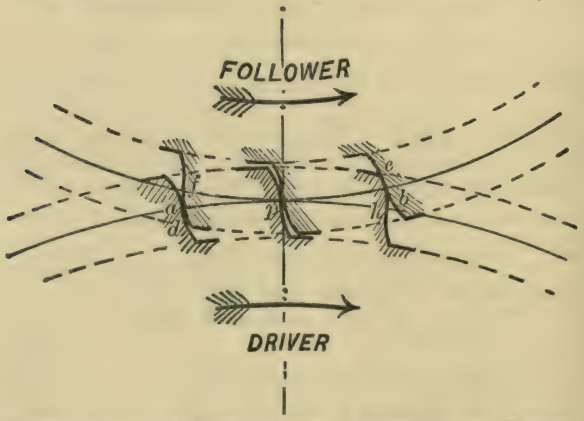
Clearance.—These proportions, as shown in the following figure, give a *side clearance* of (·52 − ·48) p = ·04 p, and a *bottom clearance* of (·4 − ·3) p = ·1 p.

Arc of Action.—Consider the action of a pair of teeth gearing together, from the instant at which contact begins to the instant at which contact terminates. During the first part of the action the *flank* of the tooth on the *driver* is in contact with the *face*



PROPORTIONS OF TEETH OF WHEELS.

of the tooth on the *follower*. This continues until the pitch point is reached, at which instant the line of contact of the teeth coincides with the line of contact of the pitch surfaces of the wheels. After passing the pitch point, *p*, the *face* of the tooth on the *driver* continues in contact with the *flank* of the tooth on the *follower*, until contact ceases. Action *begins* at *a*, the point of the tooth on the *follower*, and *terminates* at *b*, the point of the tooth on the *driver*.



ILLUSTRATING THE TERMS ARCS OF APPROACH AND RECESS.

Arc of Approach = Arc *a p*, or Arc *f p*.
Arc of Recess = Arc *p h*, or Arc *p b*.

DEFINITION.—**The arc of either of the pitch circles over which there is contact between a pair of teeth is called the Arc of Action.**

Thus, either arc *a p h*, or arc *f p b*, is called the arc of action. The arc of action is divided at the pitch point into two parts, called respectively the **Arc of Approach** and the **Arc of Recess.** By an inspection of the above figure it will be seen that the length of the arc of approach depends on the *addendum* of the teeth on the *follower;* whilst the length of the arc of recess depends on the *addendum* of the teeth on the *driver.* To increase or diminish the arc of contact, the *addendum* of the teeth must be increased or diminished, as will be shown further on.

Relation between Length of Arc of Action and Pitch of Teeth.—To insure continuous action between a pair of toothed wheels, there must, at any instant, be at least one pair of teeth in gear. Moreover, contact between one pair of teeth must not terminate before the succeeding pair comes into operation. This condition is insured by making the arc of action greater than the pitch of the teeth. In most cases, especially with heavy gearing, two or three teeth are in gear at once. *Hence, the usual rule is to make the arc of action from three to four times the pitch of the teeth.*

Clock and Watch Wheels.—In wheelwork, such as in clocks or watches, where friction is most injurious, the teeth of the wheels are usually so designed that the driving teeth have no flanks, and the driven teeth no faces. Contact, in such cases, occurs during the period of recess only, and then the arc of recess must be at least equal to the pitch. The reason for this is, that the friction, due to the sliding of the teeth on each other during action, is said to be greater during the period of approach than that during the period of recess.*

Primary Conditions for Correct Working of Toothed Wheels.—Having explained some general principles relating to toothed gearing, we shall now proceed to consider the necessary conditions to be fulfilled in order that such gearing may work correctly.

The two necessary conditions are :—

I. The radii of the pitch surfaces must be such that by rolling together they give the desired velocity-ratio.

Let R_A, R_B = Radii of pitch circles of wheels A and B.
„ ω_A, ω_B = Angular velocities „ „

Then, as a first condition we must have :—

$$R_A : R_B = \omega_B : \omega_A.$$

* Friction between the teeth of wheels will be considered in Lecture XIV.

Or, the radii of the pitch circles must be inversely as the angular velocities of the wheels.

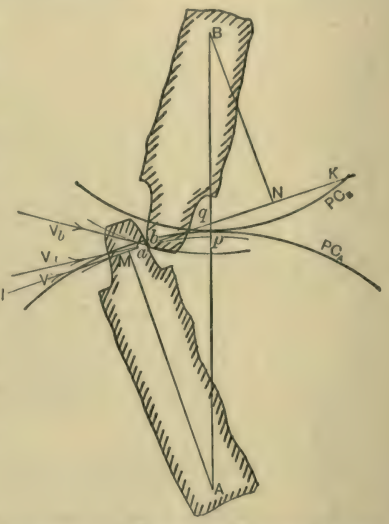
II. **The shape of the teeth of the wheels must be such, that the motion resulting from their mutual action shall be the same as that obtained by the rolling action of the pitch surfaces.**

Let A and B be the centres of the pitch circles of two wheels working together. A B the *line of centres*, and p the pitch point.

For clearness we have represented only one tooth on each wheel. The teeth are in contact at the point $a\,b$; a being the point of contact on tooth A, and b the point of contact on tooth B.

Let H K, the common normal to the curves of the teeth at their point of contact, intersect the line of centres at q. From A and B draw the perpendiculars, A M and B N, upon H K.

With centres A and B draw the circles passing through the point $a\,b$. Then, at any instant the point a is moving along the tangent to the circle passing through a, and having its centre at A; *i.e.*, the point a is moving in a direction at right angles to A a with a velocity $v_a = \omega_A \times A\,a$. Similarly, the point b is moving in a direction at right angles to B b with a velocity $v_b = \omega_B \times B\,b$. But though the points a and b are thus moving in different directions and with different velocities, yet their *component velocities along the common normal*, H K, must be equal, otherwise the teeth would either separate from or penetrate each other. For an indefinitely small movement of the wheels the only relative motion of a and b is in a direction perpendicular to H K.



Illustrating Primary Conditions for Correct Working of Wheel Teeth.

Let v denote the equal component velocities of v_a and v_b along H K. Then, with reference to wheel A:—

$$v = \omega_A \times A\,M$$

Similarly, $$v = \omega_B \times B\,N$$

$$\therefore \quad \omega_A \times A\,M = \omega_B \times B\,N,$$

$$\therefore \quad \omega_A : \omega_B = B\,N : A\,M,$$

i.e., $$R_B : R_A = B\,N : A\,M,$$

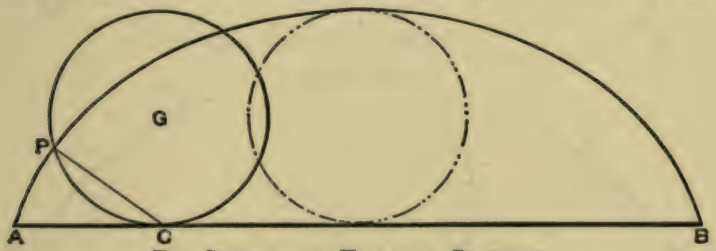
Or, $$B\,p : A\,p = B\,q : A\,q \quad \text{[By similar triangles]}$$

This can only be true when q coincides with p. Hence, the condition to be fulfilled by the curves forming the teeth of wheels is :—

The common normal to the curves at the point of contact of a pair of teeth must always pass through the pitch point.

Curves which Satisfy the above Condition.—It now remains to describe curves which shall fulfil this condition. The problem of determining the proper shape of teeth admits of many solutions. Any shape can be given to the teeth of one of a pair of wheels gearing together, so long as a corresponding shape be given to the teeth of the other wheel, to fulfil the above condition.

Two principal curves have been used by engineers for describing the teeth of wheels. These are the **cycloid** and the **involute.*** We shall now explain the nature of these curves, and then show that



THE CYCLOID AND HOW IT IS DESCRIBED.

teeth formed according to them satisfy the above condition for correct working.

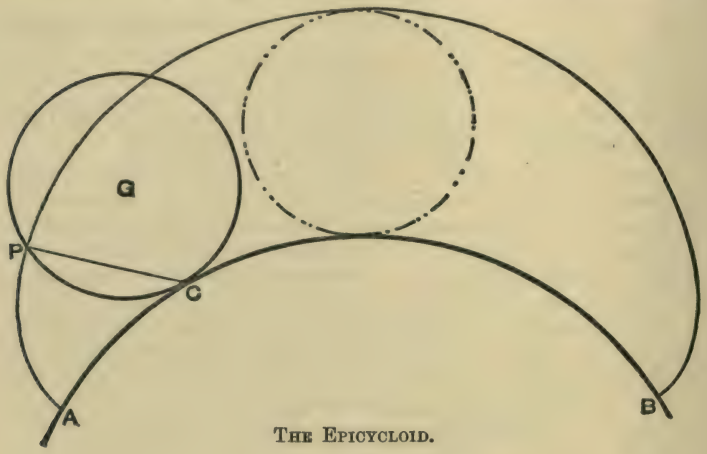
DEFINITION.—**A cycloid is a curve traced out by a point on the circumference of a circle which rolls along a straight line.**

The form of this curve will be understood from the accompanying figure. The rolling circle, G, is called the *generating circle.* The point, P, which traces out the curve is called the *tracing point.* The line, A B, on which the circle rolls is called the *base line.*

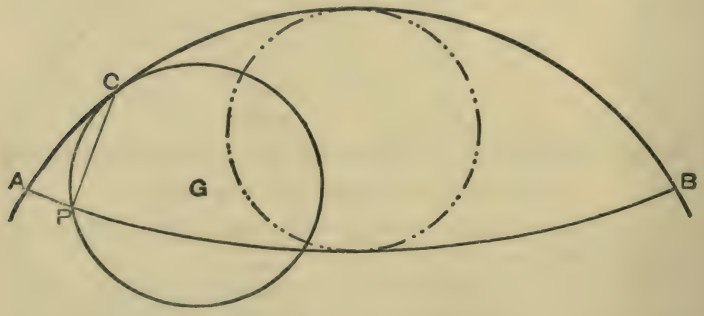
* *Cycloid* is derived from the Greek word κύκλος, signifying a *ring* or a *circle*; *Epicycloid* from ἐπί, signifying *upon* and κύκλος; and *Hypocycloid* from ὑπό, signifying *under* and κυκλος. *Involute* is derived from the Latin words *in*, signifying *upon*, and *volvo*, to *roll.*

The length of A B is equal to the circumference of the generating circle. For any position, G, of the generating circle, we have A C = arc P C of the circle.

When the base line is a *circle* the curve traced out by the tracing point is called an **Epicycloid** or a **Hypocycloid** according as



THE EPICYCLOID.



THE HYPOCYCLOID.

A C B is the Base circle, P the Tracing point, G the Generating circle.

the generating circle rolls on the *convex* or *concave* side of the base circle. We may define these curves separately, as follows :—

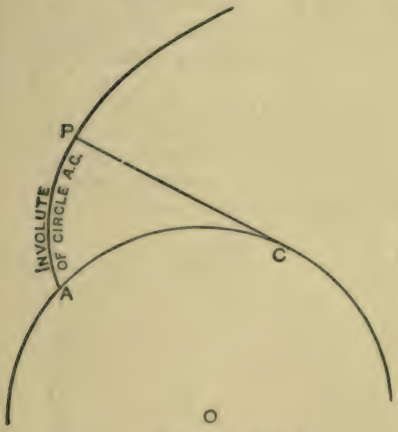
DEFINITION.—**An Epicycloid is a curve traced out by a point on the circumference of a circle which rolls on the Convex arc of another circle.**

DEFINITION.—A **Hypocycloid** is a curve traced out by a point on the circumference of a circle which rolls on the **Concave** arc of another circle.

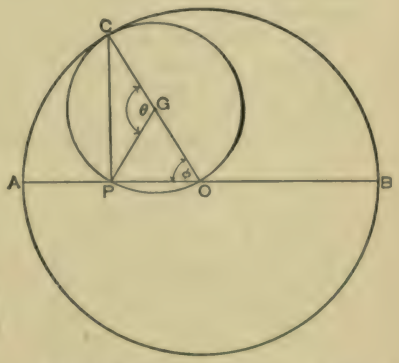
The forms of these curves will be understood from the accompanying figures.

Particular Cases—I. When the Cycloid is a Straight Line.—Suppose the radius of the generating circle to become infinitely great; then, clearly, the *cycloid* would become a *straight line perpendicular to the base line*, A B. An application of this is to be found in the case of racks having teeth with straight flanks.

II. When the Epicycloid is the Involute of the Base Circle.—If the radius of the generating circle of the *epicycloid* be infinitely great, then the arc, P C, becomes a straight line, and the epicycloid,



INVOLUTE CURVE.



HYPOCYCLOID A STRAIGHT LINE.

A C the Base circle, P the Tracing point.

A P, is now termed the **Involute of the Base Circle** or simply an **Involute.** An *involute of a circle* is the curve traced out by a point on the free end of a stretched string when the string is being unwound from the circle. The form of this curve will be understood from the left-hand figure.

III. When the Hypocycloid is a Straight Line.—If the diameter of the generating circle be equal to *half* the diameter of the circle inside which it rolls, then the *hypocycloid* traced out by the tracing point will be a straight line, and this straight line will be a diameter of the base circle. This case is shown by the figure on the right.

Let the tracing point be at A at the beginning of its motion, and at P for any position of the generating circle. Join O A, G P, and O C. O C passes through G, the centre of the generating circle

Let $\angle C G P = \theta$, $\angle C O A = \varphi$.

Then, arc P C = arc A C, by definition.

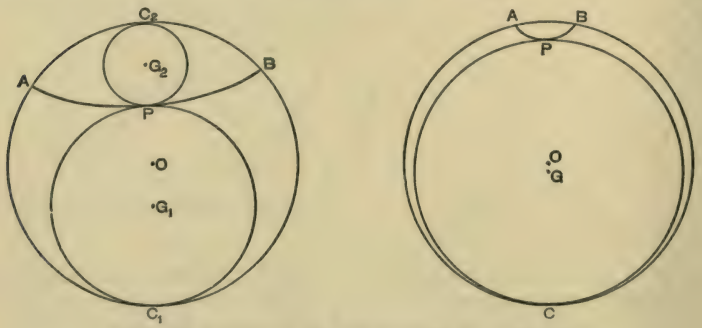
$\therefore$ $G C \times \theta = O C \times \varphi$.

But, $G C = \frac{1}{2} O C$, by hypothesis.

$\therefore$ $\theta = 2 \varphi$.

Now θ is an angle at the centre of the circle G, and φ is an angle at the circumference of the same circle. But, since $\theta = 2 \varphi$, it follows (converse of Euc. III., 20) that these two angles must stand on the same arc, P C, of the circle G. Therefore, P lies on the line O A. This being true for any position of G, we conclude that P moves along the straight line A B, which is a diameter of the base circle.

IV. **When the Hypocycloid is a Point.**—By a reference to the following left-hand figure, it appears that the same hypocycloid can be traced out by either of the generating circles, G_1 or G_2, when the diameters of these circles are such, that their sum is equal to the diameter of the circle inside which they roll.

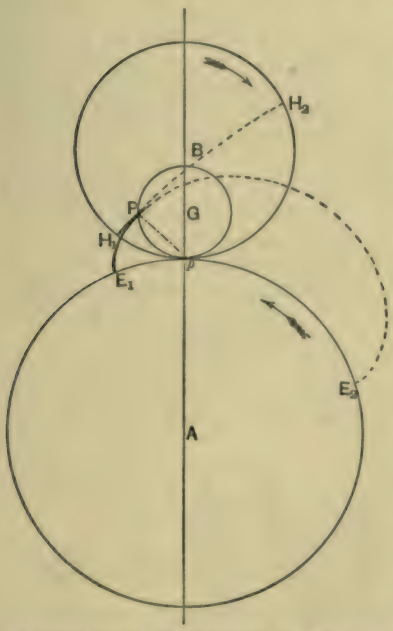


THE HYPOCYCLOID DEGENERATING TO A POINT.

Now, if the circle G_1 goes on increasing in size, the hypocycloid, A P B, becomes more and more convex towards the centre, O, until, ultimately, when G_1 becomes nearly equal in size to the base circle, the hypocycloid is a small half-loop, as shown by the right-hand figure. The same thing takes place as the circle G_2 decreases in size. Hence, when the generating circle is equal in size to the base circle, the hypocycloid degenerates into a point.

The curves of the cycloid class have many important geometrical properties, some of which are familiar to students of higher Dynamics, but the only property with which we are concerned here is that one relating to the *normal* at any point of the curve.

Referring to the above figures, let C be the point of contact of the generating circle and base line; P the tracing point. Then, at any particular instant during the rolling of the circle G, the tracing point, P, will be moving, as it were, in a circle whose radius is C P, and having C as its centre. In other words, C is the *instantaneous centre* of motion. C P is, therefore, the radius of curvature of the curve at the point P. Hence, the *normal* to the curve at the point P is in the direction P C. It is this property of the cycloidal and involute curves which fit them so well for the teeth of wheels.



ILLUSTRATING FULFILMENT OF PRIMARY CONDITIONS BY CYCLOIDAL TEETH.

Cycloidal Teeth. — Let $E_1 p E_2$, $H_1 p H_2$ be the pitch circles of two wheels gearing together, p being the pitch point. Let G be the *fixed* centre of a third circle touching the other two circles at the pitch point, p. Let P be a tracing point on the circumference of this circle.

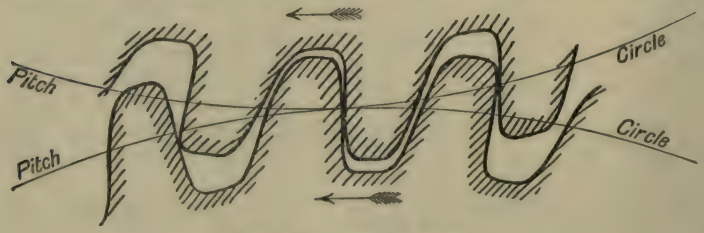
At the beginning of motion, let P, E_1, and H_1 all coincide at the point, p. Let the three circles now roll in contact with each other in the directions indicated. The point, P, will then describe simultaneously the epicycloid, $E_1 P E_2$, outside the pitch circle, A, and the hypocycloid, $H_1 P H_2$, inside the pitch circle, B. Since both curves are traced out by the *same* generating circle and tracing point, it follows, from what has already been said, that the common normal at their point of contact, P, always passes through the pitch point, p. But this is the very condition which we have

been seeking to fulfil, and we now see that teeth of the cycloidal class satisfy this condition for correct working. The part, $E_1 P$, of the epicycloid may represent the curve for the *face* of a tooth on the wheel, A, and the part, $H_1 P$, of the hypocycloid the curve for the *flank* of a tooth on the wheel, B. Hence, if the *faces* of the teeth on the one wheel, and the *flanks* of the teeth on the other be described by the *same generating circle*, the two wheels will work correctly together.

The student should observe that the action between a pair of teeth, however perfectly formed, is not wholly due to the *rolling* of one tooth on the other. An inspection of the previous figure will make this quite clear. Thus, at the beginning of the motion described, when P coincides with p, E_1 coincides with H_1. When the motion is such that P is brought into the position shown on the figure, the length of epicycloid described is $E_1 P$, and that of the hypocycloid, $H_1 P$. These arcs are not equal in length, $E_1 P$ being greater than $H_1 P$. Therefore, the amount of *sliding* is $E_1 P - H_1 P$. Hence, the action between a pair of teeth in contact is partly sliding and partly rolling.

It is not necessary that the same generating circle be employed for describing both faces or both flanks of the teeth on the same wheel, but it is very advantageous to have the teeth so described, especially if the wheels require to run in either direction.

Gee's Toothed Gearing.—When the wheels require to run in one direction only, the posterior or unacting surfaces of the teeth may be given any shape whatever. A form of toothed

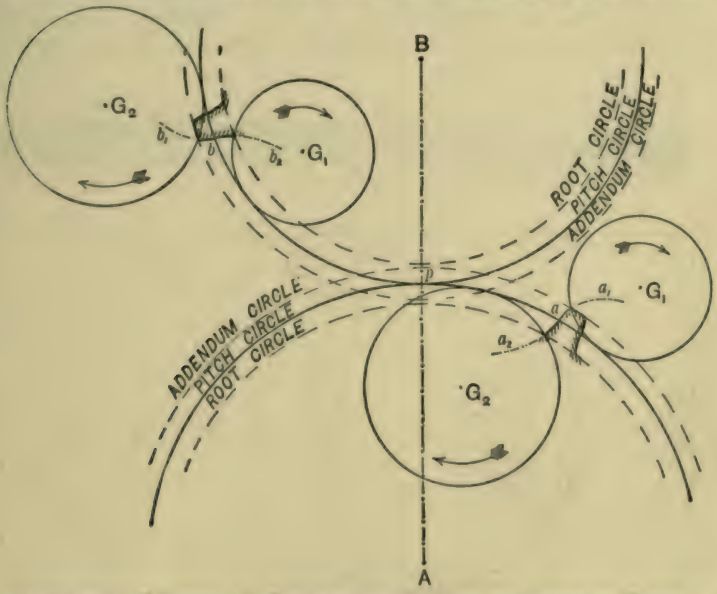


GEE'S TOOTHED GEARING.

gearing, known as Gee's, seems to fulfil the purpose very well. It is said to be 35 per cent. stronger than the ordinary form. The driving surfaces of the teeth are of the usual form, but the other surfaces are more inclined, as shown by the accompanying figure. This causes the roots of the teeth to be much thicker than with ordinary teeth, and hence the increase of strength.

Exact Method of Drawing the Curves for Cycloidal Teeth.—The method of describing the curves for the teeth of wheels will

now be easily understood. Let G_1 represent the generating circle used for describing the *faces* of the teeth on wheel A and the *flanks* of the teeth on B; G_2 the generating circle used for describing the faces of the teeth on B and the flanks of the teeth on A. Draw the addendum and root circles and divide the pitch circles into as many equal arcs as there will be teeth on the wheels. On each side of these points of division set off equal distances to represent *half* the thickness of a tooth as measured along the pitch circle.



ILLUSTRATING METHOD OF SETTING OUT CURVES FOR TEETH OF WHEELS.

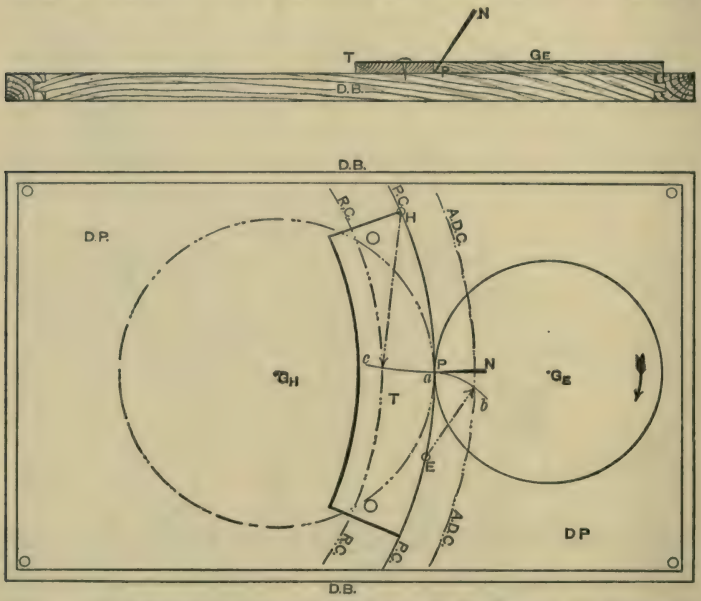
Let *a*, be a point on pitch circle of wheel A, and *b*, a point on pitch circle B, from which the curves for a tooth on each wheel have to be set out.

By placing the generating circles, G_1, G_2, in contact with the pitch circles at these points, and then tracing out the parts of the epicycloids and hypocycloids between the pitch circles and addendum and root circles as shown, the curves for a tooth on each wheel may be thus described. This process may be repeated for all the teeth on both wheels, and we thus obtain a complete representation of a pair of spur wheels having cycloidal teeth.

Practical Method of Drawing the Curves for Cycloidal Teeth.—The above method of setting out the curves for the teeth of

wheels, although mathematically exact and apparently quite simple, is found to be rather tedious in practice, and consequently in working drawings we always find the true curves represented approximately by arcs of circles. The following method of obtaining curves for the teeth of wheels is very often used in practice :—

Make a wooden template, T, having a thickness of about $\frac{1}{8}$ inch, and of such a shape that its outer and inner edges are each arcs of a circle having a radius equal to that of the pitch circle of the wheel upon which the teeth have to be described. Make also template segments of the generating circles, G_E, G_H, and pass a



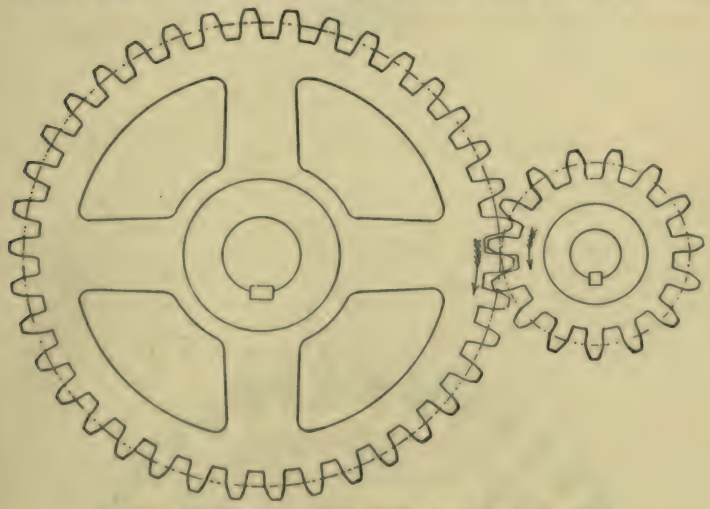
PRACTICAL METHOD OF SETTING OUT CURVES FOR TEETH OF WHEELS.

INDEX TO PARTS.

D B	represents	Drawing board.	N	represents	Needle.
D P	,,	Drawing paper.	P	,,	Tracing point of needle.
T	,,	Template.	E	,,	Centre for circular arc approximately coinciding with epicycloidal arc, *a b*.
P C	,,	Pitch circle.			
A D C	,,	Addendum circle.			
R C	,,	Root circle.			
G_E	,,	Generating circle for faces of teeth.	H	,,	Centre for circular arc approximately coinciding with hypocycloidal arc, *a c*.
G_H	,,	Generating circle for flanks of teeth.			

small pencil or needle, N P, through each, so that the point, P, coincides with the outer edge, as shown in the elevation.

Fix a sheet of drawing paper on a drawing board, D B. On this paper draw an arc of a circle having a radius equal to that of the pitch circle of the wheel. By means of nails attach the pitch circle template, T, to the drawing board in such a position that its *convex* arc coincides with the arc of the pitch circle drawn on the paper. Now take the generating circle, G_E, and bring it in contact with the convex edge of the pitch circle template, so that the point, P, coincides with the point, *a*. Roll G_E along the template, T, in the direction indicated, when the point, P, will describe an arc of an epicycloid, *a b*. The arc of the epicycloid intercepted between the pitch circle and the addendum circle represents the *face* of a tooth. Now shift the pitch circle template, T, so that its *concave* edge coincides with the pitch circle drawn on the paper. By placing the generating circle, G_H, in contact

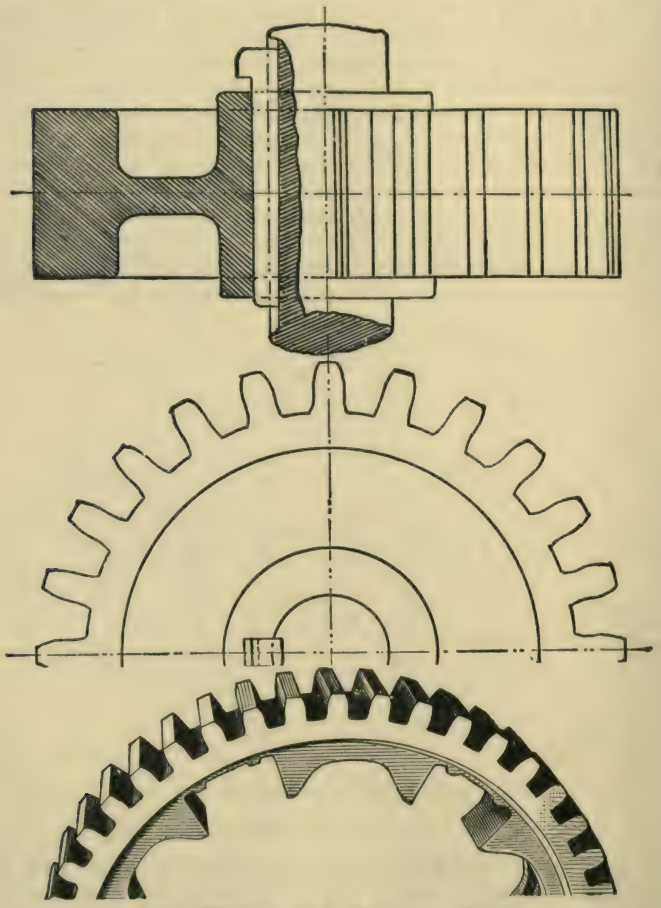


SPUR WHEEL AND PINION WITH CYCLOIDAL TEETH.

with the concave edge of T, and having the tracing point or pencil, P, coinciding with *a*, the hypocycloid, *a c*, can then be traced in the same manner. The arc of the hypocycloid intercepted between the pitch circle and the root circle will represent the *flank* of a tooth on the wheel.

Having obtained these curves, it remains to find, by trial, the radii and centres, E, H, of arcs of circles which approximately

coincide with the epicycloidal and hypocycloidal arcs respectively. These being found, approximate curves can readily be drawn to represent the faces and flanks of the teeth. This method is



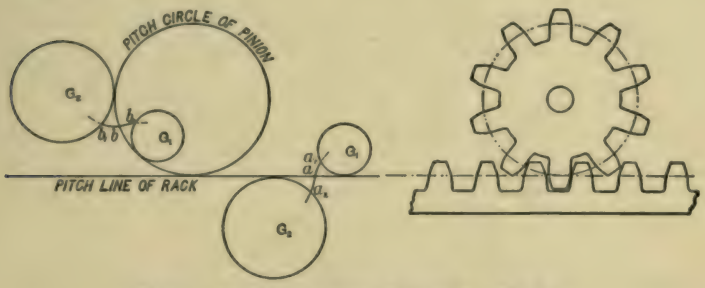
Spur Wheels with Cycloidal Teeth.

often used by patternmakers when setting out the curves for the teeth of wheels. For ordinary methods of representing on drawings the curves for the teeth of wheels, the student must consult works

on Machine Drawing. The student must be reminded, however, that true cycloidal curves can never be accurately represented by arcs of circles, however many of these may be employed in completing the drawing.

The previous figures represent spur wheels with cycloidal teeth.

Application of Preceding Principles to the Case of a Rack and its Pinion.—For our present purpose, we may consider a rack as being simply a toothed wheel having a pitch circle of infinite radius; and it therefore follows, that the preceding principles are applicable to it.



RACK AND PINION WITH CYCLOIDAL TEETH.

The figure on the right-hand side shows a rack and its pinion, the pitch lines being shown by dotted lines. The figure on the left-hand side shows the application of the preceding principles in obtaining the curves for the teeth on both rack and pinion. The generating circle, G_1, is represented describing the face of a tooth on the rack and the flank of a tooth on the pinion, while the circle, G_2, is employed in describing the flank of a tooth on the rack and the face of a tooth on the pinion. The curves forming the faces and flanks of the teeth on the rack will thus be arcs of cycloids.

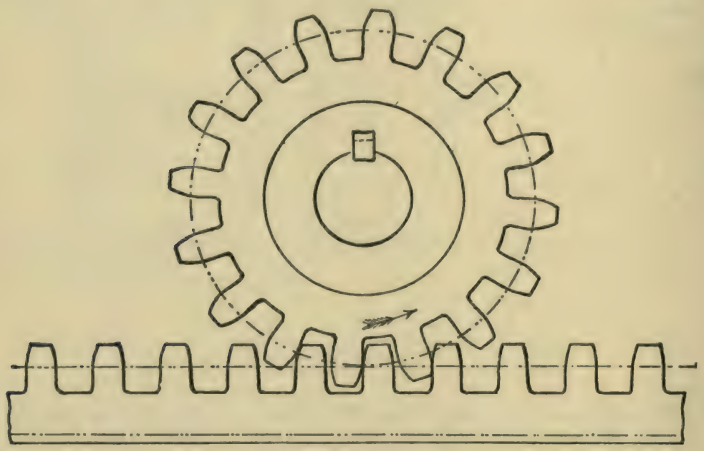
Particular Forms of Teeth as Dependent upon Changes in the Sizes of the Generating Circles Employed.—In what has preceded, we have been chiefly concerned with a discussion of the general character and shape of teeth of the cycloidal class, and we now go on to consider a few particular cases as dependent upon the sizes of the generating circles employed.

First Particular Case—When the Hypocycloid is a Straight Line.—We have already shown, that when the diameter of the generating circle is half that of the circle inside which it rolls, the hypocycloid traced out by the tracing point is a diameter of the base or pitch circle. This being the case it is easy to see, that if

the flanks of the teeth of a pair of wheels be described by generating circles, whose diameters are half that of the corresponding pitch circles, such flanks will be straight or *radial.*

The method of setting out such teeth may be briefly stated thus:—

Let A and B be the centres of the pitch circles. Take generating circles, G_A, G_B, *having diameters respectively equal to half those of*



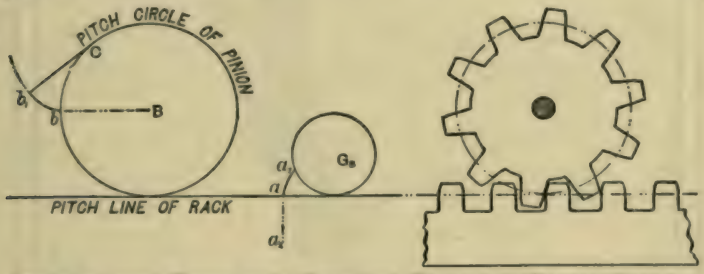
RACK AND PINION.

the pitch circles A *and* B. By rolling G_A on the convex side of pitch circle, B, an epicycloid, $b_1 b_2$, will be traced out. This curve determines the form of the faces of the teeth on B. Similarly, by rolling G_B on the convex side of pitch circle, A, the epicycloid, $a_1 a_2$, will be obtained, which will determine the form of the faces of the teeth on A.

The hypocycloids corresponding to these generating circles are straight lines or radii of the pitch circles. Hence, to complete the curves for the acting surfaces of the teeth, it is only necessary to draw the radii from the points a_1 . . . . ., b_1 . . . . ., &c.

Teeth with radial flanks are thinner at the roots than at the pitch circle, and if the wheel is small and has few teeth, it is not difficult to see that such teeth may exhibit comparative weakness at the roots, the very place where they should be strongest. With ordinary sized wheels, this need not present any serious obstacle, since the thickness at the roots may be increased by simply putting in fillets between the straight flanks and the root circle.

Rack having Teeth with Radial Flanks.—In carrying out the above idea for the case of a rack and pinion, we notice that the generating circle to be used in describing the faces of the teeth on the pinion must have a diameter *equal to that of the radius of the pitch "circle" of the rack.* But since this latter "circle" has an infinite diameter, it follows that the diameter of the generating circle just referred to must also be infinite. Now, we have already shown that the epicycloid traced out by a generating circle of infinite diameter is an *involute of the base or pitch circle*, outside which this generating circle is supposed to roll. Hence, *the faces of the teeth on the pinion must be involutes of its own pitch circle.* The faces of the teeth on the rack are *cycloids* described by a generating circle, having a diameter equal to the radius of the pinion.



PINION HAVING TEETH WITH INVOLUTE FACES AND RADIAL FLANKS.
RACK HAVING TEETH WITH CYCLOIDAL FACES AND STRAIGHT FLANKS.

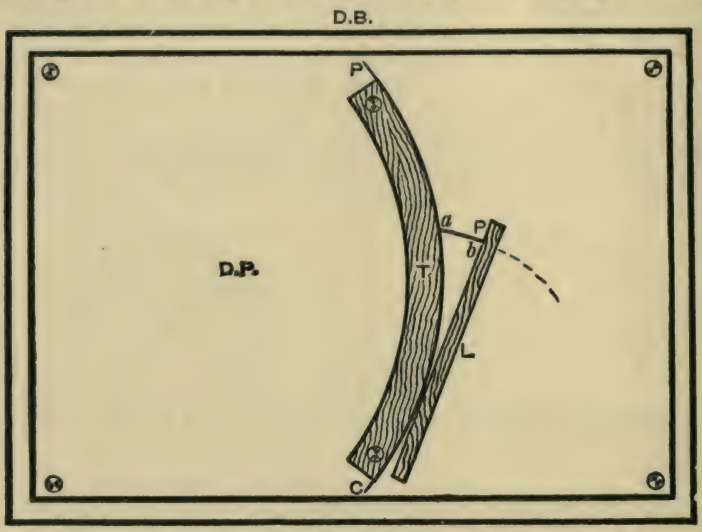
The principle of construction for this case will be understood from the left-hand figure above. The complete teeth are represented by the right-hand figure.

It must be carefully borne in mind, that the form of the acting surfaces of the teeth on the one wheel, determines the necessary form of the acting surfaces of the teeth on the other wheel in gear with it. In the case of cycloidal teeth, the only necessary condition to be observed in their construction, in order to insure correct working is, that the same generating circle be used in describing the faces of the teeth on the one wheel as that used in describing the flanks of the teeth in the other. This condition is clearly fulfilled in the two particular cases just considered.

Practical Method of Drawing Involute Curves for the Faces of Teeth on a Pinion.—The following is a simple practical method of drawing the involute curves for the faces of the teeth on the pinion, gearing with a rack having teeth with radial flanks.

D B is a drawing board, having a sheet of drawing paper, D P, fixed to it. Draw on the paper, full size, an arc, P C, of the

pitch circle of the pinion. Having made a template, T, of wood with a convex edge, struck with a radius equal to that of the pitch circle of the pinion, fix it to the board with this edge coinciding with P C, as shown by the figure. Next take a lath, L, of wood, having one of its edges perfectly straight, and carrying a small



PRACTICAL METHOD OF DRAWING INVOLUTE OF PITCH CIRCLE.

INDEX TO PARTS.

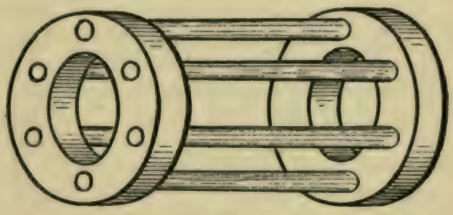
D B	represents	Drawing board.	T	represents	Wooden template.
D P	,,	Drawing paper.	L	,,	Wooden lath.
P C	,,	Pitch circle of wheel.	P	,,	Tracing point.

pencil or needle, P, projecting from the straight edge. Let the straight edge of L be placed against the convex edge of T, and let the point P coincide with the point *a*, from which the curve must start. Now allow the lath to *roll* on the edge of T, so that the straight edge of L will always form a tangent to the pitch circle, care being taken not to allow any slipping during the process. By this means, the point P will describe a curve which will be an involute of the pitch circle, P C. An arc of a circle can now be drawn, which will approximately coincide with the involute arc so found, and then the curves for the faces of the teeth may be set out.

Second Particular Case—When the Hypocycloid is a Point—Pin Wheels.—We have already shown that the hypocycloid degenerates to a point when the diameter of the generating circle is

equal to that of the base circle inside which it rolls. Hence, if the diameters of the generating circles be taken equal in size to the respective pitch circles of a pair of wheels intended to gear together, it is clear, that the teeth on both wheels will possess the peculiar property of having *no flanks*. In this particular case, the teeth on *one* of the wheels must, theoretically, be mere points. In practice the teeth must have some magnitude, and consequently we find pins instead of mere points. A wheel of this description would be called a *pin wheel*, and consists of a series of pins projecting from the face of a circular disc, as shown by the following figures. When the pins are fixed between two discs we then obtain what is called a *lantern wheel;* a form of wheel now rarely used, except in clock and watch mechanism.

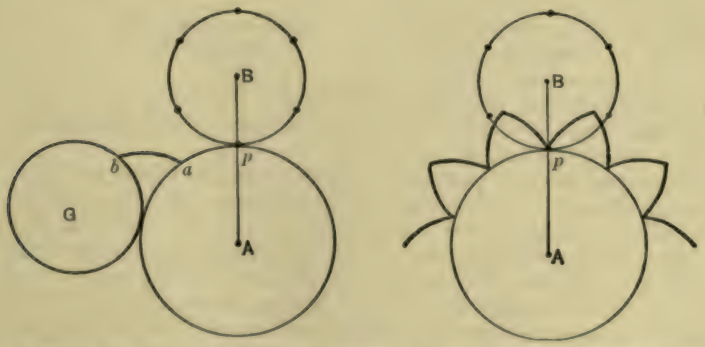
PIN WHEEL.



LANTERN WHEEL.

The problem now before us is, given a pin or lantern wheel, to describe the teeth on another wheel which shall work accurately with it.

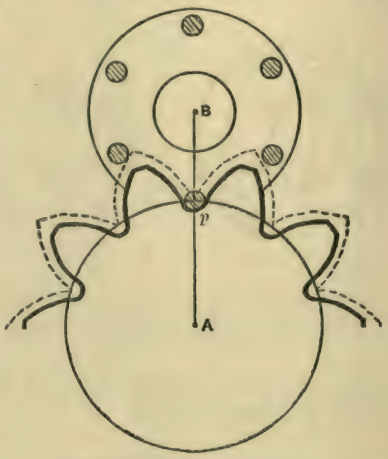
We shall first suppose the pins to have no diameter, in other words, to be mere points.



METHOD OF DRAWING TEETH ON A WHEEL TO GEAR WITH A PIN WHEEL.

Let A be the centre of the pitch circle upon which the required teeth have to be described, and B the centre of the pin wheel. The generating circle, G, used for describing the faces of the teeth on A, must be of the same size as the pitch circle, B. The shape of the complete teeth on A is shown by the figure on the right-hand side, from which it will be seen that the teeth have no flanks.

To complete the problem we must modify the above figure on the right-hand side to suit the actual case when pins of definite diameter are substituted for the points on the wheel B. Having fixed upon the diameter of the pins, draw circles to represent these round the pitch circle, B, as shown. At the points of intersection of the dotted epicycloids with the pitch circle, A, draw the small arcs inward (with a radius equal to that of the pins), to represent the recesses into which the pins enter when approaching the pitch point, *p*. Then draw curves from the ends of these small arcs parallel to the dotted epicycloids as indicated, the distance between the parallel curves being half the diameter of the pins.



TEETH TO GEAR WITH A PIN WHEEL.

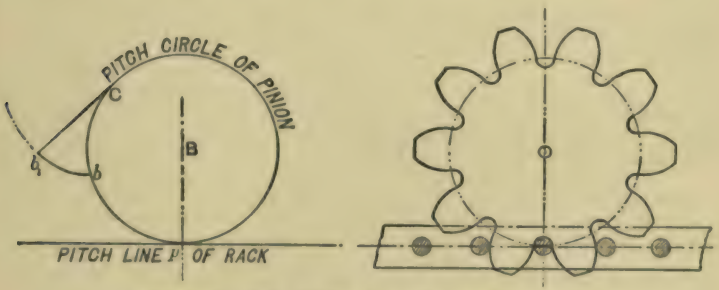
It should be noticed, however, that the parallel curves so drawn are very approximately epicycloids traced by a generating circle equal in size to the one used in describing the dotted curves. Hence, it is only necessary to draw from the ends of the small circular arcs, epicycloids with a generating circle equal in diameter to that of the pitch circle, B, and these will represent the working faces of the teeth on A.

Pins are always placed on the Follower.—When one of a pair of wheels in gear has pins instead of teeth, it is the practice to *place the pins on that wheel which is to be the follower.* The reason for this will be apparent when we remember what has been said regarding the friction between the teeth during the arcs of approach and recess. The friction during the arc of approach is said to be greater than that during recess, and if this be the case, it follows, that the arc of approach should be as small as possible

Now, in the arrangement just considered, wherein the teeth have no flanks, it is clear that there will be no arc of approach or no arc of recess according as the pin wheel is the follower or the driver. If the pin wheel be the follower, the whole of the action between the teeth on A and the pins on B will occur *after* the line of centres—*i.e.*, during the arc of recess. If B becomes the driver, then the whole of the action takes place during the arc of approach. This latter arrangement should therefore not be adopted.

Rack and Pinion.—Sometimes we find either a rack or its pinion fitted with pins instead of ordinary teeth. In any case, however, the above rule must be attended to—viz., *the pins always to be placed on the follower*. Hence two cases arise—(1) the pinion may drive the rack, or (2) the rack may drive the pinion.

(1) *Suppose the Pinion to Drive the Rack.*—In this case, the pins must be placed upon the rack. Now the pitch line of a rack has been stated to be part of a pitch circle of infinite radius, and since the faces of the teeth on the driver are supposed to be described by a generating circle having the same diameter as the pitch circle of the follower, it follows that this generating circle must also be of infinite radius. Hence, the curves for the



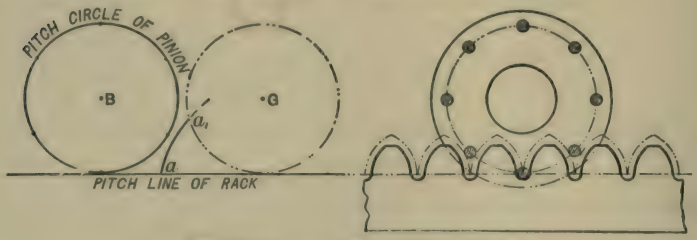
PINION IN GEAR WITH RACK FITTED WITH PINS.

faces of the teeth on the pinion will be involutes of its own pitch circle.

The principles of construction in this case will be understood from what has preceded and by a reference to the accompanying figures.

(2) *Suppose the Rack to Drive the Pinion.*—In this case, the pins must be placed upon the pinion. Hence, the faces of the teeth on the rack must be described by a generating circle, G, *equal in diameter to that of the pitch circle of the pinion*. The curves for the faces of these teeth will thus be *cycloids*. The method

of describing these as well as their appearance when complete will be easily understood from the figure.



PIN WHEEL IN GEAR WITH A RACK.

Disadvantage of Pin Wheels.—Pin wheels are now seldom used, except in clock and watch mechanism, owing to a practical disadvantage which they possess—viz., that the wheels required to gear with them have to be specially designed, and these latter can only be geared with one particular size of pin wheel, and with no other kind of toothed wheel.

Lecture XI.—Questions.

1. Explain the terms "pitch" (circular and diametral), "pitch circle," and "pitch point" as applied to toothed gearing. State the relation between the circular and diametral pitches.

2. Explain, by aid of sketches, the meanings of the terms "face," "flank," "addendum," and "clearance" as applied to toothed gearing. State the usual proportions for the addendum and clearance (side and bottom) in terms of the pitch. What is the effect of clearance on the action of the teeth?

3. What is meant by the *pitch* of a tooth in a spur wheel? What are the usual forms of teeth and how are they described? Sketch two consecutive teeth of a spur wheel, and give the relative proportions of the different parts of a tooth in terms of the pitch.

4. Design by any method you know the tooth of a spur wheel—pitch = 2 inches; diameter = 7 inches; and show by dimensions the correct proportions.

5. What is meant by the term arc of action? State the usual length of arc of action in terms of the pitch.

6. Upon what principle are teeth of wheels of the epicycloidal and hypocycloidal form constructed? Show under what conditions they will work properly. What is to be done in order that any wheels of a set may work accurately together?

7. In forming the teeth of wheels, the geometrical condition is that the common perpendicular to the surfaces of two teeth in contact shall always pass through the point of contact of the pitch circles of the wheels. Write o t a proof of this general proposition.

8 Give the theory for constructing teeth of wheels with radical flanks which shall work accurately together, the distance between the centres of the pitch circles of two such wheels being 24 inches, and the required velocity-ratio of the wheels 3 to 1, find the diameter of the rolling circles for describing the teeth of each wheel. *Ans.* 18 ins. and 6 ins.

9. A toothed spur wheel is 4 inches pitch Sketch a tooth and mark on it suitable dimensions. Draw accurately a suitable curve for such a tooth, taking the pitch line straight as in a rack, and using a describing circle of 5 inches radius.

10. What geometrical condition must be satisfied by the acting surfaces of the teeth of a pair of wheels in order that the velocity-ratio communicated may be constant? Show that this condition is fulfilled by epicycloidal and hypocycloidal curves.

11. Show, by sketches, what cycloidal curves should be used or approximated to in the faces and flanks of the teeth in the following cases:—(*a*) Pair of wheels in external contact; (*b*) Pinion and rack. It is only necessary to mention the proper curves, without attempting to draw them.

12. Define, and show roughly by sketches, the following curves:—The cycloid, epicycloid, hypocycloid, and involute. Mention one property of those curves which make them so useful for engineering purposes. Discuss the various particular forms assumed by those curves under particular circumstances, and state some of their applications.

13. Describe the form of Gee's patent wheel teeth, and mention what is their advantage.

14. In wheels with pins for teeth the pins are always placed upon the follower; will you explain this? What are the chief disadvantages of pin wheels?

15. A toothed wheel drives a pin wheel; investigate the proper form for the curves of the teeth. The diameter of each pin being known, how do you proceed to set out the teeth, preserving their theoretical outline? Sketch the necessary diagram.

Lecture XI.—A M.Inst.C.E. Exam. Questions.

1. In toothed gearing, in order to preserve a constant velocity ratio, the common normal to the acting surfaces of a pair of teeth at their point of contact must always pass through the pitch-point. Prove this, and name two forms of teeth which satisfy this condition. (I.C.E., *Oct.*, 1897.)

2. State, with proof, the single kinematic condition that has to be satisfied in order that two curved plates rotating about fixed parallel axis—such as the teeth of spur wheels—should transmit a constant angular velocity ratio. (I.C.E., *Oct.*, 1903.)

3. Prove that, if the profiles of the teeth of a pair of wheels are correctly shaped, the common normal to their surfaces at the point of contact must always pass through the pitch-point. (I.C.E., *Oct.*, 1904.)

4. In the consideration of the forms suitable for the teeth of spur wheels, state what geometrical condition has to be satisfied, and explain why. (I.C.E., *Feb.*, 1905.)

5. Define rolling contact and sliding contact, and give two examples of the practical application of each. In the case of rolling contact, what are the necessary conditions existing between two surfaces, each rotating about its own centre? Explain how every arrangement of tooth gearing has corresponding rolling surfaces. What is the name given to these surfaces? (I.C.E., *Oct.*, 1905.)

6. If the shape of the tooth for one wheel of ordinary toothed gearing be given, show by a graphic construction how the proper form of the teeth of the other wheel can be found. (I.C.E., *Feb.*, 1906.)

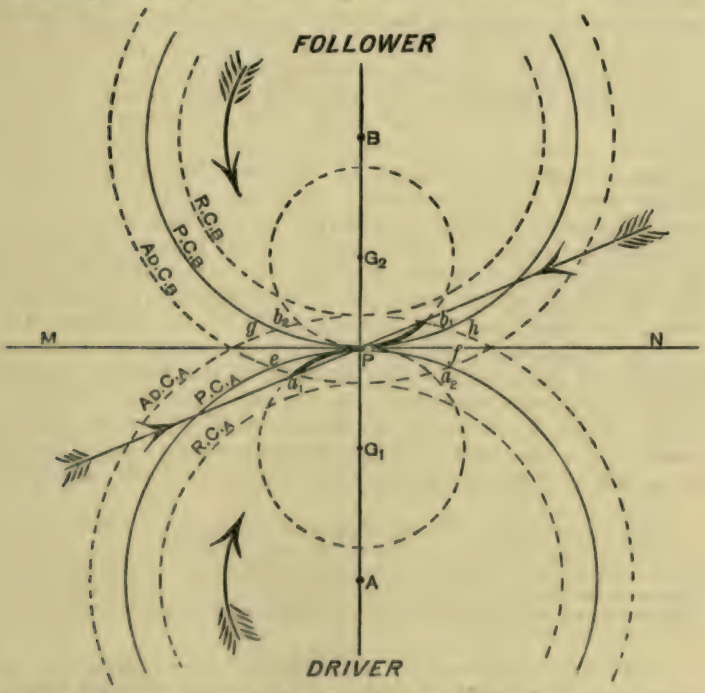
7. Find the kinematic condition which must be satisfied by the profiles of a pair of wheel teeth in order that the velocity ratio between the wheels may be constant. State the nature of the curves used in practice which satisfy this condition. (I.C.E., *Feb.*, 1907.)

8. Make a sketch showing the position of a tooth of an ordinary epicycloidal gear wheel at the beginning and at the end of contact with the tooth of the gear wheel with which it is engaged, and indicate what are known as arcs of approach and of recess and the arcs of action. Show that, if another pair of teeth must come into gear before those under consideration go out, the circular pitch of each wheel must not exceed the corresponding arc of action. (I.C.E., *Feb.*, 1908.)

LECTURE XII.

CONTENTS.—Path of Contact with Cycloidal Teeth—Obliquity of Reaction—Length of Cycloidal Teeth for given Arcs of Approach and Recess—Calculation of the Length of Cycloidal Teeth—Examples I. and II.—Diameter of Generating Circle—Least Number of Cycloidal Teeth to be placed upon a Wheel—Cycloidal Teeth for Wheels with Internal Contact—Path of Contact with Internal Gearing—Formulæ for Length of Teeth of Internal Gearing—Questions.

Path of Contact with Cycloidal Teeth.—Let the accompanying figure represent portions of two pitch circles with their addendum



circles and generating circles, G_1, G_2. From the figure, it will be seen that contact between two teeth begins at a_1 and terminates

at b_1, the points a_1 and b_1 being determined by the intersection of the addendum and generating circles. During motion the point of contact of the pair of teeth in question travels along the curve, $a_1 p b_1$, which is made up of the arcs, $a_1 p$, $p b_1$, of the two generating circles, G_1, G_2.

DEFINITION.—**The path $a_1 p b_1$, along which the point of contact of a pair of teeth moves, is called the Path of Contact.**

The whole path of contact is divided at the pitch point, p, into two parts called, respectively, the **Path of Approach**, $a_1 p$, and the **Path of Recess**, $p b_1$.*

If the direction of motion of the wheels be reversed, then the *path of contact* will be $a_2 p b_2$.

The path of contact in the case of cycloidal teeth is always circular, but in some forms of teeth, for example involute teeth, the path of contact may be a straight line. The student should examine all the preceding particular cases and ascertain the nature of the path of contact. He will then see that in teeth with involute faces part of this path is a straight line.

Obliquity of Reaction.—We have seen that (neglecting friction) the direction or line of action of the mutual pressure, or reaction, between a pair of properly constructed teeth in contact always passes through the pitch point, p. *The angle which this direction makes with the common tangent to the two pitch circles at their point of contact is called the* **Obliquity of Reaction.** Thus, in the previous figure, the direction of the mutual pressure or reaction at the beginning of contact of a pair of teeth is along $a_1 p$, and at the end of contact along $p b_1$. The obliquities of reaction at these two particular points are denoted by the angles $a_1 p$ M, $b_1 p$ N respectively. When the point of contact of the teeth reaches the pitch point, p, the direction of the reaction is along M N, the common tangent at p, and at this point the obliquity is zero. Thus, the obliquity of reaction in the case of cycloidal teeth, varies from a maximum at the beginning and end of contact of a pair of teeth, to zero at the pitch point. With such teeth,

* The student must carefully distinguish between the terms *path of contact* and *arc of contact*. The latter term refers to the *arc on either pitch circle* turned through by that pitch circle during contact of a pair of teeth, while the former refers to the *actual path* traversed by the point of contact during the same period. Nevertheless, it should be noticed, that *with cycloidal teeth*:—

Length of *arc of approach*, $e p$ or $g p$ = length of *path of approach*, $a_1 p$.
,, *arc of recess*, $p f$ or $p h$ = ,, *path of recess*, $p b_1$.

so that:—

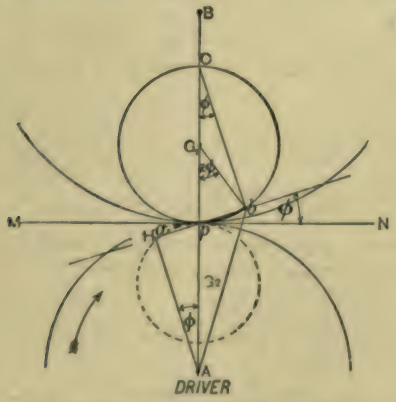
Length of *arc of contact* = length of *path of contact*.

the maximum obliquity should never be allowed to exceed 30°. In those teeth of which the path of contact is a straight line, the obliquity remains constant during contact, and should never exceed about 15°.

Length of Cycloidal Teeth for given Arcs of Approach and Recess.—When the arcs of approach and recess are given for a pair of wheels, we can then determine the lengths to be given to the teeth on the two wheels respectively. Referring to the last figure, let arcs $p\,e$, $p\,f$ represent the given lengths of the arcs of approach and recess respectively. On the given generating circles, G_1, G_2, cut off the arcs $p\,a_1$, $p\,b_1$, equal in length respectively to $p\,e$, $p\,f$. Through the points a_1, b_1 draw the circles $A\,D\,C_B$, $A\,D\,C_A$, about the centres B and A respectively. These are the addendum circles for the two wheels. After making allowances for bottom clearance, the root circles, $R\,C_A$, $R\,C_B$, can be drawn. From this construction, the sizes of the teeth on the two wheels can be determined.

Calculation of the Length of Cycloidal Teeth.—We shall now show how the previous problems may be solved by calculation.

In the accompanying figure, let wheel A be the driver, B the follower, and G_1, G_2 the generating circles. Then $a\,p\,b$ is the path of contact.



LENGTH OF CYCLOIDAL TEETH.

Let R_1 = Radius of pitch circle, A.
„ r_1 = Radius of circle, G_1, used in describing *faces* of teeth on A.
„ δ_1 = Addendum of teeth on A.
„ θ, φ = Maximum obliquities of action during approach and recess respectively.
„ α, β = Lengths of arcs of approach and recess respectively.

Then, since *Path of approach or recess = Arc of approach or recess.*

$\therefore$ α = arc $a\,p$; and β = arc $p\,b$.

Join A b, $G_1\,b$, C b, and $p\,b$. Draw A H perpendicular to $b\,p$ produced. Then clearly, $\angle\, p\,A\,H = \angle\, p\,C\,b = \angle\, b\,p\,N = \varphi$.

Since b A H is a right angled triangle, we get :—

$$A\,b^2 = A\,H^2 + H\,b^2.$$

But,
$$A\,b = R_1 + \delta_1.$$
$$A\,H = R_1 \cos\varphi.$$

And,
$$H\,b = H\,p + p\,b = R_1 \sin\varphi + 2\,r_1 \sin\varphi.$$

$$\therefore \quad (R_1 + \delta_1)^2 = R_1^2 \cos^2\varphi + (R_1 + 2\,r_1)^2 \sin^2\varphi.$$

$$\therefore \quad R_1^2 + 2\,R_1\,\delta_1 + \delta_1^2 = R_1^2(\cos^2\varphi + \sin^2\varphi) + 4(R_1 + r_1) r_1 \sin^2\varphi.$$

$$\therefore \quad 2\,R_1\,\delta_1 + \delta_1^2 = 4\,(R_1 + r_1)\,r_1 \sin^2\varphi. \quad . \quad . \quad . \quad . \quad (I)$$

Also,
$$\text{Arc } p\,b = r_1 \times 2\,\varphi.$$

i.e.,
$$\beta = 2\,r_1\,\varphi.$$

Or,
$$\varphi = \frac{\beta}{2\,r_1}. \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (II)$$

Similarly, if R_2, r_2, δ_2 apply to the follower, B.

Then,
$$2\,R_2\,\delta_2 + \delta_2^2 = 4\,(R_2 + r_2)\,r_2 \sin^2\theta \quad . \quad . \quad . \quad . \quad (I_a)$$

And,
$$\alpha = 2\,r_2\,\theta$$

Or,
$$\theta = \frac{\alpha}{2\,r_2} \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (II_a)$$

From these equations the addenda of the teeth on the two wheels can be calculated, if we know the sizes of the generating circles and the arcs of approach and recess.

When the wheels are large we may neglect δ^2 in equations (I) and (I_a), since this quantity will be small compared with R, and we get the approximate formulæ :—

$$\delta_1 = 2\left(1 + \frac{r_1}{R_1}\right) r_1 \sin^2\varphi$$

And,
$$\delta_2 = 2\left(1 + \frac{r_2}{R_2}\right) r_2 \sin^2\theta \qquad . \quad . \quad . \quad (III)$$

Again, the sizes of the generating circles are generally stated in terms of the size of the pitch circles inside which they roll.

Hence, let:— $r_1 = m_2\,R_2$, $r_2 = m_1\,R_1$.

Where m_1 and m_2 are fractions seldom greater than one-half.

Then, equations (I) and (III) become :—

$$2\,R_1\,\delta_1 + \delta_1^2 = 4\,m_2\,(R_1 + m_2\,R_2)\,R_2 \sin^2\varphi$$
$$2\,R_2\,\delta_2 + \delta_2^2 = 4\,m_1\,(R_2 + m_1\,R_1)\,R_1 \sin^2\theta \qquad . \quad (IV)$$

Or, approximately,
$$\delta_1 = 2\,m_2\left(1 + \frac{m_2\,R_2}{R_1}\right) R_2 \sin^2\varphi$$
$$\delta_2 = 2\,m_1\left(1 + \frac{m_1\,R_1}{R_2}\right) R_1 \sin^2\theta \qquad . \quad (V)$$

Let p = Pitch of teeth on wheels.
,, n_1, n_2 = Number of teeth on respective wheels.

Then, $$R_1 = \frac{n_1 p}{2\pi}, \qquad R_2 = \frac{n_2 p}{2\pi}.$$

And we get the following final equations :—

$$\left.\begin{aligned} \pi n_1 p \delta_1 + \pi^2 \delta_1^2 &= m_2 (n_1 + m_2 n_2) n_2 p^2 \sin^2 \varphi \\ \pi n_2 p \delta_2 + \pi^2 \delta_2^2 &= m_1 (n_2 + m_1 n_1) n_1 p^2 \sin^2 \theta \end{aligned}\right\} \quad \text{(VI)}$$

Or, approximately,

$$\left.\begin{aligned} \pi n_1 \delta_1 &= m_2 (n_1 + m_2 n_2) n_2 p \sin^2 \varphi \\ \pi n_2 \delta_2 &= m_1 (n_2 + m_1 n_1) n_1 p \sin^2 \theta \end{aligned}\right\} \quad \text{(VII)}$$

And, $$\left.\begin{aligned} \pi \alpha &= m_1 n_1 p \theta \\ \pi \beta &= m_2 n_2 p \varphi \end{aligned}\right\} \quad \text{(VIII)}$$

We would recommend the student to use equations (I) and (II) in working out problems, instead of attempting to remember all the above particular forms which they assume.

Example I.—The flanks of the teeth of a pair of wheels are radial. The number of teeth on the wheels are 21 and 120. The addendum to each wheel is $\frac{3}{10}$ pitch. Find the lengths of the arcs of approach and recess, supposing the small wheel to be the driver.

Answer.—Here $n_1 = 21$; $n_2 = 120$; $\delta_1 = \delta_2 = \frac{3}{10} p$.

$$\therefore \quad R_1 = \frac{n_1 p}{2\pi} = \frac{21 p}{2\pi},$$

And, $$R_2 = \frac{n_2 p}{2\pi} = \frac{120 p}{2\pi}.$$

Since the flanks are radial $r_1 = \frac{1}{2} R_2 = \frac{120 p}{4\pi}$.

And, $$r_2 = \frac{1}{2} R_1 = \frac{21 p}{4\pi},$$

From equation (I), we get :—

$$2 R_1 \delta_1 + \delta_1^2 = 4 (R_1 + r_1) r_1 \sin^2 \varphi.$$

$$\therefore \quad 2 \times \frac{21 p}{2\pi} \times \frac{3 p}{10} + \frac{9 p^2}{100} = 4 \left(\frac{21 p}{2\pi} + \frac{120 p}{4\pi} \right) \times \frac{120 p}{4\pi} \sin^2 \varphi$$

$$\therefore \quad \frac{2 \times 21 \times 3 \times \pi}{10} + \frac{9 \times 2\pi^2}{100} = 162 \times 60 \sin^2 \varphi.$$

$$\therefore \quad \sin \varphi = .0652.$$

Or, $$\varphi = 3\tfrac{3}{4}^\circ, \text{ nearly } = .065 \text{ radian.}$$

Hence, **Arc of recess** $= \beta = 2\, r_1\, \varphi$

$$\text{,,} \qquad = 2 \times \frac{120\,p}{4\,\pi} \times \cdot 065 = 1{\cdot}25\,p, \text{ nearly}$$

Next, to find the arc of approach.

From equation (I_a), we get :—

$$2\, R_2\, \delta_2 + \delta_2^2 = 4\,(R_2 + r_2)\, r_2 \sin^2 \theta.$$

$$\therefore \quad 2 \times \frac{120\,p}{2\,\pi} \times \frac{3\,p}{10} + \frac{9\,p^2}{100} = 4\left(\frac{120\,p}{2\,\pi} + \frac{21\,p}{4\,\pi}\right) \times \frac{21\,p}{4\,\pi} \sin^2 \theta.$$

$$\therefore \quad 2 \times 12 \times 3 \times 2\pi + \frac{9 \times 4\,\pi^2}{100} = 261 \times 21 \sin^2 \theta.$$

$$\therefore \qquad \sin \theta = \cdot 2885.$$

$$\therefore \qquad \theta = 16\tfrac{3}{4}^\circ \text{ nearly} = \cdot 292 \text{ radian.}$$

Hence, **Arc of approach** $= \alpha = 2\, r_2\, \theta$

$$\text{,,} \qquad = 2 \times \frac{21\,p}{4\,\pi} \times \cdot 292 = \cdot 976\,p, \text{ nearly.}$$

Had we neglected δ^2 in the above solution, and taken the approximate formulæ (III), we would have got :—

$$\sin \varphi = \cdot 0638, \text{ instead of } \cdot 0652.$$

Now the difference in those two angles is only about 5 minutes, the first sine corresponding to an angle of 3° 39′, the second corresponding to an angle of 3° 44′.

Again, the exact value of θ is 16° 46′, while the approximate value (neglecting δ^2) would be 16° 42′.

Had we, therefore, assumed the approximate formulæ the results would practically not have been different from what we have just obtained.

EXAMPLE II.—In Example I., find the addenda, when the arcs of approach and recess are each equal to half the pitch.

ANSWER.—Here, $\alpha = \beta = \frac{1}{2}p.$

From equation (II_a), we get:—

$$\theta = \frac{\alpha}{2\, r_2} = \frac{\frac{1}{2}p}{2 \times \frac{21\,p}{4\,\pi}} = \frac{\pi}{21}.$$

$$\therefore \qquad \theta^\circ = \frac{\pi}{21} \times \frac{180}{\pi} = 8^\circ 34'.$$

$$\therefore \qquad \sin \theta = \cdot 149.$$

And from II,
$$\varphi = \frac{\beta}{2 r_1} = \frac{\frac{1}{2} p}{2 \times \frac{120 p}{4 \pi}} = \frac{\pi}{120}.$$

$$\therefore \qquad \varphi^\circ = \frac{\pi}{120} \times \frac{180}{\pi} = 1^\circ 30'.$$

Or,
$$\sin \varphi = \cdot 0262.$$

Hence, taking the approximate formulæ (III), we get :—

$$\delta_1 = 2 \left(1 + \frac{r_1}{R_1}\right) r_1 \sin^2 \varphi$$

$$\text{,,} = 2 \left(1 + \frac{60}{21}\right) \frac{120 p}{4 \pi} \times \cdot 0262^2 = \cdot 05 p$$

Similarly,
$$\delta_2 = 2 \left(1 + \frac{r_2}{R_2}\right) r_2 \sin^2 \theta.$$

$$\text{,,} = 2 \left(1 + \frac{21}{240}\right) \frac{21 p}{4 \pi} \times \cdot 149^2 = \cdot 08 p.$$

In this example we might have further simplified our calculations by writing $\sin \theta = \theta$ and $\sin \varphi = \varphi$, since these angles are very small. Doing this and combining equations (I) and (II), we get the following approximate formulæ :—

$$\left.\begin{aligned} \delta_1 &= \tfrac{1}{2} \left(\frac{1}{R_1} + \frac{1}{r_1}\right) \beta^2 \\ \delta_2 &= \tfrac{1}{2} \left(\frac{1}{R_2} + \frac{1}{r_2}\right) \alpha^2 \end{aligned}\right\} \quad \ldots \ldots \quad \text{(IX)}$$

Substituting for α, β, R_1, R_2, r_1, r_2, we get :—

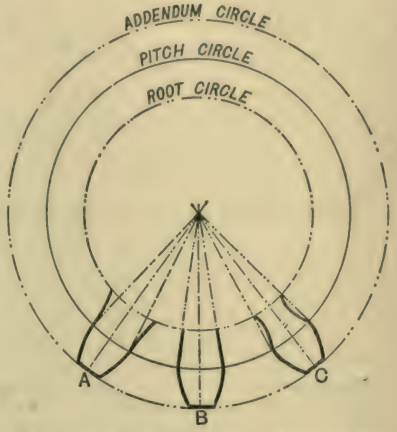
$$\delta_1 = \tfrac{1}{2} \left\{ \frac{2 \pi}{21 p} + \frac{4 \pi}{120 p} \right\} \times \left(\frac{p}{2}\right)^2 = \cdot 05 p,$$

$$\delta_2 = \tfrac{1}{2} \left\{ \frac{2 \pi}{120 p} + \frac{4 \pi}{21 p} \right\} \times \left(\frac{p}{2}\right)^2 = \cdot 08 p,$$

which are exactly the same results as before.

Diameter of Generating Circle.—We have already had instances of the effects produced on the form of cyloidal teeth, by the size of the generating circle employed. We have seen that if the size

of the generating circle be half that of the pitch circle inside which it rolls, the flanks of the teeth so described are radial. If the generating circle be larger than this, the flanks described by it will be undercut and comparatively weak at the roots. The accompanying figure illustrates the influence of the size of the generating circle on the form of the teeth. The flanks of the teeth, A, B, and C, are described by generating circles, having respectively diameters less than, equal to, and greater than the radius of the pitch circle. From these figures it is evident that the generating circle employed in describing the flanks of the teeth on a wheel should never have a diameter greater than half that of the pitch circle of the wheel.



INFLUENCE OF SIZE OF GENERATING CIRCLE ON THE FORM OF TEETH.

If a set of wheels, such as the change wheels for a screw-cutting lathe, have to gear together in different arrangements, it is clear that the same generating circle must be used for describing both faces and flanks of the teeth of every wheel in the set. This being the case, it follows from what has been said above that **the diameter of the generating circle employed in describing the faces and flanks of the teeth of a set of wheels must not be greater than half that of the pitch circle of the smallest wheel in the set,** otherwise the flanks of the teeth on the smallest wheel would be undercut and weak at the roots.* Since it is desirable to have as large a generating circle as possible, it is usual to construct the teeth on the smallest wheel with radial flanks. *The size of the generating circle must then have a diameter equal to the radius of the smallest wheel in the set.*

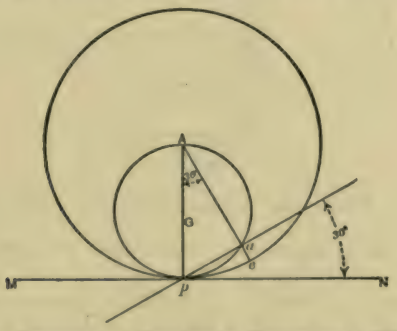
Least Number of Cycloidal Teeth to be placed upon a Wheel.—It has been pointed out, that with any pair of wheels gearing together, *there should never be fewer than two pairs of consecutive*

* Cases are not wanting where the flanks have been described by a generating circle larger than the radius of the pitch circle; but in such cases fillets are made at the roots in order to strengthen the teeth.

teeth in action at any time, and, further, *that the maximum obliquity of reaction should never exceed* 30°, in the case of cycloidal teeth. These conditions being premised, it is easy to determine the least number of teeth which must be placed upon the smallest wheel of a set.

Let A be the centre of the pitch circle of the smallest wheel in the set; G the generating circle, the diameter of which is half that of pitch circle, A.

Let contact between a pair of teeth begin at a, then $\angle a\,p\,N = 30°$. When one pair of teeth are in contact at a, the preceding consecutive pair will be in contact at the pitch point, p. Make arc $p\,e$ = arc $p\,a$. Then in this case, $p\,e$ is the arc of approach, and is equal to the pitch of the teeth. Join A e. This line will



LEAST NUMBER OF CYCLOIDAL TEETH ON A WHEEL.

pass through the point a; hence, it is easy to see that $\angle p\,A\,e = \angle a\,p\,N = 30°$. We then get:—

Arc $p\,e$ = arc of 30° on pitch circle A,

" = $\frac{1}{12}$ of circumference of pitch circle A.

But arc $p\,e$ = pitch of teeth,

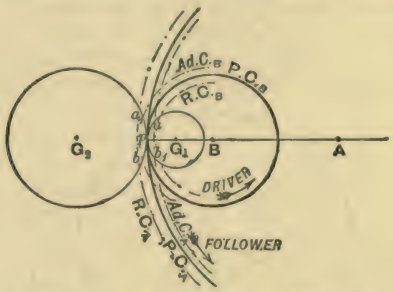
$\therefore$ Pitch of teeth = $\frac{1}{12}$ of circumference of pitch circle A,

i.e, the least number of teeth on the smallest wheel of a set must be 12.

In a similar way, it can be shown that the least number of pins to be placed upon a pin or lantern wheel is 6.

Cycloidal Teeth for Wheels with Internal Contact.—Let A be the centre of an internal toothed wheel; B the centre of a pinion gearing with wheel A. The faces of the teeth on A and the flanks of the teeth on B are described by the same generating circle, G_1, while the flanks of the teeth on A and the faces of the teeth on B

are described by the generating circle G_2. Since the *faces* of the teeth on an internal toothed wheel lie *inside* the pitch surface, and the *flanks outside* the pitch surface, it is clear that the curves for the former are arcs of *hypocycloids* obtained by rolling G_1 inside the pitch circle A, while the curves for the flanks are *epicycloids* obtained by rolling G_2 outside the pitch circle A. Hence, the curves for the faces of the teeth on A and the flanks of the teeth on B are hypocycloids, while the curves for the flanks of the teeth on A and the faces of the teeth on B are epicycloids. The size of



CYCLOIDAL TEETH FOR WHEELS WITH INTERNAL CONTACT.

generating circle, G_1, must not exceed half that of pitch circle of the pinion, B, otherwise the roots of the teeth on the pinion will be undercut and weak. The size of generating circle, G_2, may be anything we like, since the curves described by it on both pitch circles are epicycloids.

Path of Contact with Internal Gearing.—Let P C, Ad C, and R C, with the suffixes A and B, denote the pitch, addendum, and root circles of wheels A and B respectively. Let the pinion be the driver. Contact between a pair of teeth begins at a, the point of intersection of circles G_1 and Ad C_A, and terminates at b, the point of intersection of circles G_2 and Ad C_B; $a\,p\,b$ is, therefore, the path of contact. If the annular wheel were the driver, then the curve, $a_1\,p\,b_1$, would represent the path of contact, the direction of motion being the same as before. The student should experience little difficulty in applying all the preceding principles to internal toothed gearing if he has followed intelligently what has already been said regarding wheels with external contact.

Formulæ for Length of Teeth of Internal Gearing.—The student should now prove the following formulæ for internal gearing, the method of arriving at the results being similar to that previously given.

(1) *When the Pinion is the Driver.*

For Pinion:— $2\,R_1\,\delta_1 + \delta_1^2 = 4\,(R_1 + r_1)\,r_1 \sin^2 \varphi$

And, $2\,r_1\,\varphi = \beta.$

For Annular Wheel, $2\,R_2\,\delta_2 - \delta_2^2 = 4\,(R_2 - r_2)\,r_2 \sin^2 \theta$

And, $2\,r_2\,\theta = \alpha.$

(2) *When the Pinion is the Follower.*

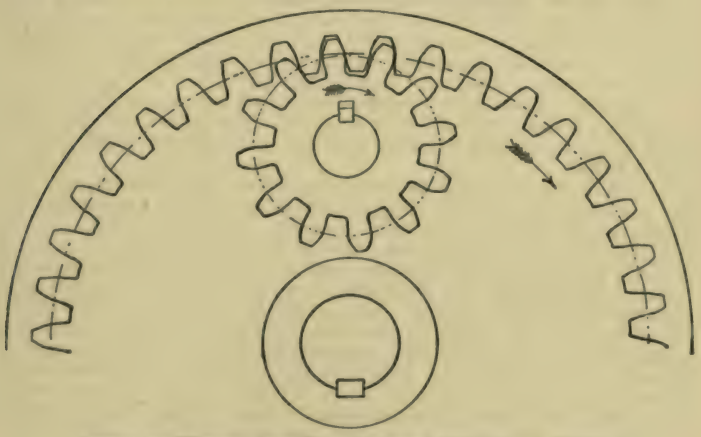
For Pinion:— $2\,R_2\,\delta_2 + \delta_2^2 = 4\,(R_2 + r_2)\,r_2 \sin^2 \theta$

And, $2\,r_2\,\theta = \alpha.$

For Annular Wheel, $2\,R_1\,\delta_1 - \delta_1^2 = 4\,(R_1 - r_1)\,r_1 \sin^2 \varphi$

And, $2\,r_1\,\varphi = \beta.$

The various symbols have the same meanings as before.



ANNULAR WHEEL AND PINION WITH CYCLOIDAL TEETH.

Lecture XII.—Questions.

1 Distinguish between the terms *arc of approach* or *recess* and *path of approach* or *recess*. Given the pitch circles of a pair of wheels, sizes of generating circles, and arcs of approach and recess, show by a construction how to set out the path of contact.

2. Upon what principle are teeth of wheels of the epicycloidal and hypocycloidal form constructed? Show under what conditions they will work properly. What is to be done in order that any wheels of a set may work accurately together?

3. In the consideration of the form suitable for the teeth of spur wheels, state:—(*a*) What geometrical condition should be satisfied as to the position of the common normal at the point of contact of two teeth, and explain why that condition should be satisfied; (*b*) within what limits it is desirable to keep the obliquity of the line of action of the pressure between two teeth, and why within those limits; (*c*) the least number of pairs of teeth which it is desirable should be engaged at the same time. Explain also (*d*) why it is undesirable that the action between two teeth should extend far from the pitch point. By a graphical construction, determine the arc of action, and the greatest obliquity of the line of action of a pair of cycloidal teeth, according to the following data, and state how many pairs of teeth will be in action at the same time:—Pitch of teeth = 2 inches; number of teeth in wheels = 30 and 50; diameter of rolling or describing circles = $8\frac{3}{4}$ inches; addenda of teeth = $\frac{5}{8}$ inch. *Ans.* 4 inches; 13·5°; 3.

4. Show by construction how you would determine the correct form for the teeth of a spur wheel 4 feet in diameter. The diameter of the smallest wheel in the train being 8 inches, what sized rolling circle would you use? *Ans.* 4 inches.

5. A pair of wheels have 25 and 130 cycloidal teeth respectively. Find the addendum of each wheel, that the arcs of approach and recess may each be equal to the pitch, the flanks being radial. *Ans.* ·17 p; ·28 p.

6. The diameter of the pitch circle of an annular wheel by means of which a water-wheel communicates motion to a mill is to differ as little as possible from 24 feet. The pitch of the teeth is to be 4 inches. Find actual diameter and number of teeth. If the velocity of the periphery be $5\frac{1}{2}$ feet per second, and the pinion in gear with the wheel is required to make 30 revolutions per minute, find the necessary diameter of the pinion and the number of teeth. Again, if both faces and flanks of all the teeth be described by a constant generating circle 12 inches in diameter, find the arcs of approach and recess, the addendum of both wheels being $1\frac{1}{4}$ inches. *Ans.* 23·98 ft.; 226 teeth; 3·5 ft.; 33 teeth; $a = 3{\cdot}42$ ins.; $\beta = 3{\cdot}96$ ins.

Lecture XII.—A.M.Inst.C.E. Exam. Questions.

1. What is meant by a "set" of cycloidal-toothed wheels? Under what circumstances are the flanks of cycloidal teeth straight? (I.C.E., *Feb.*, 1898.)

2. How do we find the shape of a tooth which will gear with a given tooth? Prove the truth of your rule. (I.C.E., *Oct.*, 1898.)

3. Select either the involute or cycloidal form of tooth for a spur wheel, and show that it satisfies the required geometrical condition. Point out on a diagram the "path of the point of contact," and explain how the two ends of the path are determined. (I.C.E., *Oct.*, 1904.)

LECTURE XIII.

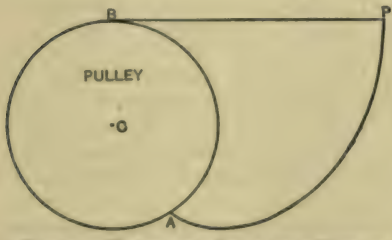
CONTENTS.—Involute Teeth—Size of Base Circle to be employed—Length of Involute Teeth for given Arcs of Approach and Recess—Calculation of the Length of Involute Teeth—Least Number of Involute Teeth to be placed upon a Wheel—Rack and Pinion with Involute Teeth—Wheels with Involute Teeth and Internal Contact—Calculations for Involute Teeth with Internal Contact—Examples I. and II.—Bevel Wheels—Teeth of Bevel Wheels—Mortice Wheels—Machine Cut Gearing—Spur Gear Cutting Machine—Correct Form of Bevel Wheel Teeth—Gibson's Bevel Wheel Cutting Machine—Bevel Wheel Cutting Machine—Worm Wheels—Worm Wheel Cutting Machine—Small Wheel Cutting Machine—Rawhide Pinions—Questions.

Involute Teeth.—In a previous Lecture we have shown that an involute curve is a particular case of an epicycloid, and with those particular conditions we have had instances of teeth with *involute faces*. But no case has yet been considered wherein both faces and flanks are involute in form. Involute teeth—*i.e.*, those forms of teeth whose faces and flanks are described by involutes of circles—possess certain peculiar properties, and on that account may be studied as a class independent of all other forms.

In this Lecture, we shall first show that the involute form of tooth satisfies all the primary conditions for correct working, and we shall thereafter explain its unique properties.

In order to properly understand what follows, we shall first explain how an involute curve can be drawn.

Let C represent the centre of a thin pulley with a fine string wound round its circumference. Fix a sheet of drawing paper to one of the faces of the pulley, and tie a pencil, P, to the free end of the string. By keeping the string taut and unwinding it from the pulley, the pencil at P will trace on the drawing paper the involute, A P.



HOW TO TRACE AN INVOLUTE CURVE.

For our present purpose it is much better to conceive the curve described in the following manner:—

Let the pulley and its attached paper be capable of turning round C as an axis. Take hold of the pencil, P, and pull it along the straight line, B P. By this means the pulley and

paper will be made to rotate about C, while the pencil will trace out the involute, A P, on the moving paper just as before. Similarly, if the string be wound on to the pulley, by rotating the pulley and the paper, the pencil will retrace the curve P A, if P be made to move in the direction, P B.

The circle of which the curve is the involute is called the *base circle.*

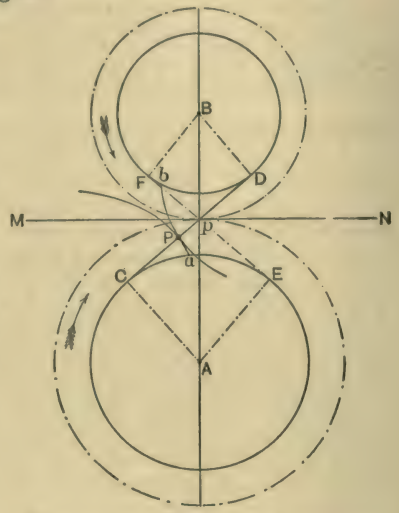
In the next figure, let the dotted circles represent the pitch circles of a pair of wheels in gear.

With A and B as centres, describe the circles C *a* E, D *b* F, of such sizes that

$$A C : B D = A p : B p.$$

These are the base circles for the pair of wheels.

Now imagine these base circles to represent pulleys over which a *crossed* string, C D F E, is stretched. Then, clearly, the motion transmitted by means of these pulleys and the crossed string will be identically the same as that obtained by the rolling of the pitch circles.



TRACING THE CURVES FOR INVOLUTE TEETH.

Suppose a sheet of paper to be fixed to pulley, A, and capable of rotating with it. Then a pencil, P, anywhere on the string, C D, will, during rotation in the direction shown, describe the involute, *a* P, on the rotating paper. The curve, *a* P, is an involute to the base circle, C *a* E. Similarly, by supposing a sheet of paper to have been fixed to pulley, B, the pencil would have, simultaneously described the involute, *b* P, to the base circle, D *b* F.

The two involutes, *a* P, *b* P, being simultaneously described by the tracing point, P, will always be in contact at that point, and have a common normal, C D. The locus of P is C D, and therefore the common normal always passes through the pitch point, *p*. Hence the principal condition for the curves of all properly-constructed teeth is satisfied by involutes traced in the above manner.

By fixing a pencil to the string, E F, and proceeding as before, the curves for the opposite working surfaces of the teeth can be drawn.

To complete the curves representing the working surfaces of the teeth, it is only necessary to describe the addendum and root circles; the parts of the involutes intercepted between these circles will represent the acting surfaces of the complete teeth.

The following properties of involute teeth should be noted :—

(1) *Both face and flank of an involute tooth form one continuous curve.*

On this account it will be much easier to set out the curves for involute teeth than for cycloidal teeth, since, in the latter, the curves for face and flank are always of opposite convexity.

(2) *With involute teeth the centres of the wheels can be pushed further apart or brought closer together without affecting their velocity-ratio or smoothness of action.*

This is a most valuable and unique property of such teeth. The reasons for this property will be apparent from an inspection of the previous figure. Pushing the wheels further apart, or bringing them closer together, alters the sizes of the pitch circles without altering their ratio, but does not alter the sizes of the base circles, and, therefore, does not affect the curvature of the involutes. It does, however, affect the direction of the normal thrust between the teeth. The direction of this thrust is always along the common tangent to the base circles. More will be said about this immediately.

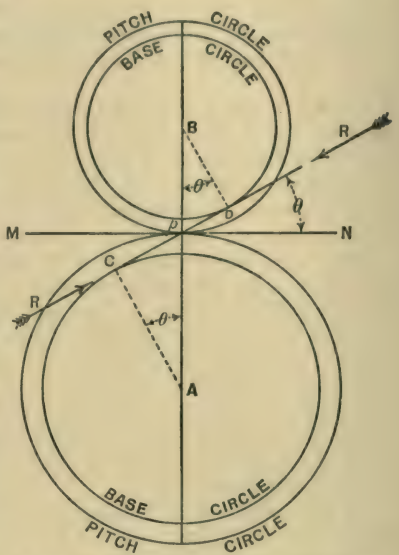
(3) *All wheels with involute teeth of equal pitch and obliquity of action work accurately with each other.*

The reasons for this will also be apparent from what has just been said.

From the above properties it will be seen, that involute teeth are singularly well suited for most purposes, and in our opinion it would be well if engineers would universally adopt this form, and thus save endless expense and trouble in patterns, &c. As matters now stand, each maker of toothed wheels has his own method of constructing the teeth; the result being, that the wheels of one maker will not work correctly, nor approximately correct, with those of equal pitch by other makers. The same state of affairs with regard to screw threads existed prior to the establishment of a standard thread by the late Sir Joseph Whitworth. At that time, if the nut of a bolt were lost, it was ten chances to one if another could be found to fit it, with the result that many good bolts had to be thrown into the scrap heap for want of nuts. The nuts, too, were of all sizes, even for the same size of bolt, and as a consequence every fitter had to be supplied

with a multitude of spanners whenever he had a series of bolts to deal with.

One objection which has been urged by some engineers against involute teeth is the normal thrust between the teeth, which is always constant in direction, and, being oblique to the common tangent at the pitch point, tends to push the wheels out of gear. We are of opinion, however, that too much importance has been attached to this; for, if the obliquity of reaction be kept less than 15°, no very serious results will follow. With cycloidal teeth, the obliquity of reaction varies from zero, when the pair of teeth are in contact at the pitch point, to a maximum, when the teeth are just beginning or just ending contact.



SIZE OF BASE CIRCLE FOR INVOLUTE TEETH.

Size of Base Circle to be employed.—We have shown that the line of action of a pair of involute teeth in contact is always along a common tangent to the base circles. The direction of the mutual thrust, or, in other words, the obliquity of reaction, is constant.

Let R = Thrust between a pair of teeth in contact.
,, θ = Obliquity of reaction = $\angle$ D p N.
,, $R_1 R_2$ = Radii of pitch circles, A and B.
,, $r_1 r_2$ = ,, base ,, ,,

Then, *Component of* R *along* M N = R cos θ.

Component of R *along* A B = R sin θ.

The former of these represents the effective pressure causing motion, while the latter tends to throw the wheels out of gear, and has, therefore, to be resisted by the bearings at A and B. To reduce this separating force to a minimum, θ must be made as small as possible. In practice, θ never exceeds $14\frac{1}{2}°$ or 15°.

Taking θ at 15°, we can easily calculate the size of base circle for any given size of pitch circle.

Referring to the above figure, we see that :—

$$\angle p\,A\,C = \angle p\,B\,D = \theta.$$

Hence, $r_1 = R_1 \cos \theta.$

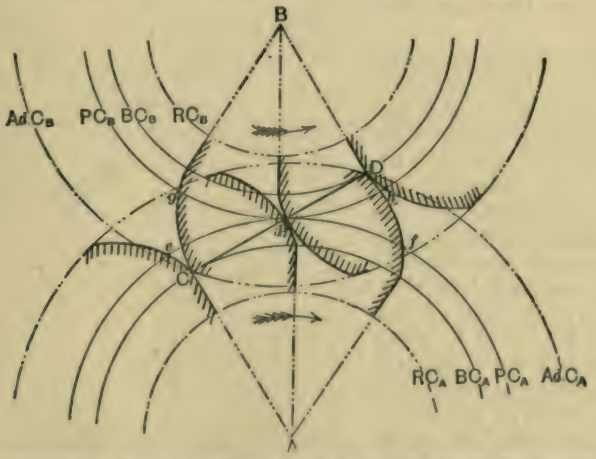
$r_2 = R_2 \cos \theta.$

But, $\cos \theta = \cos 15° = \cdot 966$ or $\frac{63}{65}$, nearly.

$$\therefore \quad r_1 = \cdot 966\ R_1 = \frac{63}{65} R_1 . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (I)$$

Or, Diameter of base circle $= \frac{63}{65}$ (diameter of pitch circle).

Length of Involute Teeth for given Arcs of Approach and Recess.—With involute teeth the path of contact is a straight line, which is a tangent to the base circles. In the accompanying figure (which is much exaggerated for the sake of clearness) three pairs of teeth are shown in contact. One pair is beginning contact



LENGTH OF INVOLUTE TEETH FOR GIVEN ARCS OF APPROACH AND RECESS.

at C, a second pair is in contact at the pitch point, p, while the third pair is terminating contact at D. Contact cannot commence before C, nor be carried beyond D. If, therefore, the maximum length of contact be utilised, then the addendum circles for A and

B must be drawn through the points D and C respectively. After allowing for bottom clearance, the root circles, R C_A, R C_B, can be drawn.

From the figure it will be seen, that the root circles fall inside the base circles and are concentric with them. Now the curves forming the working surfaces of the teeth, being involutes of the base circles, do not pass beyond those base circles, and consequently those parts of the roots of the teeth lying between the base and root circles must be formed by some line straight or curved. Since, however, those parts are not portions of the working surfaces they are usually made straight and radial.

If arc $e\,p$ = arc $g\,p$ = arc of approach, and arc $p\,f$ = arc $p\,h$ = arc of recess, then, manifestly, e, f, g, and h are points of intersection of the curves of the teeth with the pitch circles.

We are now able to find the addenda of the teeth for given arcs of approach and recess; or conversely, to find the arcs of approach and recess when the addenda of the teeth are given.

To Find the Addenda of Involute Teeth.

Draw the pitch circles for the two wheels. Through p draw the line of mutual pressure, C D, making an angle, θ, with M N. As already explained, θ should not exceed 15°. With A and B as centres draw the base circles tangential to C D. Along pitch circle A set off arc $p\,e$ equal to given arc of approach, and on pitch circle B set off arc $p\,h$ equal to given arc of recess. Join A e, B h. These radii cut the base circles at c and d respectively. Then arc $c\,m$ is the arc turned through by base circle, A, during approach, while arc $n\,d$ is the arc turned through by base circle, B, during recess.

Along C D set off $p\,a$ = arc $c\,m$, and $p\,b$ = arc $d\,n$. Then the straight line $a\,p\,b$ is the path of contact.

Through b and a draw the addendum circles as shown. After allowing for clearance the root circles can be drawn in.

The solution of the converse proposition should not present much difficulty to the student.

Calculation of the Length of Involute Teeth.—Suppose wheel A to be the driver.

Let R_1, R_2 = Radii of pitch circles of wheel A and B.
„ n_1, n_2 = Number of teeth on „ „
„ p = Pitch of teeth.
„ δ_1, δ_2 = Addenda of teeth on A and B.
„ α, β = Arcs of approach and recess respectively.
„ θ = Obliquity of mutual pressure.

Referring to the previous figure, join A to b,* then, A b C is a right-angled triangle.

$$\therefore \quad A\,b^2 = A\,C^2 + C\,b^2.$$

But,
$$A\,b = R_1 + \delta_1,$$
$$A\,C = R_1 \cos\theta,$$

And,
$$C\,b = C\,p + p\,b = R_1 \sin\theta + p\,b.$$

Now,
$$p\,b = \text{path of recess}$$
$$\therefore \quad \text{„} = \text{arc } n\,d \text{ of base circle B.}$$

But,
$$\frac{\text{Arc } n\,d}{\text{Arc } p\,h} = \frac{\text{Radius of base circle B}}{\text{Radius of pitch circle B}} = \cos\theta.$$

$$\therefore \quad \text{Arc } n\,d = \beta \cos\theta.$$

$$\therefore \quad C\,b = R_1 \sin\theta + \beta\cos\theta.$$

$$\therefore \quad (R_1 + \delta_1)^2 = R_1^2\cos^2\theta + (R_1 \sin\theta + \beta\cos\theta)^2.$$

$$\therefore \quad 2R_1\delta_1 + \delta_1^2 = 2R_1\beta\sin\theta\cos\theta + \beta^2\cos^2\theta$$
$$\text{„} \quad = R_1\beta\sin 2\theta + \beta^2\cos^2\theta.$$

Or,
$$2R_1\delta_1 + \delta_1^2 = (R_1 \sin 2\theta + \beta\cos^2\theta)\beta$$
Similarly,
$$2R_2\delta_2 + \delta_2^2 = (R_2 \sin 2\theta + \alpha\cos^2\theta)\alpha \quad \Big\} \quad . \quad . \quad \text{(II)}$$

Generally δ is small compared with R, therefore we may neglect δ^2, and we get the approximate formulæ :—

$$2R_1\delta_1 = (R_1 \sin 2\theta + \beta\cos^2\theta)\beta$$
$$2R_2\delta_2 = (R_2 \sin 2\theta + \alpha\cos^2\theta)\alpha \quad \Big\} \quad . \quad \text{(III)}$$

* The line A b has been accidently omitted, and the student should now draw it on the figure before proceeding further.

Again, since $R_1 = \frac{n_1 p}{2\pi}$, $R_2 = \frac{n_2 p}{2\pi}$, and usually $\theta = 15°$.

$$\therefore \quad \sin 2\theta = \sin 30° = \frac{1}{2}$$

And, $$\cos^2 \theta = \cos^2 15° = \frac{2 + \sqrt{3}}{4}.$$

Making these substitutions in (III), we get :—

$$2 \times \frac{n_1 p}{2\pi} \times \delta_1 = \left(\frac{n_1 p}{2\pi} \times \frac{1}{2} + \beta \times \frac{2 + \sqrt{3}}{4}\right) \beta.$$

$$\therefore \quad n_1 p\, \delta_1 = \left(\frac{n_1 p}{4} + \beta \times \frac{3{\cdot}732 \times 3{\cdot}1416}{4}\right) \beta$$

$$\text{,,} \quad = \left(\frac{n_1 p}{4} + 2{\cdot}931\, \beta\right) \beta.$$

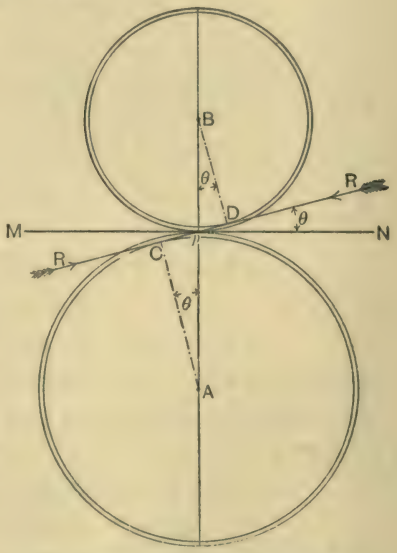
$$\therefore \quad \delta_1 = \left(\frac{1}{4} + \frac{3\beta}{n_1 p}\right) \beta$$

$$\text{Similarly,} \quad \delta_2 = \left(\frac{1}{4} + \frac{3\alpha}{n_2 p}\right) \alpha \qquad \text{. . . . . (IV)}$$

Least Number of Involute Teeth to be placed upon a Wheel.—Let A and B be the centres of a pair of wheels in gear. Set out the obliquity line, C D, making an angle, θ, with the common tangent, M N, to the pitch circles. Draw the base circles tangential to C D. Then C D is the maximum length of path of contact.

Let B be the smaller wheel, then p D is less than p C.

Now, in order that there may not be less than two pairs of teeth in contact at any instant, it is clear that if one pair of teeth be just ending contact at D, another pair must be just



To Find the Least Number of Involute Teeth for a Wheel.

beginning contact at some point near C, while an intermediate pair is in contact at p.

Let r = Radius of base circle of smaller wheel B.
„ n = Least number of teeth on „ „

Then the minimum number of teeth on B will occur when the whole path p D is utilised and when θ has its maximum value.

Hence, $n \times p\,D$ = Circumference of base circle B = $2\pi r$

But, $p\,D = B\,D \tan \theta = r \tan \theta$.

$\therefore$ $n r \tan \theta = 2\pi r$.

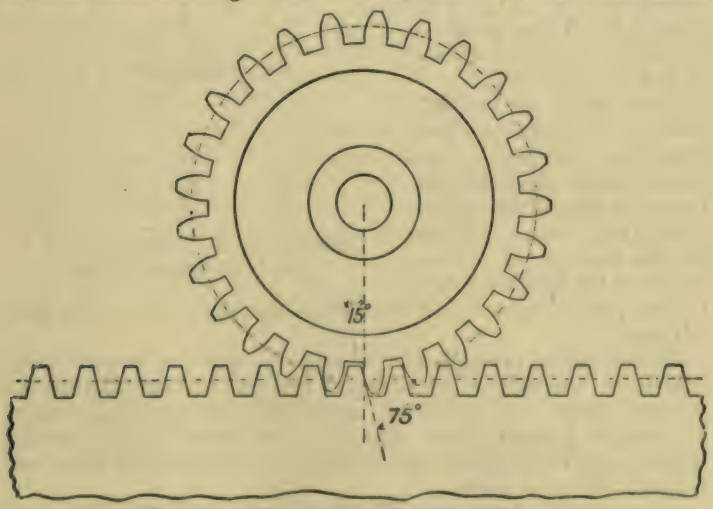
$\therefore$ $$n = \frac{2\pi}{\tan \theta}.$$

Since θ must not exceed 15°, we get :—

$$n = \frac{2 \times 3{\cdot}1416}{{\cdot}2679} = 23{\cdot}45.$$

i.e., The least number of involute teeth to be placed upon a wheel is 24.

Rack and Pinion with Involute Teeth.—When a pinion with involute teeth has to gear with a rack, then the teeth of the latter



RACK AND PINION WITH INVOLUTE TEETH.

must also be involute. Now, the pitch circle of the rack is infinite in size, hence its base circle is also infinite in size, and, therefore,

the involute corresponding to it will be a straight line. Hence, the face and flank of a tooth on the rack will be a straight line.

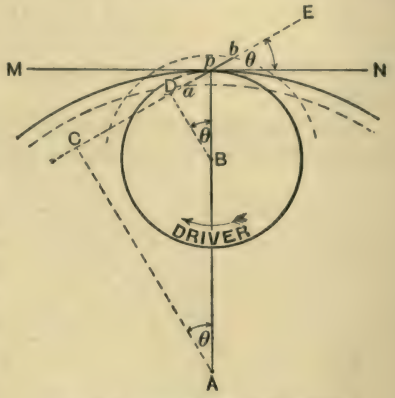
Since the working surface of the teeth during contact must be perpendicular to the line of obliquity; and, since this line makes an angle of about 15° with the common tangent to the pitch circles, it follows that the working surfaces of the teeth on the rack (being straight) make constant angles of 75° with this common tangent. And clearly, in the case of a rack and pinion, the common tangent coincides with the pitch line of the rack.

The previous figure represents a rack and pinion with involute teeth.

Wheels with Involute Teeth and Internal Contact.—Draw the pitch circles of the annular wheel, A, and its pinion, B. Let M N be the common tangent to the pitch circles at the pitch point, p. Through p draw the obliquity line, C p E, making an angle, θ, with M N.

From A and B draw the perpendiculars A C, B D upon C E. These are radii of the base circles for wheels A and B respectively.

Next draw in the addendum circles for the two wheels as indicated. Let these circles intersect the obliquity line in the points a and b respectively. Then, for the direction of motion shown in the figure, $a\,b$ will be the path of contact, $a\,p$ the path of approach, and $p\,b$ the path of recess.



WHEELS WITH INVOLUTE TEETH AND INTERNAL CONTACT.

The preceding principles for wheels with external contact apply equally to the case of wheels with internal contact, so that the student should not experience much difficulty in applying them. He should, however, note that the base circle for the annular wheel must be less than its addendum circle, which, in this case, is inside the pitch circle.

Calculations for Involute Teeth with Internal Contact.—The student should now prove the following formulæ for internal gearing, the method of arriving at the results being similar to that previously given.

Let the symbols with the suffixes 1 and 2 refer to the driver and follower respectively. Then :—

(1) *When the Pinion is the Driver.*

$$\left.\begin{aligned} 2\,R_1\,\delta_1 + \delta_1^2 &= (R_1 \sin 2\theta + \beta \cos^2\theta)\,\beta \\ 2\,R_2\,\delta_2 - \delta_2^2 &= (R_2 \sin 2\theta - \alpha \cos^2\theta)\,\alpha \end{aligned}\right\} \quad . \quad . \quad . \quad (V)$$

(2) *When the Annular Wheel is the Driver.*

$$\left.\begin{aligned} 2\,R_1\,\delta_1 - \delta_1^2 &= (R_1 \sin 2\theta - \beta \cos^2\theta)\,\beta \\ 2\,R_2\,\delta_2 + \delta_2^2 &= (R_2 \sin 2\theta + \alpha \cos^2\theta)\,\alpha \end{aligned}\right\} \quad . \quad . \quad . \quad (V_a)$$

The approximate formulæ corresponding to these are :—

(1)
$$\left.\begin{aligned} \delta_1 &= \left(\frac{1}{4} + \frac{3\,\beta}{n_1\,p}\right)\beta \\ \delta_2 &= \left(\frac{1}{4} - \frac{3\,\alpha}{n_2\,p}\right)\alpha \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (VI)$$

(2)
$$\left.\begin{aligned} \delta_1 &= \left(\frac{1}{4} - \frac{3\,\beta}{n_1\,p}\right)\beta \\ \delta_2 &= \left(\frac{1}{4} + \frac{3\,\alpha}{n_2\,p}\right)\alpha \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (VI_a)$$

The student should also notice that the formulæ for internal gearing are at once deduced from those for external gearing by considering the radius of the annular wheel as negative.

EXAMPLE I.—A pair of wheels with involute teeth have 30 and 120 teeth respectively. The addendum to each wheel is $\frac{3}{10}$ pitch. Find the lengths of the arcs of approach and recess, supposing the obliquity to be 15°, and the small wheel the driver.

ANSWER.—Here $n_1 = 30$; $n_2 = 120$; $\delta_1 = \delta_2 = \frac{3}{10}\,p$.

From equation (IV), we get :—

$$\delta_2 = \left(\frac{1}{4} + \frac{3\,\alpha}{n_2\,p}\right)\alpha.$$

$$\therefore \quad \frac{3}{10}\,p = \left(\frac{1}{4} + \frac{3\,\alpha}{120\,p}\right)\alpha.$$

$$\therefore \quad \alpha^2 + 10\,p\,\alpha - 12\,p^2 = 0.$$

$$\therefore \quad \alpha = \frac{-10 \pm \sqrt{10^2 + 4 \times 12}}{2}\,p = 1{\cdot}08\,p.^*$$

i.e., Arc of approach = 1·08 × pitch.

* The positive sign for the radical must be taken, since neither α nor β can be negative.

Again, $$\delta_1 = \left(\frac{1}{4} + \frac{3\beta}{n_1 p}\right)\beta$$

$$\therefore \quad \frac{3}{10}p = \left(\frac{1}{4} + \frac{3\beta}{30\,p}\right)\beta$$

$$\therefore \quad 2\beta^2 + 5p\beta - 6p^2 = 0.$$

$$\therefore \quad \beta = \frac{-5 \pm \sqrt{5^2 + 4 \times 2 \times 6}}{2 \times 2}\,p = \cdot 885\,p\,*$$

i.e., Arc of Recess = ·885 × pitch.

EXAMPLE II.—With the same sizes of wheels as in last example, find the addenda of the wheels, the arcs of approach and recess being each $\frac{7}{8}$ of the pitch.

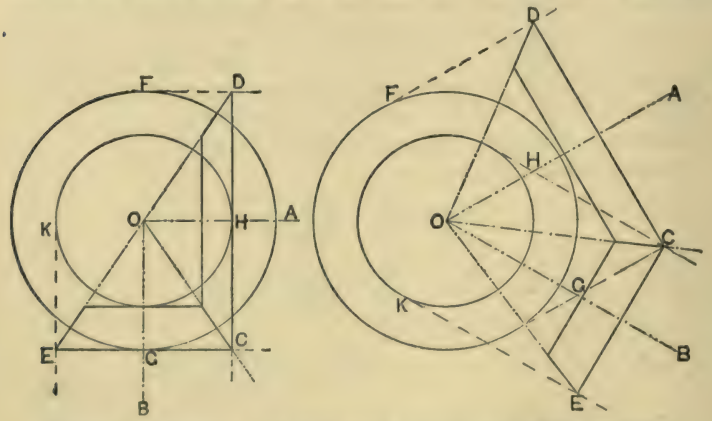
ANSWER.—Here $\alpha = \beta = \frac{7}{8}p$.

From equation (IV), we get :—

$$\delta_1 = \left(\frac{1}{4} + \frac{3 \times \frac{7}{8}p}{30\,p}\right) \times \frac{7}{8}p = \cdot 295 \times \text{pitch.}$$

Again, $$\delta_2 = \left(\frac{1}{4} + \frac{3 \times \frac{7}{8}p}{120\,p}\right) \times \frac{7}{8}p = \cdot 237 \times \text{pitch.}$$

Bevel Wheels. — The teeth on bevel wheels are constructed upon precisely the same principles as those on spur wheels. In the case of bevel wheels, however, the pitch surfaces are conical,



METHOD OF SETTING OUT THE PITCH CONES FOR BEVEL WHEELS.

* The positive sign for the radical must be taken, since neither α nor β can be negative.

and on this account it becomes necessary to give a brief description of the application of the preceding principles to such cases. In the first place we shall explain how to set out the pitch cones for a pair of bevel wheels whose angular velocity-ratio is given.

Two principal cases are shown by the foregoing figure—(1) when the axes of the shafts intersect at right angles, and (2) when they intersect at an acute angle. The letters on the two diagrams are so arranged that the following description is applicable to both.

Let O be the intersection of the axes, O A, O B, of the shafts. Suppose the angular velocity-ratio to be 2 : 3 ; *i.e.*, let :—

Angular velocity of shaft, O A : *Angular velocity of shaft*, O B = 2 : 3.

When the size of one of the wheels is given, that of the other wheel can be found in the usual way when the angular velocity-ratio is known. Thus :—

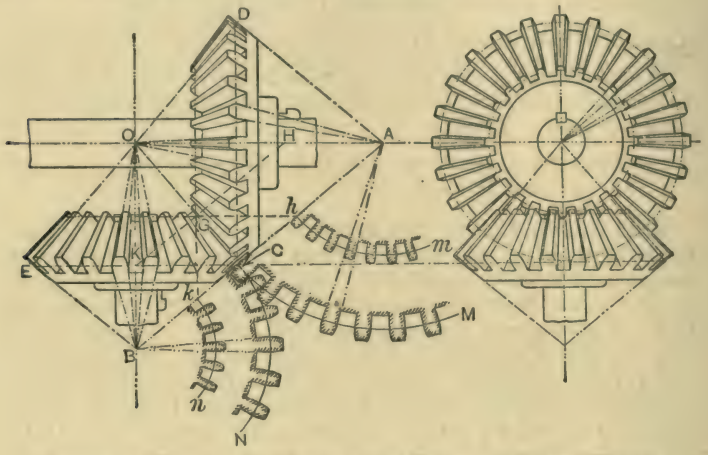
$$\left.\begin{matrix}\textit{Diameter of wheel on shaft, O A} : \textit{Diameter} \\ \textit{of wheel on shaft, O B}\end{matrix}\right\} = 3 : 2.$$

With centre, O, draw two circles of diameters equal or proportional to those of the wheels. From the larger circle draw the tangents, F D, G C, parallel to the axis, O A ; and from the smaller circle draw the tangents, H C, K E, parallel to the axis, O B. The tangents, G C, H C, intersect at C. The line, O C, is then the line of contact of the two pitch cones. By drawing C D and C E perpendicular to the axes, O A, O B, respectively, and meeting the tangents, F D, K E, in the points, D and E, and by joining O D, O E we get the complete pitch cones, C O D, C O E. In practice, frusta only of the pitch cones are used. These are shown by full lines on the figures.

Another method of setting out the pitch cones is as follows :—Along the axes, O A, O B, measure off distances, O H, O G, respectively proportional to the angular velocities of the shafts, O A and O B ; *i.e.*, in this particular case, let O H : O G = 2 : 3. Complete the parallelogram, O H C G ; then the diagonal, O C, is the line of contact of the pitch cones as before. The pitch cones can then be completed by making $\angle$ A O D = $\angle$ A O C, and $\angle$ B O E = $\angle$ B O C. The remainder of the construction is obvious.

Teeth of Bevel Wheels.—We shall now give a brief explanation of the usual method adopted in setting out the teeth for a pair of bevel wheels in gear. Let O A, O B be the axes of the shafts. Having drawn the pitch cones, C O D, C O E, draw A C B through C perpendicular to the line of contact, O C, and join A D, B E. Imagine C A D, C B E to be conical surfaces whose vertices are A and B respectively. Now, if we further imagine these conical

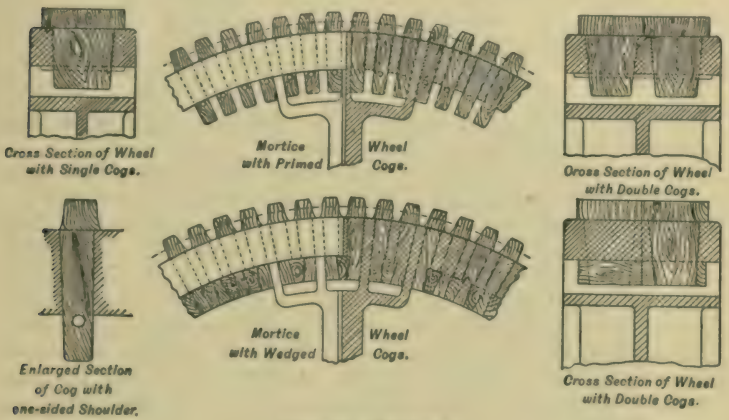
surfaces *developed*—that is, flattened out into circular segments, A C M, B C N—then these segments would roll, without slipping, upon their circular edges, C M, C N, during the rotation of the wheels. Hence, we may look upon A C M, B C N as portions of the pitch circles, with reference to which the *outer* ends of the teeth are described. On these *virtual* pitch circles the curves for the outer ends of the teeth are to be described in the usual way. The teeth may be either cycloidal or involute. From the figures it will be seen that the teeth taper towards the point, O.



METHOD OF SETTING OUT CURVES FOR TEETH ON BEVEL WHEELS.

In a similar way to the above we could draw the virtual pitch circles for the inner ends of the conical frusta, and then construct the curves for the inner ends of the teeth. These inner virtual pitch circles are usually drawn concentric with the outer ones as follows:—Let C G be the breadth of the face of the wheels. Through G draw H G K perpendicular to O G, and meeting O A in H and O B in K. Then, just as before, H G and K G are the radii of the virtual pitch circles for the smaller ends of the frusta. It is more convenient to draw these circles about A and B as centres than about H and K as centres. Hence project G on to A C and B C, by drawing G h, G k parallel to O A and O B respectively. Then, A h = H G, and B k = K G. With A and B as centres, draw the virtual pitch circles $h\,m$, $k\,n$. The curves for the inner ends of the teeth are set out on these virtual pitch circles. The dimensions of the teeth on $h\,m$, $k\,n$ are reduced in the proportion A h : A C, or B k : B C.

Mortice Wheels.—When ordinary toothed wheels are run at a nigh speed their mutual actions become very rough and noisy, and severe vibrations are usually set up. To obtain a smoother and more regular action the method is sometimes adopted of placing wooden teeth on one of each pair of wheels in gear. These wooden teeth, or "*cogs*" as they are called, are *morticed* into the iron rims of the wheels, and hence such wheels are termed **mortice wheels.** The action of mortice wheels is very smooth and noiseless. Some of the common methods of securing the cogs to the rims are shown by the accompanying figures. In the upper


Cross Section of Wheel with Single Cogs.
Mortice Wheel with Primed Cogs.
Cross Section of Wheel with Double Cogs.
Enlarged Section of Cog with one-sided Shoulder.
Mortice Wheel with Wedged Cogs.
Cross Section of Wheel with Double Cogs.

MORTICE WHEELS.

figures the cogs are secured by pegs or pins passing through the tenons of the cogs in a direction parallel to their length. In the lower figures they are shown secured by dove-tailed wooden keys driven into correspondingly-shaped grooves at the projecting ends of the tenons. Double cogs are sometimes used when the wheels are very broad; these are shown by the cross sectional views on the right-hand side. The cogs are usually made with side shoulders, as shown by the longitudinal sections of the rims of the wheels, but occasionally there is only one side shoulder, as shown by the lower left-hand figure.

The cogs may be cycloidal or involute, according to the shape of the iron teeth in gear with them, and are usually shaped by hand. To prevent undue wear of the cogs by the iron teeth, it is usual to "pitch" and "trim" the acting surfaces of the latter by carefully chipping, and afterwards filing, them to a high degree of smoothness. With machine-moulded teeth, which are much

smoother, more uniform and perfect in shape than those cast from a pattern in the ordinary way, it is sufficient to merely file the teeth to smoothness.

The cogs are made of hard, tough wood, such as oak, beech, hornbeam, holly, or apple tree. Since the material of the cogs is softer and weaker than that of cast iron, it follows, as a matter of economical distribution of material, that the thickness of the cogs should be greater than that of the iron teeth. The following are the usual proportions for the teeth of mortice gearing:—

Thickness of wood cogs at pitch circle	=	0·60 ×	pitch.
„ *iron teeth*	„	= 0·40	„
Height of teeth above	„	= 0·25	„
Depth „ *below*	„	= 0·30	„

From these proportions it will be seen that there is no side clearance between the teeth when new.

Bevel wheels can also be fitted with wooden cogs, which are secured to the rim by methods similar to those for spur wheels.

Machine Cut Gearing.—As we have seen, one of the first things to consider in designing wheel gearing, is the precise form of the teeth to be employed; because, upon this depends very much the smooth working and the efficiency of the gearing, since all toothed gearing must slide as well as roll, more or less, when transmitting motion and power. When two cylinders the axes of which are parallel roll on each other, they will not transmit rotary motion with regularity, unless their surfaces are provided with teeth meshing with each other; and these must be shaped so as to produce the same uniformity of motion between the two cylinders as would be transmitted by friction, without slipping. The perfect form and pitch of toothed wheels cannot be easily or cheaply obtained, by either machine moulding or by any other process of moulding and casting the same; because, of the irregularities which naturally arise from either the warping of patterns, the distortion of the mould, or the unequal contraction of the parts of the wheel casting, when cooling. Engineers can now, however, cut the teeth of wheels from blank castings by the aid of machinery, so as to produce a practically perfect and highly efficient gearing. The teeth may have an absolutely correct form, and thereby fulfil all the requirements of smooth and positive action. The Admiralty and many of the most important buyers of wheel gearing specify, that the "teeth of all wheels be machine cut, accurately shaped and well fitted, so as to reduce backlash to a minimum."

Spur Gear Cutting Machine.—The wheel, **W**, on which the teeth are to be cut, is supported on the spindle, **S**. This spindle is provided at its further end with a worm wheel, which is called a "dividing wheel." One complete turn of the spindle, **S**, carries round the wheel, **W**, and the dividing wheel through one revolution. An arrangement of change wheels gearing with the

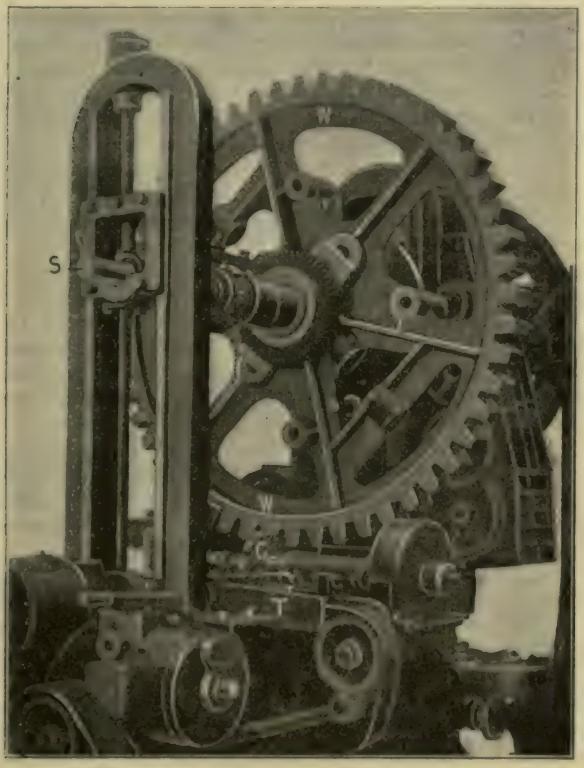


FIG. 1.—SPUR GEAR CUTTING MACHINE.
(As used by David Brown & Sons, Huddersfield).

"dividing wheel" gives the desired pitch of teeth on the wheel, **W**. The sliding table, **T**, which carries the cutter, **C**, has a slow forward movement during the operation of cutting a tooth space, and a quick return. The forward feed is kept stationary during the short time that the wheel, **W**, is being turned through the

pitch of a tooth, before the next space is begun to be cut. All these operations are automatically effected when the machine has been set and started to work.

In setting out a pair of spur wheels, the profile of the tooth should be obtained by rolling circles in the usual way, and then each and every cut made parallel to the axis of the wheel for the whole breadth of the tooth.

Correct Form of Bevel Wheel Teeth.*—The right hand of Fig. 5, on the next full-page diagram of "Gibson's Machine for Cutting Teeth of Bevel Wheels," gives a bird's-eye view of the essential features of a bevel wheel tooth. Suppose the wheel blank, W, to be made of any plastic, easily-cut material, and that a "space template" or a "former," F (same as 11 on Gibson's Machine), is mounted at a convenient distance from the apex of the pitch cone. Then, with a long, thin, rigid, narrow, sharp needle or knife, K, pivoted at the apex of the pitch cone, and having the other end guided hard over the enlarged profile of the tooth, F, the knife, K, would sweep out from the blank a perfect bevel wheel space. Every peculiarity of the profile of the "former," F, would thus be faithfully reproduced in ever-decreasing miniature as the apex or pivot is approached. By bringing the sides of this "space template," F, closer, the pitch of the teeth could be reduced to suit any required number of teeth in a wheel. Compare such an outline of a perfect bevel wheel tooth with the following Figs., 2 to 4. Here Fig. 2 shows properly shaped teeth, but Figs. 3 and 4 show tooth spaces in bevel wheels cut out by rotary cutters, whose sides are radiused to approximate to the curves of face and flank. These spaces are gapped out by "angling" or "wobbling" the wheel, first to one side and then to the other of the middle plane, and running the cutter through. The result is as indicated, viz., a cycloidal tooth at the outer end, merging in a mongrel involute at the other. There is not a single position where a pair of teeth bear evenly along their whole length.

To within a few years ago, nearly all the machine-cut gears—more especially bevel and worm gears—suffered from the most serious defect, that the teeth were not properly shaped. We know, that the contact between the two smooth rolling cones is a straight line, passing through their common apex. Hence, it

* I am indebted to "The Council of the North-East Coast Institution of Engineers and Shipbuilders" for their kind permission to reproduce several figures, and make extracts here from a paper "On the Machine Cutting of Accurate Bevel and Worm Gears," by J. H. Gibson. I have pleasure in referring students who desire further information upon the subject to that paper and discussion. See *Trans.*, vol. xiii., 1897.

will be seen that the cross-section of the teeth to be formed for a pair of perfect bevel gear wheels, must uniformly diminish towards the apex of their pitch cones. Therefore, by assuming

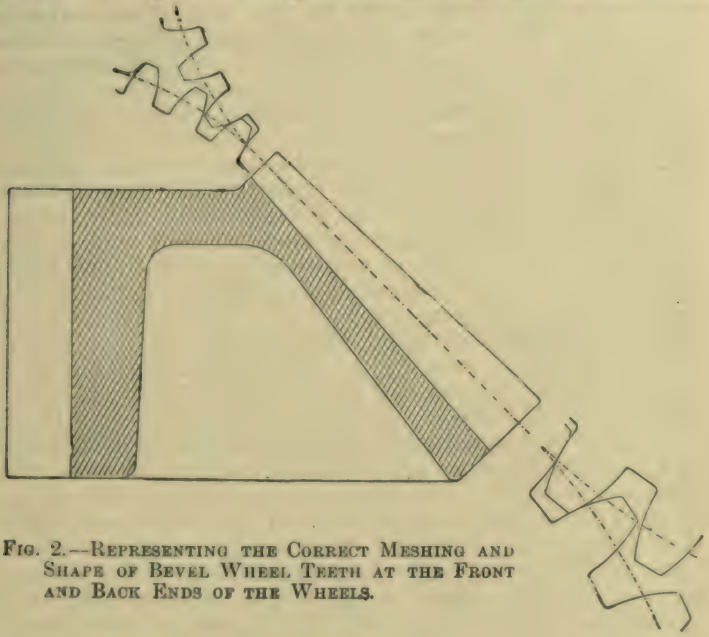


FIG. 2.—REPRESENTING THE CORRECT MESHING AND SHAPE OF BEVEL WHEEL TEETH AT THE FRONT AND BACK ENDS OF THE WHEELS.

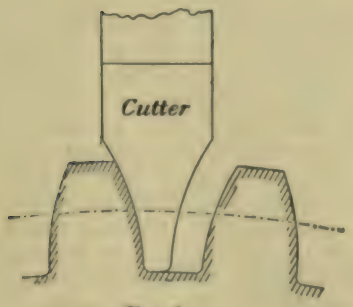


FIG. 3.

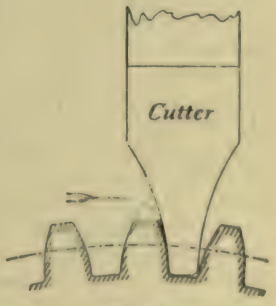


FIG. 4.

SHOWING CORRECT FORM OF TOOTH FOR BACK END, FIG. 3, AND OF NECESSITY AN INCORRECT FORM OF TOOTH FOR THE FRONT END, FIG. 4, WHEN SHAPED THROUGHOUT THE WHOLE LENGTH BY ONE REVOLVING CUTTER.

the curves of the faces and the flanks to be arcs of circles, the radii at the inner ends of their teeth arcs, must be relatively shorter than at their outer ends. Too often the faces and flanks are struck with a radius to suit the eye, without any regard to the smooth rolling of the pitch cones; and the pair of wheels "mesh" all right, being machine divided, but each tooth has to be eased by filing until they gear all round, as shown by Fig. 4. The bearing part between a pair of teeth may only be at a point, instead of having a good bearing surface the full breadth of the teeth. All rotary cutters produce the same defect, for they cut both ends of the tooth to the same radius. They are very expensive, and there is always a tendency to make the one cutter come in for quite another pitch, slightly more or less, irrespective of the angle of the wheel or of the ratio of the gearing. *It is, therefore, practically impossible to cut bevel wheel teeth correctly, with rotary cutters.*

Gibson's Bevel Wheel Cutting Machine.—We shall now describe the accompanying three illustrations of a machine, Fig. 5, in which it will be observed, that the true principles for the correct cutting of bevel teeth are carried out. The bevel wheel blank, 1, to be cut is mounted on a mandril, 2, one end of which pivots in a small trunnion bearing, 3, at the apex of the pitch cone. The other end is capable of movement in a vertical plane, and can be fixed on a graduated quadrant, 4, at any required angle. A dividing gear, 5, turns the wheel blank into successive positions for cutting the required number of teeth; and, while each tooth is being cut, a pinching screw, 6, helps the dividing wheel to hold the wheel blank, 1, firmly in its place. A slider arm, 7, carrying a ram, 8, and a tool box, 9, is also pivoted, so as to have a universal motion about the apex of the pitch cone. The outer end of 7 is connected to the feed gear and carries a roller, 10, the centre of which is constrained to trace out the enlarged profile of tooth, by means of the "space template" or "former," 11, feed gear, 12, and spring, 13. The ram, 8, must be set so as to reciprocate in a definite line, according to the size and angle of wheel to be cut. The slot bracket, 14, is pinched to 8, by bolts. The stroke of 8, can be varied to suit the breadth of tooth, by altering the radius of crank pin, 15. The tool box, 9, is capable of lateral adjustment by the screw, 17, so as to bring the cutting corner of the tool, 18, on to the straight line joining the centre of the roller, 10, to the apex of the pitch cone. If the left side of a space is to be cut, the roller is made to roll down the left-hand profile of 11, and the left corner of the tool is thus brought on to the centre line. A simple depth and side gauge, enables the corner of the tool to be set with ease and

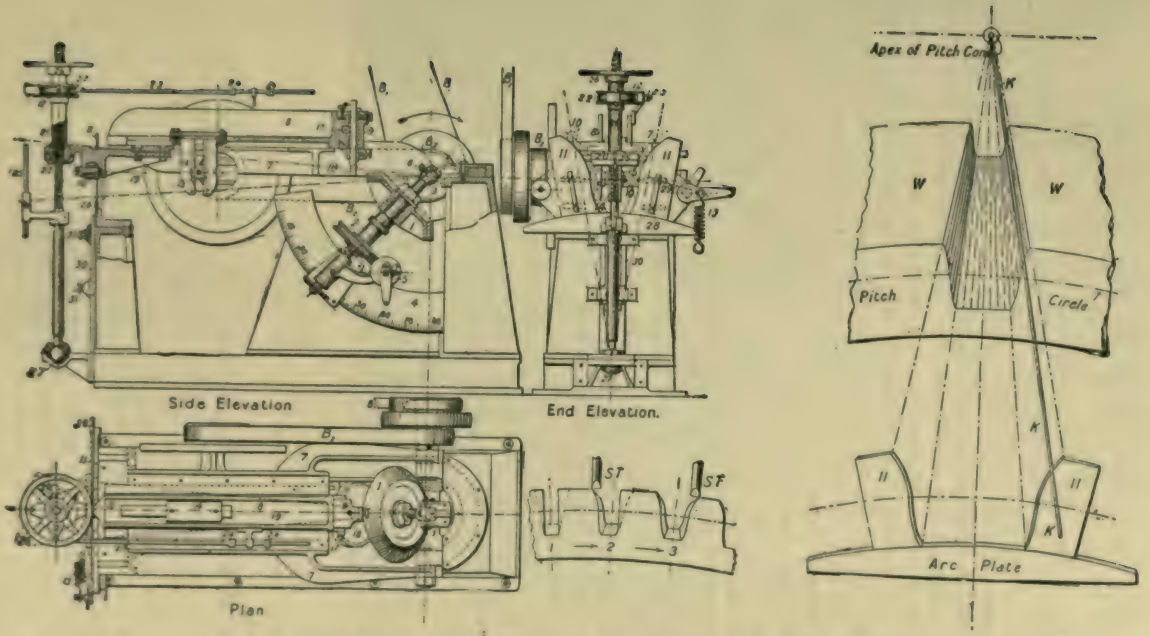


FIG. 5.—GIBSON'S MACHINE FOR CUTTING THE TEETH OF BEVEL WHEELS.

accuracy. The outer end of the slider arm, 7, terminates in a socket, 20, and engages a ball, which forms one piece with the feed nut, 21. To the upper end of the feed nut is keyed a ratchet wheel, 22, which is actuated by the tappet rod, 23. Thus, by simply adjusting the tappets, 24, any desired feed can be imparted to the outer end of the slider arm, 7, by the motion of the ram, 8. When the tool, 18, reaches the root of the tooth, the pawl engages the adjustable rod, 25, and stops the feed. Then, the hand wheel, 26, enables the nut to be brought to the top of the feed screw. The wheel blank, 1, is then turned through another pitch, the pawl is sprung into gear with the ratchet, 22, and the next tooth is operated upon. The bottom end of the feed screw, terminates in a ball and socket joint, 27. This allows the feed screw to accommodate itself to any position taken up by the end of slider arm, 7. The "space templates," 11, are mounted on an arc plate, 28, which is struck from the same centre as the development of the pitch circle, 29. The templates, 11, are so pitched, that the distance between them, as measured on the template pitch line, is equal to ·5 of the pitch + the diameter of the roller and any clearance, say ·01 inch. The templates are correct for the same angle of wheel, whatever its diameter and pitch may be, provided the same generating circles are used to roll or develop these curves. If it be required to "gap out" the spaces in the wheel blank by this machine, the vertical roller guide, 30, is lifted and clamped in position by the set bolts, 31. The gaps are then cut out as shown by the left-hand space below the End Elevation of Fig. 5. The width of the tool is made exactly the same as the root-space at its inner or front end. It is found to be better, as a rule, to cast the blank wheel, 1, with gaps of the V shape, as indicated by the dotted lines. This method, consequently, leaves the machine to simply shape the teeth, as shown by the central part of this figure. The finishing cut to the teeth is shown by the right-hand part of the same figure.

Bevel Wheel Cutting Machine.—A wheel is supported on the spindle, S, which is provided with the usual dividing wheel and change gears, so that the wheel, W, to be cut, may be divided into the desired number of teeth. A radial arm, B, carries a slide with the cutting tool, C. The outer end of this arm rests on and is guided by the "former," F, and being set to the proper pitch angle of the wheel to be cut, the tool, C, describes the tooth curve as determined by the "former," F. The tooth of the wheel, W, is therefore a reduced facsimile of F, with a corresponding reduction of any error in the "former," as

previously explained in the description of Fig. 5, Gibson's machine for cutting the correct form of bevel wheel teeth.

Worm Wheels.—It is difficult to correctly draw a worm wheel, since the teeth are thinner at the tips than at their middle—more especially so with coarse pitches. Also, the angle of the teeth are greater at their roots than at their tips. The process of cutting a worm wheel will, however, be easily understood from the following illustration and description.

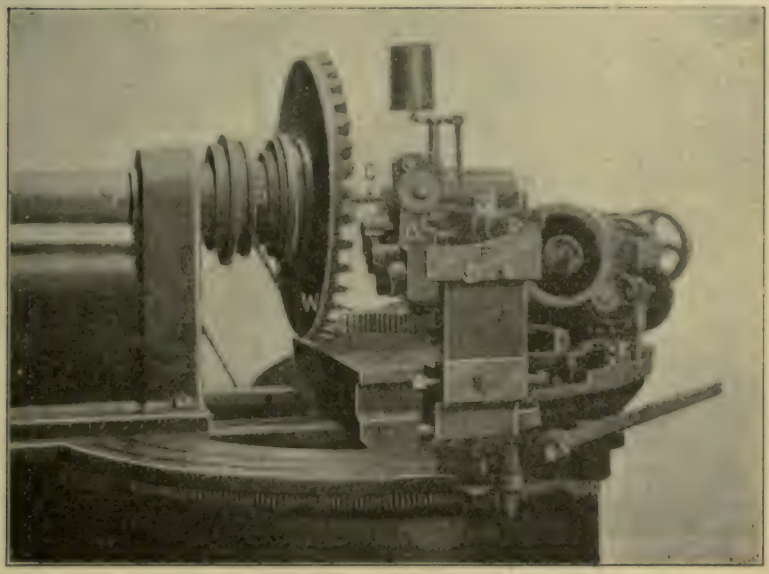


FIG. 6.—BEVEL WHEEL CUTTING MACHINE.
(As used by David Brown & Sons, Huddersfield.)

Worm Wheel Cutting Machine.—The wheel, **W**, to be cut is supported on the vertical spindle, **S**, which is fixed to a travelling table, **T**. The hob or cutter, **C**, is fixed on the horizontal spindle, **B**. Both spindles, **S** and **B**, are connected by worm and spur gears in such a manner that the cutter, **C**, makes so many revolutions while the wheel, **W**, makes one revolution. In fact, just in the same way as the worm in the finished gearing will do, when turning the wheel through one revolution. The worm or cutter, **C**, travels in a tangential direction relatively to the worm wheel, **W**, and cuts teeth thereon just like a screw tap.

The machine works automatically when set and started. This plan of using a solid hob formed from tool steel, having one complete thread and three to four roughing threads, is preferable to that of using simply a single cutting tool; because, the latter has a great tendency to "spring" whilst cutting, and thus leave the work with an imperfect finish, or take much longer time to accomplish.

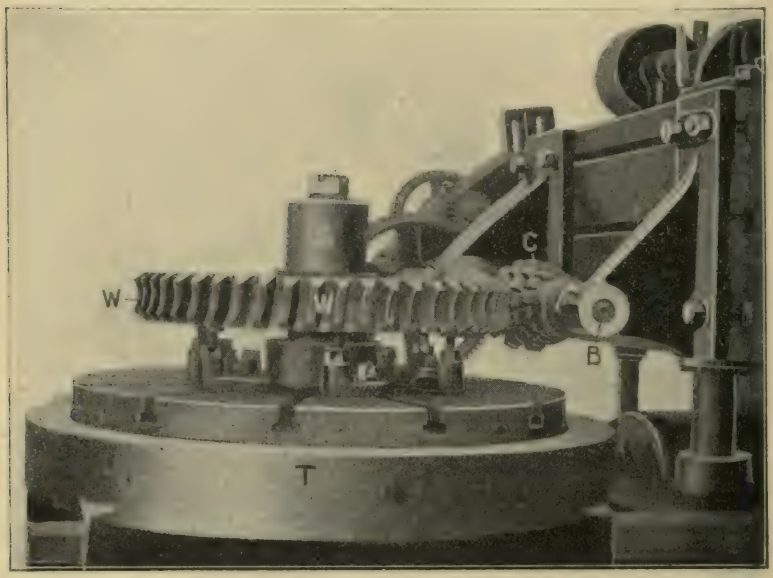


FIG. 7.—WORM WHEEL CUTTING MACHINE.
(As used by David Brown & Sons, Huddersfield.)

Small Wheel Cutting Machine.—As a final illustration upon this subject, there is shown, by Fig. 8, a handy machine for cutting comparatively small, straight, bevel, or helical gears, with a steel division plate for wheels up to 6 inches diameter. The machine may also be fitted with a rack-cutting attachment for taking in blanks up to 13 inches in length. This machine is more suitable for the lighter class of wheel cutting, as carried out by opticians, instrument makers, and mechanicians generally, than those previously described for the heavier types of wheels used by engineers.

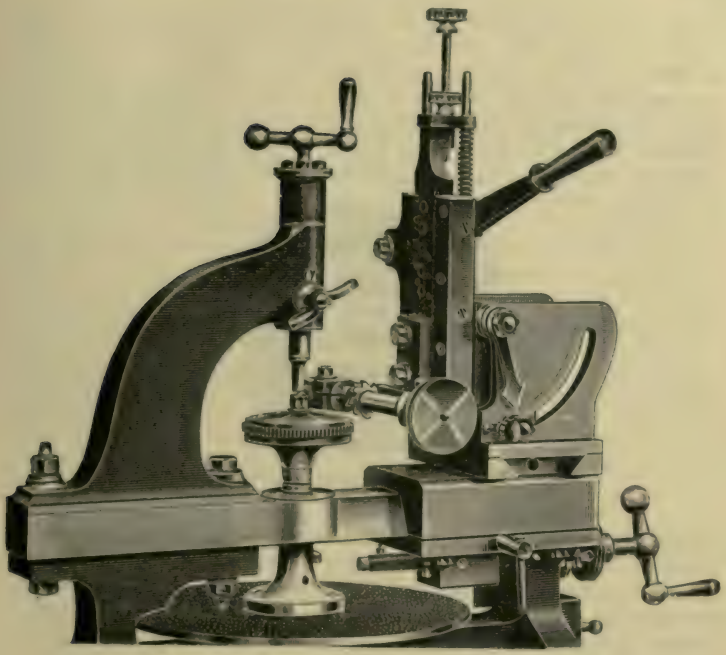


FIG. 8.—SMALL WHEEL CUTTING MACHINE.
(By Grimshaw & Baxter, London.)

Rawhide Pinions.—When freedom from noise is desirable, it is found that this may be attained by making the smaller of each gearing pair of rawhide. As may be seen from Fig. 1,

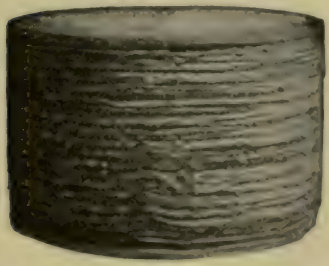


FIG. 1.—ROUGH RAWHIDE BLANK.

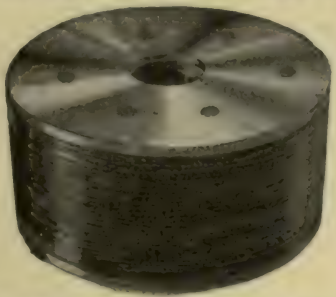


FIG. 2.—FLANGED RAWHIDE BLANK.

these pinions are built up of washers from a number of cut rawhides, with thin glue between them. The built up blank is then subjected to very great hydraulic pressure, to squeeze out all superfluous glue, and the pressure is maintained for some considerable time. They are afterwards allowed to dry thoroughly. Two gun-metal plates of sufficient thickness and diameter form the end flanges to the blanks, and these are

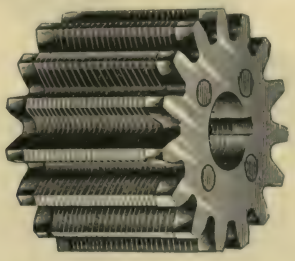


FIG. 3.—FINISHED RAWHIDE PINION.

firmly connected with each other and the hide by riveted or screwed bolts, as shown by Fig. 2. These flanged blanks are then bored, faced, turned, and keyed before the teeth are cut in the manner previously described, and as shown by the finished pinion in Fig. 3. Bevel pinions are also made in this manner by David Brown & Sons, of Huddersfield, to whom we are indebted for these figures.

Lecture XIII.—Questions.

1. A pair of wheels have 25 and 130 involute teeth respectively. The addenda of the teeth in both cases is $\frac{1}{3}$ the pitch. Find the lengths of the arcs of approach and recess, assuming the obliquity to be 15°, and the larger wheel the driver. *Ans.* $a = \cdot 925\,p$; $\beta = 1\cdot 19\,p$.

2. Prove the formulæ for the addenda of involute teeth in terms of the arcs of approach and recess, &c. Hence show that, if the arcs of approach and recess are each equal to the pitch, the addenda should be calculated from the formula:—Addendum $= \left(\frac{1}{4} + \frac{3}{n}\right) \times$ pitch, where n represents the number of teeth on wheel.

3. What are bevel wheels? Two axes intersect at an angle of 60°, and it is required to connect them by bevel gearing, so that their angular velocities shall be as 3 : 2. Construct the pitch cones of the bevel wheels.

4. Under what conditions will two cones roll together? Motion is to be communicated between two shafts inclined at an angle of 90°, and one is to make three rotations while the other makes four. Set out the pitch cones in a diagram, marking dimensions.

5. Two shafts intersecting at right angles are connected by bevel wheels with 22 and 44 teeth respectively, of 1 inch pitch. Draw, to a scale of $\frac{1}{4}$, the pitch surfaces of the wheels, and find the development of the conical surfaces on which the shape of the ends of the teeth are set out. Having given the shape of the end of a tooth, explain how the shape of the surface of the tooth is determined.

6. Draw the pitch cones for two bevel wheels in gear, having 60 and 45 teeth respectively with 2½ inches pitch, measured at larger end of conical frustum, the shafts to make an angle of 60° with each other. Explain fully the method adopted by engineers in setting out the teeth for a pair of bevel wheels, by applying it to the example given.

7. Sketch to scale a pair of bevel wheels in gear with each other, the gearing ratio to be 3 to 1, the mean pitch to be 1¼ inches, and the number of teeth in the smaller wheel to be 16. Make sufficient sketches to show the construction of the wheels and the shape of the teeth fully. Given the shape of the tooth of a rack in a set of interchangeable wheels, show how to develop by graphic construction the proper shape of tooth for a wheel of any given number of teeth.

8. Distinguish between a bevel wheel, a mitre wheel, and a mortice wheel. Draw a section of two mitre wheels in gear. Sketch a mortice tooth for (1) a spur wheel, (2) a bevel wheel, and describe with sketches two methods of fixing it in position.

9. Give two views of a tooth of a mortice spur wheel, showing how it is fitted into the rim of the wheel and held in position. Under what circumstances would you use mortice wheels?

Lecture XIII.—A.M.Inst.C.E. Exam. Questions.

1. A bevel wheel, making 93 revolutions per minute, and having 40 teeth, 2-inch pitch, 5-inch face, drives a mortice wheel at 120 revolutions a minute; sketch the wheels in section roughly correct as to the scales of the various parts. (I.C.E., *Feb.*, 1899.)

2. Select either the involute or cycloidal form of tooth for a spur wheel, and show that it satisfies the required geometrical condition. Point out on a diagram the "path of the point of contact," and explain how the two ends of the path are determined. (I.C.E., *Oct.*, 1904.)

3. What are the chief peculiarities of involute teeth? Explain how to set out an involute tooth, and sketch one in roughly to scale suitable for a wheel with thirty teeth of 4-inch pitch. (I.C.E., *Oct.*, 1906.)

4. Explain the relative advantages of involute and cycloidal teeth for wheels. Show how to construct the cycloidal teeth for a pair of wheels 6 inches and 14 inches in diameter, sketching one tooth in position. (I.C.E., *Oct.*, 1903.)

LECTURE XIV.

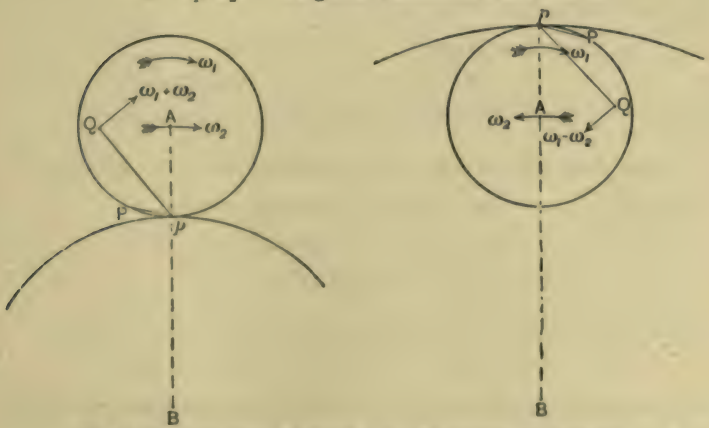
CONTENTS.—Friction of Toothed Gearing—The Principle of Combined Rotations—Example I.—Strength of Wheel Teeth—Case I.—Strength of Teeth when Contact between the Various Pairs of Teeth in Gear is Perfect—Case II.—Strength of Teeth when Contact is Imperfect—Breadth of Wheel Teeth—Example II.—Flanged or Shrouded Teeth—Hooke's Stepped Gearing—Helical Gearing—Double Helical Wheels—Wüst Double Helical Gear—Questions.

Friction of Toothed Gearing.—The action between a pair of teeth in contact is partly rolling and partly sliding, but the frictional resistance of the former is small compared with that of the latter, and may, therefore, be conveniently neglected. The sliding between a pair of teeth takes place in a direction perpendicular to the common normal at their point of contact, and in order to find its amount we require to know the relative motion of the teeth in this direction. We proceed, in the first place, to determine this relative motion of the teeth, and in doing so shall make use of:—

The Principle of Combined Rotations.—Let A and B be the centres of two spur wheels in gear, p being the pitch point.

Let R_1, R_2 = Radii of wheels A and B.

,, ω_1, ω_2 = Angular velocities of A and B.



TO ILLUSTRATE THE PRINCIPLE OF COMBINED ROTATIONS.

The wheels will rotate in the opposite or in the same direction, according as the contact is external or internal. It is convenient to indicate this distinction by the sign attached to ω. Thus, in the case of external contact, let the angular velocities be $+\omega_1$ and $-\omega_2$; for internal contact, let these quantities be $+\omega_1$ and $+\omega_2$.

The *relative* angular velocities of wheels A and B will not be altered, if to each we impart an equal angular velocity. Thus, suppose each of the wheels in external contact receives an angular velocity $+\omega_2$ about the axis, B. Then the angular velocity of B will be $\omega_2 - \omega_2 = 0$; *i.e.*, it will be at rest. The wheel A will be rotating about B with angular velocity, ω_2, and about its own axis with angular velocity, ω_1. Hence, the *resultant* angular velocity of wheel A is $(\omega_1 + \omega_2)$ about some axis which we are about to determine. During the rotation of the wheel A about B, that line on A, which is, for the instant, in contact with B, is at rest. At that instant, the wheel A is rotating about this line as an *instantaneous axis*. Clearly, this instantaneous axis passes through the pitch point, p, and is parallel to the axis of A or B. Hence, at a given instant any point, P, on wheel A is rotating about the axis through p, with an angular velocity $(\omega_1 + \omega_2)$.

By similar reasoning, if each of the wheels in internal contact receive an angular $-\omega_2$, then wheel B will be brought to rest, and any point on wheel A will be rotating about the instantaneous axis through p, with an angular velocity $(\omega_1 - \omega_2)$.

If, then, P be a point of contact between a pair of teeth, P p is the direction of the common normal to the two teeth at that point, and the velocity of P perpendicular to P p is the velocity of sliding between the teeth.

Let V = Velocity of pitch circles.

,, v = Velocity of P perpendicular to P p.

,, r = Distance of P from pitch point, p.

Then, from what has been demonstrated above, we get :—

Or, $v = (\omega_1 + \omega_2)\, r$, for external gearing,

,, $= (\omega_1 - \omega_2)\, r$, ,, internal ,,

But, $V = \omega_1 R_1, = \omega_2 R_2$.

$$\therefore \quad v = \left\{ \frac{1}{R_1} \pm \frac{1}{R_2} \right\} r\, V.$$

Let arc p P be denoted by x, and suppose the wheels to receive a small circular displacement, $d\,x$, as measured along the pitch

circles. During this displacement, let ds represent the distance through which the teeth slide on one another.

Then, $$ds = v\,dt = \left\{\frac{1}{R_1} \pm \frac{1}{R_2}\right\} r\,V\,dt,$$

$$\text{,,} = \left\{\frac{1}{R_1} \pm \frac{1}{R_2}\right\} r\,dx.$$

We may take r as being approximately constant, since, for a small movement ds, at right angles to $p\,P$, it does not perceptibly change.

Let P_n = Normal pressure between the teeth.
,, μ = Coefficient of friction.
,, α, β = Length of arcs of approach and recess respectively.

Then, *Work lost in friction during displacement, dx,* $= \mu\,P_n\,ds.$

$$\text{,,} \qquad \text{,,} \qquad = \mu \left\{\frac{1}{R_1} \pm \frac{1}{R_2}\right\} P_n\,r\,dx.$$

∴ Total work lost in friction during whole arc of contact $= W = \mu \left\{\frac{1}{R_1} \pm \frac{1}{R_2}\right\} \left\{\int_0^{\alpha} P_n\,r\,dx + \int_0^{\beta} P_n\,r\,dx\right\}$ (I)

The law according to which P_n varies is not definitely known, being dependent upon the number of teeth in contact, the state of the teeth, and other causes. Its magnitude may not vary much, and probably has a mean value somewhere between $\frac{1}{2}$ P and P, where P denotes the driving or tangential force at the pitch circles of the wheels.

Taking $P_n = \frac{2}{3}$ P, which is quite a legitimate assumption, and putting chord $p\,P$ = arc $p\,P$, or $r = x$, we get the following approximate equations :—

$$W = \tfrac{2}{3}\,\mu\,P \left\{\frac{1}{R_1} \pm \frac{1}{R_2}\right\} \left\{\int_0^{\alpha} x\,dx + \int_0^{\beta} x\,dx\right\}$$

$$\text{,,} = \tfrac{1}{3}\,\mu\,P \left\{\frac{1}{R_1} \pm \frac{1}{R_2}\right\} (\alpha^2 + \beta^2). \quad . \quad . \quad . \quad (II)$$

Let N_1, N_2 = Number of teeth on wheels A and B respectively.
,, p = Pitch of teeth.

Then, $$W = \tfrac{2}{3}\,\pi\,\mu\,P \left\{\frac{1}{N_1} \pm \frac{1}{N_2}\right\} \frac{\alpha^2 + \beta^2}{p}. \quad . \quad . \quad . \quad . \quad (II_a)$$

If, further, we suppose the arcs of approach and recess each equal to the pitch, we get :—

$$W = \tfrac{4}{3}\pi\mu P \left\{ \frac{1}{N_1} \pm \frac{1}{N_2} \right\} p. \quad . \quad . \quad . \quad . \quad (II_b)$$

From these equations, we learn that the greater the number of teeth (*i.e.*, the smaller the pitch), and the shorter the arcs of approach and recess, the smaller is the loss due to friction.

The above results are equally true for bevel gearing.

The loss due to the friction of toothed gearing is usually very small, being about 3 per cent. It has been stated by some authorities that the friction during approach is greater than that during recess. This explains why, in some kinds of wheelwork (such as in watches and clocks), the teeth are shaped so that there is no arc of approach.

Example I.—In a pair of spur wheels with external contact, the number of teeth on the wheels is 30 and 70 respectively. Assuming the arcs of approach and recess each equal to the pitch of the teeth, and taking the coefficient of friction at ·1, find the efficiency of the gearing.

Answer.—The *work lost* by friction during the action between one pair of teeth is given by equation (II_b), viz. :—

$$W = \frac{4}{3}\pi\mu P \left\{ \frac{1}{N_1} + \frac{1}{N_2} \right\} p.$$

The *total work expended* during the same action (arc of approach + arc of recess) is:—

$$W_T = P_n \text{ (arc of approach + arc of recess)}$$

$$\text{,,} \quad = \frac{2}{3} P \times 2p = \frac{4}{3} P p.$$

$$\therefore \quad \textit{Efficiency} = \frac{\textit{Useful work done}}{\textit{Total work expended}}$$

$$\text{,,} \quad = \frac{W_T - W}{W_T} = 1 - \frac{W}{W_T}$$

$$\text{,,} \quad = 1 - \pi\mu \left\{ \frac{1}{N_1} + \frac{1}{N_2} \right\}$$

$$\text{,,} \quad = 1 - \frac{22}{7} \times \frac{1}{10} \left\{ \frac{1}{70} + \frac{1}{30} \right\}.$$

Or, **Efficiency** = 1 − ·015 = ·985, or, **98·5 per cent.**

This example serves to show how small is the loss due to the friction of the teeth of wheels.

Strength of Wheel Teeth.—The power which can be transmitted by toothed gearing depends upon the circumferential speed of the wheels and the strength of the teeth. Thus :—

Let H.P. = Horse-power transmitted.
„ P = Tangential pressure in lbs. at pitch circle.
„ V = Velocity in feet per minute at pitch circle.

Then,
$$\text{H.P.} = \frac{\text{P V}}{33{,}000} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(III)}$$

We now proceed to determine P in terms of the dimensions of the teeth, &c.

The strength of a tooth depends upon the manner in which the pressure on that tooth is distributed, and this latter depends upon the accuracy with which the wheels are made and adjusted in gear. Two cases occur, according as the contact between a pair of teeth is perfect or imperfect. We shall consider these cases separately.

Case I.—Strength of Teeth when Contact between the Various Pairs of Teeth in Gear is Perfect.—When the wheels are accurately adjusted in gear, and the teeth well formed, any pair of teeth should be in contact along a line across the breadth of the teeth. In such a case the mutual pressure between the teeth will probably be uniformly distributed along that line. Let this line be taken at the end or point of the tooth which we are about to consider, so that the bending moment due to the distributed pressure may be a maximum. We may also neglect the curved form of the tooth, and assume it to be a rectangular block fixed to the rim of the wheel. It may then be looked upon as a short beam fixed at one end (the root) and loaded uniformly along a transverse line at the other or free end.

Let P_n = Total pressure acting on the tooth.
„ b, l, t = Breadth, length, and thickness of the tooth.
„ f = Safe stress for the material.

If the material of the tooth be of uniform strength throughout, then the tendency of P_n will be to break the tooth along the root E F G H.

The bending moment at section E F G H is, B.M. = $P_n l$.

The resisting moment offered by the material at section E F G H is :—

$$\text{R.M.} = \frac{1}{6} b\, t^2 f.$$

$$\therefore \quad P_n l = \frac{1}{6} b\, t^2 f.$$

$$\text{Or,} \quad P_n = \frac{1}{6} \left(\frac{b\, t^2}{l} \right) f.$$

Now, the magnitude of P_n is not exactly known; but, if there are never fewer than two pairs of teeth in contact at once, it seems quite a fair assumption to take $P_n = \frac{2}{3} P$. Hence:—

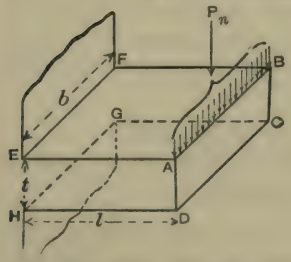
$$P = \frac{1}{4} \left(\frac{b\, t^2}{l} \right) f \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(IV)}$$

Usually the dimensions of a tooth are stated in terms of the pitch of the teeth, and the ordinary proportions for *new* teeth were stated at the beginning of Lecture XI., Vol. V. Making an allowance for wear, we may take:—

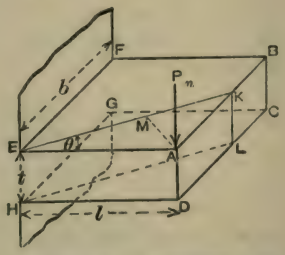
$l = \cdot 7\, p$ for iron teeth. $\quad$ $t = \cdot 36\, p$ for iron teeth.
$l = \cdot 6\, p$ for wooden teeth. $\quad$ $t = \cdot 45\, p$ for wooden teeth.

The breadth, b, varies considerably, the average being, $b = 2 \cdot 5\, p$. In the meantime, denote the breadth by $n\, p$. Making these substitutions in equation (IV), we get:—

$$\left.\begin{aligned} P &= 0{\cdot}0463\, n\, p^2 f, \textit{ for iron teeth} \\ P &= 0{\cdot}0844\, n\, p^2 f, \textit{ for wooden teeth} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad \text{(V)}$$



Illustrating Distributed Pressure on Tooth.



Illustrating Concentrated Pressure on Tooth.

From these equations we see that *the driving force*, P, *varies as the square of the pitch of the teeth*.

By substituting these values for P in equation (III), the maximum H.P. transmitted can be obtained.

Case II.—**Strength of Teeth when Contact is Imperfect.**—When the wheels are badly adjusted in gear, or the teeth badly shaped, contact between the latter will most likely be very imperfect. Instead of the teeth bearing along a line, as in the case just considered, they may bear at a few points, or perhaps at one point only. This state of affairs may be caused by one or more of the following defects :—(1) in spur gearing, the shafts may not be strictly parallel ; or, in bevel gearing, the shafts may not be coplanar—*i.e.*, in the same plane. Although these defects may not exist when the gearing is newly erected, the subsequent wear of the shaft bearings may ultimately bring about this state of affairs. (2) The severe stresses to which the parts of the gearing (especially at the shaft supports) are sometimes subjected cause imperfect contact between the teeth. (3) The teeth may have been badly shaped to begin with. With wheels which have been moulded from a pattern in the ordinary way, the teeth are slightly tapered across their breadth, caused by the pattern which is purposely made thus to allow its being withdrawn from the mould. Hence, care is needed in the erection of such wheels, to see that they are so placed that the thick parts of the teeth on the one come in contact with the thin parts of the teeth on the other. Attention to this rule is not always given. This defect does not exist with machine-moulded or machine-cut teeth.

The worst, and most likely, case occurs when the one tooth presses upon a corner of the other. We shall, therefore, consider this case.

Let P_n act at the corner A. Then its tendency is to break off a triangular portion, E A K, along a section, E K L H, passing through E H.

Let θ = angle A E K. Draw A M perpendicular to E K.

Then bending moment about section, E K L H, is :—

$$\text{B.M.} = P_n \times A\,M = P_n l \sin \theta.$$

Resisting moment of material at section, E K L H, is :—

$$\text{R.M.} = \frac{1}{6} . \text{E K} . t^2 f = \frac{1}{6} . l \sec \theta . t^2 f.$$

$$\therefore \quad \frac{1}{6} l \sec \theta . t^2 f = P_n l \sin \theta$$

$$\therefore \quad f = \frac{3 P_n}{t^2} . \sin 2\theta.$$

This gives the stress in the material along the section, E K L H, in terms of P_n, and will be a maximum when $\theta = 45°$, or sin $2\theta = 1$. Hence, P_n tends to break off the portion, E A K, along a plane, E K L H, inclined at an angle of 45° with E A or E F.

$$\therefore \qquad P_n = \frac{1}{3} . t^2 f.$$

Or, putting $P_n = \frac{2}{3} P$, we get :—$P = \frac{1}{2} . t^2 f.$ . . . . (VI)

This equation shows that in this case P is independent of the breadth and length of the tooth.

Substituting for t its value in terms of the pitch, p, we get :—

$$\left.\begin{array}{l} P = 0{\cdot}065\, p^2 f, \textit{ for iron teeth} \\ P = 0{\cdot}100\, p^2 f, \quad \textit{,, wooden ,,} \end{array}\right\} \quad . \ . \ . \quad (VII)$$

Equations (VII) again show that *the driving force*, P, *varies as the square of the pitch of the teeth.*

From what has been said above regarding the uncertainty of the distribution of the pressure on the teeth, it will be evident that the results expressed by equations (VII) should be taken in the design of wheel teeth.

Hence, combining equations (III) and (VII) we get the following :—**When the teeth of different wheels are proportioned according to the same rules, the power which they are capable of transmitting is proportional to the pitch circle velocity and to the square of the pitch of the teeth.**

Or, $$\text{H.P} \propto \text{V} p^2.$$

$$\therefore \qquad p \propto \sqrt{\frac{\text{H.P.}}{\text{V}}}.$$

The value of f in equations (VII) varies according to circumstances. In machinery subjected to shocks, vibrations, or sudden reversals (as in pumping and rolling-mill gears) a larger factor of safety, and, therefore, a smaller value of f must be employed, than in those other cases (such as hand-worked or slow moving machinery), which are not so severely stressed. The following average values for f are given by Prof. Unwin :—

Iron teeth subjected to little shock, f = 9,600 lbs. per sq. in.

,, ,, moderate ,, f = 6,100 ,, ,,

,, ,, excessive ,, f = 4,300 ,, ,,

For wooden teeth we may take $f = 2{,}740$ lbs. per square inch, since these should never be subjected to severe shocks.

Breadth of Wheel Teeth.—The greater the breadth of the teeth the greater is their durability. When, however, the teeth are made too broad, there is a difficulty in fixing the wheels accurately in gear, since the slightest amount out of truth may cause the mutual pressure between the teeth to act over a very limited area. The breadth varies from $2p$ to $4p$ in ordinary gearing, the average being $2{\cdot}5\,p$.

The above formulæ for the strength of teeth, though deduced for the case of spur wheels, are equally true for bevel gearing. In the latter case, however, the velocity, V, and the pitch, p, are to be measured at a pitch circle half way between the larger and smaller ends of the conical pitch frustum.

EXAMPLE II.—A spur wheel of 2 inches pitch and 4 inches width of face transmits 30 H.P. when its pitch line velocity is 10 feet per second. What power could be transmitted by a spur wheel of 4 inches pitch and 8 inches width of face with a pitch line velocity of 3 feet per second? (S. and A. Mach. Const. Hons. Exam., 1881.)

ANSWER.—Let the various quantities in the two cases be distinguished by the suffixes 1 and 2 respectively.

From equation (III) $\quad \text{H.P.} = \dfrac{\text{P V}}{33{,}000}$.

Now, assuming contact between the teeth to be perfect, as explained in the text, we get:—

From equation (IV) $\quad \text{P} = \frac{1}{4}\left(\dfrac{b\,t^2}{l}\right)f.$

Or, „ (V) $\quad \text{P} = {\cdot}0463\,b\,p\,f.$

$$\therefore \quad \text{H.P.} = \frac{{\cdot}0463}{33{,}000}\,f\,b\,p\,\text{V}.$$

Hence, assuming f to be the same in both cases, we get:—

$$\frac{\text{H.P.}_2}{\text{H.P.}_1} = \frac{b_2 p_2 \text{V}_2}{b_1 p_1 \text{V}_1}.$$

$$\therefore \quad \frac{\text{H.P.}_2}{30} = \frac{8 \times 4 \times (3 \times 60)}{4 \times 2 \times (10 \times 60)} = \frac{6}{5}.$$

$$\therefore \quad \text{H.P.}_2 = \frac{6}{5} \times 30 = 36.$$

Flanged or Shrouded Wheels.—Sometimes the rims of toothed wheels are made broader than the teeth, and extend to the pitch circle or even to the points of the teeth. The wheels in such cases are said to be "*flanged*" or "*shrouded.*" Shrouding has the effect of increasing the strength of the teeth, and it is for this purpose they are so made. With ordinary proportions of teeth, shrouding to the points may have the effect of nearly doubling the strength of the teeth. It is clear, however, that only one of a pair of wheels can be strengthened in this way. When the two wheels in gear are about equal in size, both may be shrouded to near their pitch circles. In other cases, it is usual to fully shroud the smaller wheel only, since the teeth of this wheel are subjected to greater wear than those on the larger one. Sometimes both wheels are shrouded to the points of their teeth on one side only, the shrouded side of the one being opposite the unshrouded side of the other. This

Longitudinal Section of Wheel Shrouded to Points of Teeth. *Shrouded to the Points.* *Shrouded on one Side.* *Shrouded to Pitch Line.* *Longitudinal Section of Wheel Shrouded to the Pitch Line.*

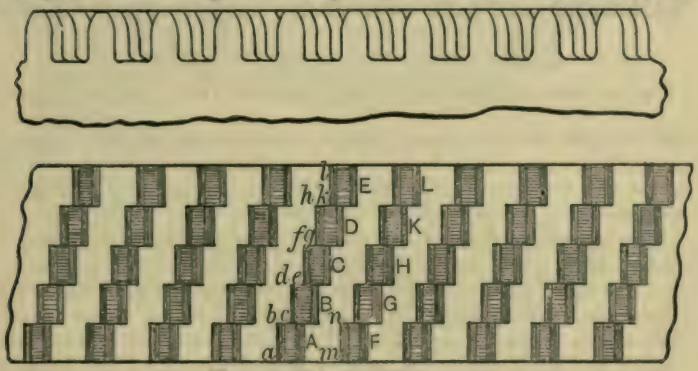
SHROUDED TEETH.

method is also adopted in those cases where the wheels are required to be thrown out of gear, by sliding one of the wheels along its shaft. These three methods of shrouding will be easily understood from the accompanying figures. Owing to the difficulty in moulding shrouded wheels, they are never adopted except in heavy machinery subjected to severe shocks.

Hooke's Stepped Gearing.—The smoothness of action with toothed gearing depends upon the number of pairs of teeth which are in contact simultaneously, the greater the number the sweeter and smoother the motion. This is of the greatest importance with some kinds of machinery, and noisy action should be avoided, as far as possible, in every case. We have seen in a previous Lecture that the number of pairs of teeth which are in action at once may be increased, either by reducing the pitch of the teeth, or by increasing the length of the path of contact. But reducing the pitch of the teeth reduces their strength, as we have just shown ; and increasing the length of the path of contact causes an increase in the length of the teeth, which has also the effect of reducing their strength. Hence, neither of these methods can be advantageously adopted. To overcome these difficulties, Dr. Hooke invented his *stepped gearing*, which results in the smoothness of action due to fine pitched teeth without the reduction in strength.

To understand this form of gearing, imagine an ordinary spur

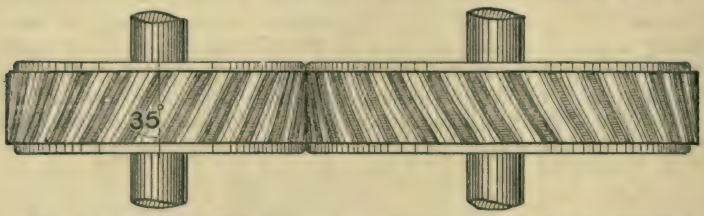
wheel built up of n (five on the figure) narrow spur wheels rigidly fixed together side by side, the pitch and other dimensions of the teeth being proportioned by the ordinary rules. Instead of the teeth being placed end to end in a straight line parallel to the shaft, let them be arranged in steps, as shown at A, B, C, D, E, so that each successive tooth is a short distance behind the previous one. The action with a pair of such wheels will clearly be similar to that of ordinary wheels wherein the pitch is only $b\,c$. The number of steps, n, and the length of the steps, $b\,c$, may be anything to suit circumstances, but the former is generally so arranged that the face of the last tooth, E, may just be to the left of the face of the first tooth, F, on the next series of teeth, by the length of a step, $b\,c$. The length of a step, $b\,c$, is then one nth of the pitch



HOOKE'S STEPPED GEARING.

of the teeth, and should be small compared with the thickness of the teeth, otherwise the teeth will be weakened for want of sufficient connection with those on either side. If the teeth are designed in the usual way, so that two consecutive teeth on any of the rings composing a wheel are always in contact with two consecutive teeth on the corresponding ring of the other wheel, it is evident that for the two wheels there will never be fewer than $2\,n$ pairs of teeth in contact. Thus, under ordinary circumstances, with five steps on each wheel, there would always be at least ten pairs of teeth in contact. The motion would, therefore, be much smoother and sweeter than with ordinary gearing. Stepped gearing is sometimes used for the rack and pinion arrangements for moving the tables of planing machines, which require to be very uniformly and steadily moved. This form of gearing might be conveniently adopted in many other cases where regularity and smoothness of motion are of primary importance.

Helical Gearing.—If we suppose bc to become infinitely small, and, therefore, the number of steps, n, infinitely great, the broken line $abcde\ldots$ would become a continuous curve, which is clearly a helix, or screw line, traced on the pitch surface of the wheel. The series of stepped teeth, A, B, C, D, E, would then form a single helical tooth. Wheels having their teeth formed in this manner are called **Helical Wheels.** When accurately made



SINGLE HELICAL WHEELS IN GEAR, SHOWING OPPOSITE OBLIQUITIES.

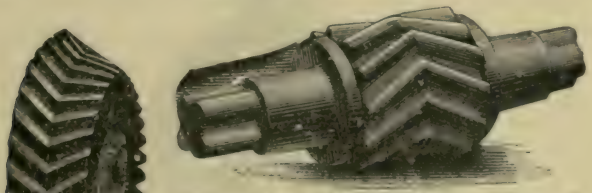
and erected, there is almost perfect line contact between the pairs of teeth in gear, thus augmenting the strength of the gearing, while the smoothness of action is about the greatest attainable with ordinary materials.

The obliquity of the teeth is the angle which their directions make with a plane containing the axis of the shaft, and in ordinary circumstances is about 35°. Just as screws are right-handed or left-handed, according to the direction of the helical thread on the bolt, so we speak of right-handed or left-handed obliquities with respect to helical teeth. A pair of helical wheels to gear together must have right- and left-handed obliquities respectively, as shown by the above figures.

Double Helical Wheels.—Owing to the oblique direction of the teeth their mutual pressure tends to separate the wheels axially, and thereby produces considerable lateral pressure on the bearings. To neutralise this prejudicial action, the teeth are now formed in two equal parts with opposite obliquities, and united at a common section, as shown by the accompanying figures. Such wheels are termed **Double Helical Wheels.** They are remarkable for their strength and smoothness of action. The pinion shown is shrouded to the pitch circle, which further increases its strength. No reliable information has as yet been forthcoming regarding the relative strength of helical and ordinary gearing, but it is believed that, under similar circumstances, the former are at least 30 per cent. stronger than the latter, and they are certainly more durable and easier in their action. It is, however, more difficult to obtain perfect bearing between the teeth of helical wheels than with

ordinary ones. The smallest displacement of the middle planes of the wheels from coincidence may cause the action between the teeth to be confined to one half of each tooth, thus virtually diminishing their strength. The method of overcoming this difficulty is to allow one of the shafts a small amount of lateral motion in its bearings, so that the wheel which it carries may always accommodate itself to the other.

Helical teeth are designed in the same way and according to the same rules as ordinary teeth, by setting off their dimensions,



PINION WITH DOUBLE HELICAL TEETH.*

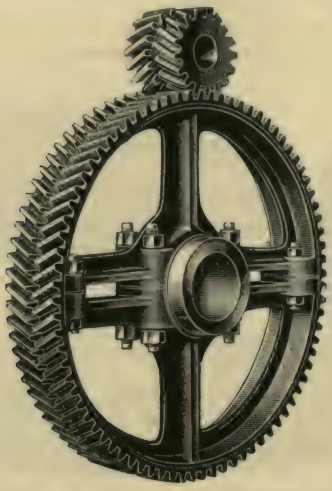
BEVEL WHEELS WITH DOUBLE HELICAL TEETH.*

&c., along a pitch circle of the wheel. The teeth, however, must lie along the face of the wheel in a helical direction.

Helical wheels are extensively used in the construction of heavy gearing, such as that required for large cogging and rolling mills and submarine cable machinery, and they are said to give great satisfaction. The casting of such wheels presented considerable difficulties at first, but these have been overcome, and now helical bevel wheels can be as easily cast as spur wheels. Their cost is little more than that of ordinary toothed wheels.

* We are indebted to Messrs. P. R. Jackson & Co., Ltd., Manchester, and Messrs. Bodley Bros & Co., Exeter, for these figures.

*Wüst Patent Double Helical Gear.**—The spur gear shown in the accompanying figure, is unlike the usual form of cut double helical gear—which usually consists of two half wheels bolted together—since the teeth of both sides are cut entirely from the solid blank. The teeth of the wheel are so arranged, that those on one side alternate with those on the other. Hence,



Spur Wheels with Double Helical Teeth.

there is a difference of half the pitch between the position of the teeth on opposite sides of this new and improved form of wheel gear. The larger wheel is shown split for use in a tramcar gear. The advantages of this gear are, that the engaging surface between the pinion and wheel is much greater than can be obtained with any other arrangement of teeth; whilst backlash and noise are reduced to a minimum, without the necessity of using rawhide pinions.

* The author is indebted to Marryat & Place, London, E.C., for the above figure.

Lecture XIV.—Questions.

1. Deduce an expression for the work lost by friction between the teeth of a pair of wheels in gear. In a pair of spur wheels with external contact, the number of teeth on the wheels are 25 and 75 respectively, and the coefficient of friction is 0·1. Assuming the arcs of approach and recess each equal to the pitch, find the efficiency of the gearing. *Ans.* 98·3 %.

2. In spur-wheel gearing, explain how to estimate the pitch of the teeth to transmit a given horse-power with a given speed of periphery. Show that under some circumstances the pitch should be proportional to $\sqrt{P}$, and under other circumstances to $\frac{P}{b}$, where P is the pressure between two teeth, and b is the breadth of the face of the teeth.

3. A toothed wheel, 18 inches diameter, makes 150 revolutions per minute, and transmits 30 horse-power, what is the maximum pressure on one tooth, assuming it to take the whole load? If the width of the teeth is 2 inches, what pitch should you adopt? *Ans.* P = 1,400 and p = 1·4 inches.

4. A cast-steel spur wheel transmits 80 horse-power. The pressure comes upon one tooth, and may be supposed to act uniformly along its point—that is to say, the tooth may be regarded as a cantilever loaded at the extremity. Diameter of spur wheel, 3 feet; number of revolutions per minute, 150; length of tooth, 1½ inches; width of tooth, 4 inches; safe tensile and compressive strength of cast steel (allowing for vibrations) per square inch, 8,000 lbs. Find thickness for root of tooth. *Ans.* ·72 inch.

5. Sketch the rim of a spur wheel with shrouded teeth. Explain the object of shrouding the teeth.

6. Give sketches showing Hooke's stepped gearing, and explain its construction and action. What advantages are derived from making toothed gearing of this form?

7. Give sketches showing single and double helical wheels. Explain the principles upon which they are constructed and act. Why are double helical wheels used in preference to single helical wheels? Discuss the advantages and disadvantages of helical and ordinary toothed wheels.

8. Distinguish between a spur-wheel, a bevil-wheel, a worm-wheel, and a rack. What is the velocity ratio of two wheels? If a bar of cast iron 1 inch square and 1 inch long when secured at one end, breaks transversely with a load of 6,000 lbs. suspended at the free end, what would be the safe working pressure, employing a factor of 10, between the two teeth which are in contact in a pair of spur-wheels, whose width of tooth is 6 inches, the depth of the tooth, measured perpendicularly from the point to the root, being 2 inches, and the thickness at the root of the tooth 1½ inches? (S. & A. Adv. Exam., 1896.) *Ans.* 4,050 lb.

9. Describe how we find the shape of the teeth of a worm-wheel.* (S. & A. Hons. Exam., 1897.)

10. Why is the engineers' theory of bending not applicable to short beams. Show how it is made use of to help in finding the proper pitch for a spur-wheel tooth. (B. of E. H., Part I., 1902.)

* See Prof. Rankine's *Machinery and Millwork*, Section IV., and Prof. Unwin's *Machine Design*, Part I., chap. xi.

LECTURE XIV.—A.M.INST.C.E. EXAM. QUESTIONS.

1. Define the following forms of gearing:—Ordinary tooth gearing, shrouded gearing, helical gearing, screw gearing; and make hand sketches of each. Design and draw a suitable screw to one-fifth scale to suit the case when the shafts are at right angles and a speed reduction of 10 to 1 is desired; the driving shaft is rotating at 1,000 revolutions per minute, and the power to be transmitted is 10 H.P. The diameter of both shafts, for reasons unconnected with the gearing, is 5 inches. (I.C.E., *Oct.*, 1905.)

LECTURE XV.

CONTENTS.—Belt, Rope, and Chain Gearing—Materials for Belting—Curing, Cutting, and Splicing Leather for Belts—Different Methods of Jointing Leather Belts—Average Strength of Leather Belt Joints—Manufacture of Long and Broad Leather Belting—Which Side of the Leather should Face the Pulley—Double and Treble Belting—Compound Belting—Link Chain Belting—Victoria Belting—Waterproof Canvas Belts—India-rubber Belts—Guttapercha and Composite Guttapercha Belts—Strength of, Working Tension in, and Horse-power Transmitted by, Belts—General Requirements for Belting—Rope Gearing—Sizes of Ropes and Pulleys—Strength of Cotton and Hemp Ropes—Rope Pulleys—Multigroove Rope Drives—Speed of, and Horse-power Transmitted by, Ropes—Power Absorbed by Rope Driving—Telodynamic Transmission—Pulleys—Wire-Rope Haulage and Transport—Questions.

Belt, Rope, and Chain Gearing.—The transmission of power between distant shafts is usually effected by means of pulleys and belts, ropes, or chains.* Although belts are most commonly

* The transmission of power between distant shafts is often effected by means of dynamos, wires, and motors, but this case is evidently outside of the range of the present work. The following is a list of books and papers treating of belt and rope driving:—

E. & F. Spon's *Dictionary of Engineering* (E. & F. Spon, London).

Paper on "Transmission of Power by Wire, Ropes, and Turbines," by H. M. Morrison. *Proceedings of Inst. of Mech. Engineers*, 1874, p. 56.

Paper on "Rope Gearing for Transmission of Large Powers in Mills and Factories," by J. Durie. *Proceedings of Inst. of Mech. Engineers*, 1876, p. 372.

Sir W. Siemens' prize "Essay on Machine Belting," by A. H. Barendt, read before the Liverpool Polytechnic Society, January 29, 1883.

Paper on "Belt Driving," by J. Tullis, communicated to the Convention of British and Irish Millers in Glasgow, June 17, 1885. See Messrs. Tullis & Son's *Guide to Belt Driving*, 1891.

"Rope Driving," by Chas. W. Hunt, of N. Y. City. See vol. of *Trans. Am. Soc. of Mech. Engs.* for 1890.

A Treatise on the Use of Belting for the Transmission of Power, by John H. Cooper. Fourth edition, 1891. Published by Edward Meeks, Walnut Street, Philadelphia, U.S.A., and E. & F. Spon, London.

"Experiments on the Transmission of Power by Belting," made by Messrs. Wm. Sellars & Co. See vol. vii. *Am. Soc. Mech. Engineers.*

Notes on the "Introduction and Development of Rope Driving," by Abram Combe. See *Proc. Inst. Mech. Engs.*, 1896.

Paper on the "Transmission of Power by Ropes," by Edwin Kenyon. See *Trans. Inst. Engs. and Ships*, Glasgow, vol. xlviii., 1905.

employed for this purpose, yet in certain cases, where great power has to be transmitted between two distant shafts at high speeds, rope gearing is preferred on account of its flexibility, lightness, quiet smooth working, easy repair, small first cost, and the facility with which the desired tension can be regulated by means of a tightening pulley when one continuous rope is used.*

Chain gearing is employed in cases where the motion is slow and the power greater than could be safely transmitted by narrow belts or ropes. It is also used where slipping is inadmissible, and where, as in the case of rolling mills, &c., belts or ropes would soon become perished or burned by the heat from the materials being acted upon by the rolls.

Materials for Belting. — The most common and, generally speaking, the best material for belting is leather; although many substitutes, such as cotton, india-rubber, Dick's canvas and balata, woven gut, camels' and Llama hair, have been devised, and found very serviceable under special conditions.

Curing, Cutting, and Splicing Leather for Belts. — The best leather for belting is made from skins taken from the backs of full-grown Highland oxen. The hides are thoroughly cured by being immersed for a long time in an "orange tan" liquid,† which possesses the property of condensing and contracting the raw hide instead of causing it to swell and become heavier, as is the case when they are tanned with "oak bark." This process produces a

Paper on "Belting for Machinery," by H. A. Mavor, M.Inst.E.E., read before the Institution of Engineers and Shipbuilders, Glasgow, on February 21, 1893. See *Proceedings of the Institution.*

Elements of Machine Design, Part I., "Belt and Rope Gearing," by Prof. W. C. Unwin, F.R.S. (Longman, Green & Co., London).

Pocket Diary of the *Mechanical World* (Emmot & Co., Manchester).

The *Practical Engineer* Pocket-Book and Diary (Technical Publishing Co., Manchester).

Paper by Prof. W. C. Unwin, F.R.S., being the Howard Lectures delivered before the Society of Arts, 1893. Printed in the *Society of Arts Journal*, and reprinted in the *Practical Engineer*, December, 1893, and January, 1894.

A series of excellent articles on "Rope Driving," by Prof. J. J. Flather in *The Electrical World* (W. J. Johnston Co., Limited, New York City), from October 21, 1893, to April, 1895.

* When two or more independent ropes are employed, tightening pulleys are found to be impracticable.

† From the Patent Specification No. 8,165 of 1884 we learn that this liquid consists of "'Spanish extract,' having borax or saltpetre dissolved therein; the said extract being an astringent solution obtained by grinding and boiling the rind of the orange and lemon. The quantities of borax and saltpetre may be considerably varied. After the hides, when thus saturated, are treated by the ordinary operation of currying, it converts them into leather fit for commercial use."

thin, firm, practically stretchless and very light, strong leather, which is particularly well suited for the transmission of power. The thickness of the leather thus obtained is generally about $\frac{3}{16}$ inch, its extreme length is about 4 feet 6 inches, and width about 4 feet. The tanned hides are usually cut up into parallel strips of the required breadth, and these strips are then joined end to end by overlap tapered splices in order to make up the desired length of belting.

In forming these splices the ends of two strips are *first* carefully tapered to a thin edge by planing machines.* *Secondly*, the tapered faces are covered with glue. *Thirdly*, they are cemented together under great pressure. *Fourthly*, as an additional precaution the splices are laced with untanned ox-hide, riveted with copper rivets and washers, or sewn with wax thread and copper wire.

If a greater thickness should be required than that afforded by a single strip of the hide (as in the case of double or treble belting), then two or three such strips are thoroughly glued together under great pressure. The final ends are left square until the roll of belting is taken from the store for the purpose of making up an endless belt of the required length for any particular set of pulleys. A sufficient length is then cut from the roll and stretched before making the final joint by one or other of the following methods.

Different Methods of Jointing Leather Belts.—Fig. (1) shows a joint which is generally made in a belt factory where the necessary scarfing planes and compressing gear are available. Cemented joints made in this way are nearly as strong as the other joints. They last longer and drive better than when cut up by sewing or riveting (see following table).

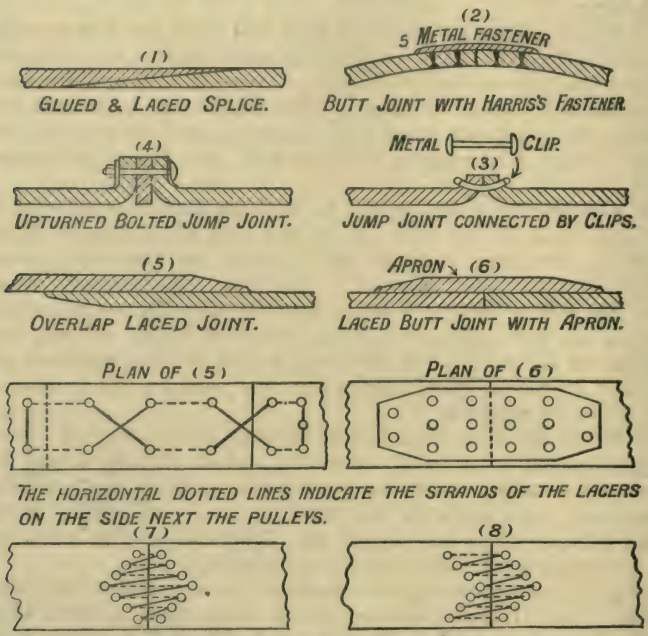
Fig. (2) represents a good connector for new and stout leather belts. This Harris's fastener is laid down upon an iron block with the teeth upwards. First one end of the belt is driven home upon one half of the teeth, and then the other end upon the other half, care being taken not to disturb the curve of the fastener as this gives the necessary holding bite to the teeth.

Fig. (3). The tough yellow metal fasteners shown serve very well for belts running at high speeds over small pulleys. The slits for these clips are made by a special form of cutting pliers and should be cut fully $\frac{1}{4}$ inch from the end of the belt, being so spaced that the crossheads of the fasteners come close to each other across

* Care should be taken to plane the one strip from the hair side and the other strip from the flesh side, so that the belt as a whole may present a uniform surface on each face.

the strap. In order to prevent the body of the metal clip from being bent up and down when passing over the pulleys the fastener should be as short as practicable.

Fig. (4). The strength of an upturned joint of this form is increased by inserting a leather strip or washer between the turned up ends as shown. For this kind of joint Jackson's patent bolt and washer fasteners are used by Messrs. John Tullis & Son, of



TWO METHODS OF LACING BUTT JOINTS.

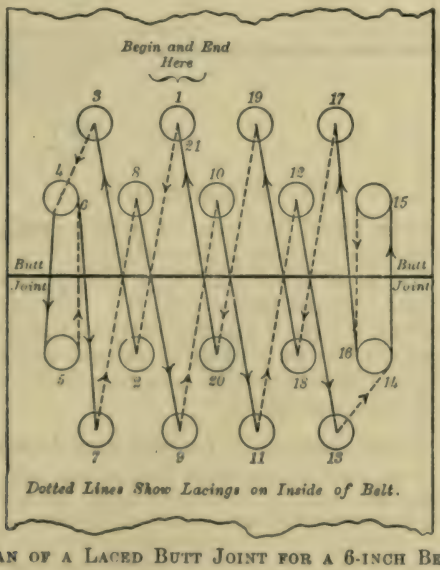
DIFFERENT METHODS OF JOINTING LEATHER BELTS.

Glasgow. They are also recommended by them for cotton and Llama hair belting.

Fig. (5) shows the ordinary overlap laced joint in both section and plan. Care should be taken to taper down and curve the ends to suit the smallest pulley over which the belt has to pass, otherwise the joint will be stiff, and, consequently, every time it travels on and off the pulley a sort of hinge action takes place, accompanied by a shock, which not only shortens the life of the belt, but also communicates a jar to the shaft and machine being driven.

Fig. (6). For heavy broad leather belts, where a scarfed cemented joint cannot be used, this form of joint with leather apron makes a very good connection.

Figs. (7) and (8) show the best methods of lacing simple butt joints. The holes for the lacer are punched so as to form a diamond or pointed figure, whereby there are never more than two holes in line across the belt ; and, consequently, it is possible to retain almost the entire strength of the belt without reducing its flexibility. Another plan of lacing a butt joint, which is very suitable for dynamo driving and ensures steady smooth running, is illustrated by the following figure :—

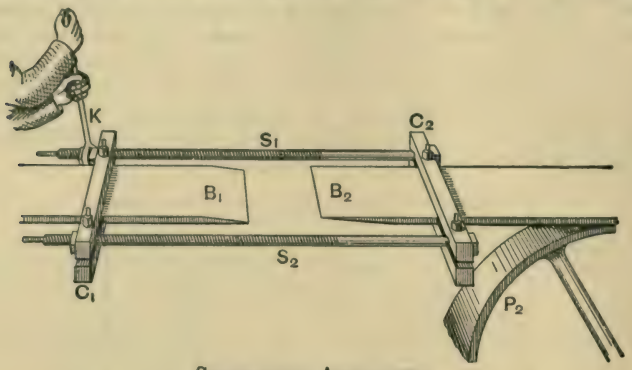


PLAN OF A LACED BUTT JOINT FOR A 6-INCH BELT.

The joint is made by a Helvation leather lacer of $\frac{1}{2}$-inch width in one length, by beginning at hole marked 1, and continuing through the several holes in the order, 2, 3, 4, &c., as shown by the figure, ending with hole 1, which is also marked 21.

In order to facilitate the making of any one or other of the above-mentioned joints in belts more than 6 inches wide, it is usual to employ a "drawing-up frame" of the form shown by the accompanying figure. The two ends, B_1, B_2, of the belt, after being passed over the driving and driven pulleys, are brought towards each other and gripped by the clamps, C_1, C_2. The

spanner, K, is then applied alternately to the nuts of the long screws, S_1, S_2, until the tapers of the splice are brought fairly over each other, or until the ends come together fairly and squarely in the case of a butt joint. The lacing can then be done



STRETCHING APPARATUS.

without any other effort than that of merely passing the thong through the holes and drawing it home tightly.

The chief objects to be attained by a good joint are—*first*, to maintain a close approximation to the belt strength; and *second*, to keep the thickness and flexibility of the joint as nearly equal to that of the belt as possible in order to prevent jumping. From the following table it will be seen how far the first of these objects is attained in practice:—

AVERAGE STRENGTH OF LEATHER BELT JOINTS.

And percentage strength as compared with the average strength of leather belts, taken as 4,132 lbs. per square inch. From tests made by A. H. Barendt at the Liverpool School of Science.

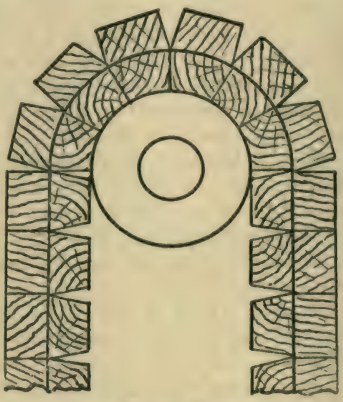
Kind of Joint.	Average Strength in lbs. per square inch.	Per Cent. Strength.
Cemented (only), ...	2,254	52·12
Ordinary white thong, without rivets, .	2,404	59·4
Waxed thread, without rivets, . .	3,240	78·4
Wire sewn or clinched, without rivets, .	3,272	79·18
White thong, with two rivets, . .	2,262	54·74
Waxed thread, with two rivets, . .	2,802	67·83
Copper wire sewn, with two rivets, .	3,050	75·81
White thong, with one rivet, . . .	3,226	78·06

Manufacture of Long and Broad Leather Belting.—By the ordinary plan of cutting up hides into straight strips, their length is limited to about 5 feet, thereby necessitating a corresponding number of splices. Messrs. Sampson & Co., of Stroud, have recently devised a method of making long continuous strips of leather. A disc of leather, 4 to 4½ feet in diameter, is cut from the hide. After being tanned and cured in the usual way, it is cut in a spiral direction from the outer edge to within about 9 inches of the centre. This spiral strip is then stretched and well rubbed until it becomes quite straight. It is found that the strength of the leather is not apparently diminished by this process. Of course, the length of the strip depends on its breadth. If the breadth be only 1·5 to 2 inches, then a strip of about 100 feet can thus be cut from a disc 4½ feet in diameter. By sewing a number of such strips together side by side, while stretched, a belt of any required width can be obtained. Belts 75 inches broad, ½ inch thick, and over 150 feet in length, have thus been built up for transmitting power in mills, directly from the flywheels of large engines to the main shafting. Long belts should never be made heavy, because the greater their weight, the greater becomes their tendency to oscillate up and down, and swing from side to side.

Which Side of the Leather should Face the Pulley.—The grain or hair side of the leather is naturally much smoother than the flesh side, and engineers differ in opinion regarding which of these sides should run in contact with the pulley. In this country, the rough or flesh side is almost invariably placed in contact with the pulley; but in America it is the smooth side. It is claimed for the latter method, that the life and efficiency of a belt is thereby increased. Whichever side of the belt is in contact with one pulley, the same side should be in contact with all the pulleys over which it passes, so as not to bend it alternately in different directions. When the flesh side is next the rim of the pulley, it should receive one coating of currier's dubbing, and three coatings of boiled linseed oil every year, in the case of a belt having to endure continuous hard work. This has the effect of rendering it about as smooth as the hair side, and its efficiency as a whole is said to be thereby increased. When the hair side is in contact with the pulley, it is usual to give that side an occasional coating of castor oil in order to render it more flexible, and, consequently, more durable than it would be in its natural state. Rosin or cobbler's wax should *never* be employed, as they gather dirt, and form lumps on the pulley and belt.

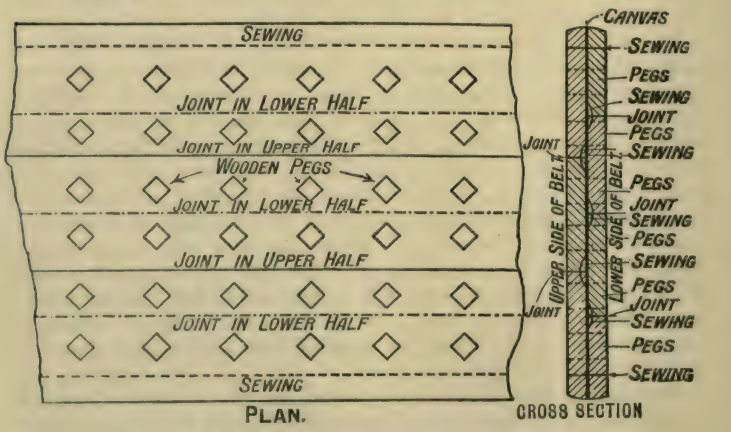
Double and Treble Belting.—Sometimes the breadth of a single belt necessary to transmit a given power would be inconveniently great, and hence double or treble belts are used. These are made

by cementing, and then riveting or sewing, two or three thicknesses of belting together. Owing to the greater rigidity of such belts, they do not work so satisfactorily as single ones, and are not well adapted for running at high speeds, or over pulleys having a diameter of less than 3 or 4 feet, or in cases where the pulleys are close together. This is clearly seen from the accompanying illustration, which shows the change that is continually going on in thick belts, especially when passing over small pulleys. The compression on the inner face of the belt is shown by the necessary reduction in the size of the inside blocks, and the stretching of the outer face by the parting of the blocks of the outside row, when passing over the pulley.



ACTION ON THICK BELTS PASSING OVER SMALL PULLEYS.

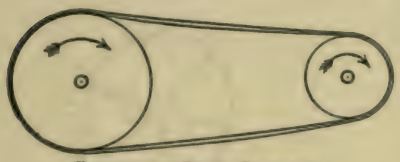
Messrs. Sampson & Co. build their double belts by sewing together and pegging with hard wood pins, a number of specially cut narrow leather strips, with a layer of canvas between



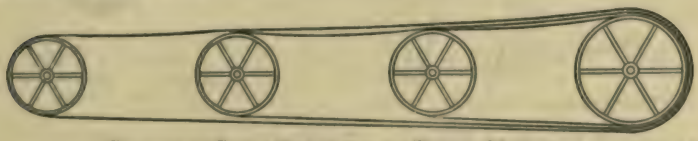
SAMPSON & CO.'S DOUBLE LEATHER BELTING.

them, as illustrated by the accompanying figure. There is always one more strip on the lower or working side than on the upper half. They claim the advantage of being able to produce in this way a belt of very nearly uniform weight throughout its length, having a hard, smooth surface formed on the flesh side, due to the hammering required to drive home and clench the numerous hard wooden pegs with which the two single belts are fixed together. Further, since the sewing is not exposed on either surface of this belt, the leather must be nearly worn through before the sewing is damaged.

Compound Belting.—In order to avoid the internal straining action of thick double or treble belting, it has been found advisable to simply put two or three thin single belts on the top of each other, without any cementing or riveting together, as shown by the accompanying figure. This gives perfect freedom of action to each belt to accommodate itself to the curves over which it passes without stressing its neighbours. The friction between the surfaces of these several belts is not found to have any observable objections, and it is said that 70 per cent. more power can be transmitted by compounding two single belts in this manner. By the addition of a third belt, still more power may be conveyed. Messrs. Tullis & Co. find that by placing plain double belting on the top of their linked chain belts (when arched to fit the curve of the pulley), twice as much power may be transmitted than by either of these alone.



COMPOUND BELT DRIVING.



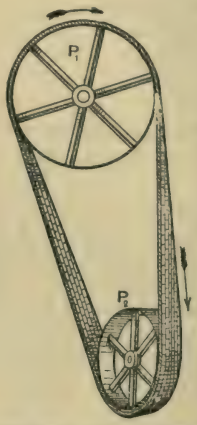
COMPOUND BELT DRIVING FOR THREE MACHINES.

A simple modification of this compound belt drive is illustrated by the above figure. The power is transmitted from one main driving pulley to two or more driven pulleys by separate belts, all moving in the same plane, and in a direct line with each other. This arrangement saves considerable room in a workshop or factory. It has been proved to be very handy for driving several dynamos direct from one flywheel, instead of from independent drums.

It is often urged as an argument against single belting, that it does not work well under the action of shifting guide forks. This may be avoided by running the belt at a sufficient speed, say between 2,000 and 3,000 feet per minute, and, in all cases, by placing the guiding fork at least 1 foot from the point where the belt touches the ongoing side of the pulley, so as to give freedom to the belt to assume its new position.

Link Chain Belting.—Since flat belts fail to take a perfect grip, due to their retaining a cushion of air between them and their pulleys, Messrs. Tullis & Son, of Glasgow, have devised a complete

THICK-SIDED CHAIN BELTING.



TULLIS'S THICK-SIDED LEATHER CHAIN BELT, WORKING QUARTER TWIST.

system of link chain belting. This form of belting permits of the escape of the air through the spaces between the links, and thus enables the leather to bear uniformly over the face of the pulley.

It is composed of a series of short leather links bound together by steel pins and washers. These belts possess considerable flexibility, and can be made with a flexible central row of links to automatically suit the arch of any pulley. If desired, they may be specially curved to suit any camber. They further possess the advantage of being easily shortened and rejointed by simply bringing the ends together with a drawing-up frame, interlocking

the links until a row of rivet holes are brought fair in a line and then inserting the rivet. When one side of this belting is made thicker than the other it is well adapted for working on tapered cones and for quarter twist driving.

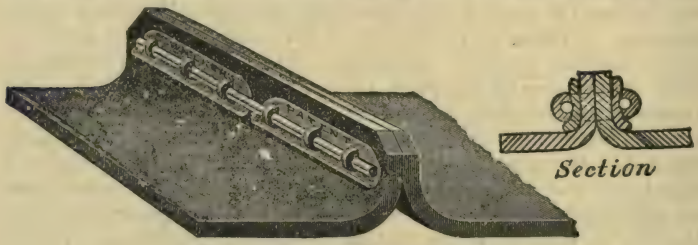
Victoria Belting.—A successful kind of belting has of late been introduced under the name "*Victoria Belting.*" This is simply ordinary belting from which the spongy parts of the leather on the flesh side have been pared away by aid of machinery, and the whole reduced to a uniform thickness. A belt of this description is more flexible than an ordinary one, and, although thinner, is about equal in strength. Double belts, made by cementing and riveting two thicknesses of Victoria belting, are little thicker than ordinary single belts, and are quite as flexible and very much stronger. The hair side of each of the parts cemented together being kept outside, these will always run in contact with the rims of the pulleys and guide pulleys over which they pass.

Waterproof Canvas Belts.—Next to leather, waterproof canvas belting of one kind or another is the most popular. It is especially well suited for paper-mills, dye-works, and other factories where moisture or steam is prevalent, or for working in the open air under varying climatic conditions. The best canvas belting is made from selected Egyptian cotton. After being folded to the required breadth and number of plies, it is stretched and then stitched throughout its length along the open seam. After being dried it is thoroughly saturated with linseed oil, painted with red silicate to make it grip, mangled to squeeze out the superfluous oil, and finally stretched for about a fortnight to render it as inextensible as possible under ordinary working conditions. It can be made in one piece of any required length, breadth, or thickness, and it has a greater breaking stress per square inch than oak tanned leather in the ratio of about 6,700 lbs. for canvas to 4,100 lbs. for the leather. It forms excellent main driving belts, but once it begins to wear and give way at any place it cannot be repaired or patched so readily as leather, neither does it so well endure cross driving or being acted upon by shifting forks. The latter defect is to a large extent overcome by protecting it with stitched leather edges throughout its entire length.

India-Rubber Belts.—These belts present a very smooth, well finished, and excellent gripping surface. They are made by taking stout canvas of the required breadth direct from the roll, passing it over a steam heated cylinder in order to expel the damp inherent in the canvas, which would otherwise rot the first layer of rubber. The web is next passed between two heated rollers. From the upper one it receives a thin plastic coating of rubber. It is then folded double; or, if the canvas is of the same breadth as the belt,

two or more plies similarly treated are pressed together. It is then entirely wrapped in rubber, trimmed along the edges, and vulcanised in steam heated moulds under considerable pressure. They do not wear nearly so well as leather belts, for the life of a hard worked one does not exceed eighteen months, and once the rubber begins to peel off the working side and the canvas heart to wear they suddenly break straight across. They do not give so much trouble as might be expected from stretching, and they can be taken up and jointed by the methods shown in Figs. 1, 4, and 6 at the beginning of this Lecture. They cannot be recommended for shifting belts, as the edges which come into contact with the shifting fork soon become ragged and cut.

Guttapercha and Composite Guttapercha Belts.—Solid guttapercha belts have been tried, but being expensive, liable to perish, and to stretch they have been abandoned. A very successful composite belt has, however, been introduced of late by Messrs. R. & J. Dick of the Greenhead Works, Glasgow, which possesses all the advantages of water-proofed canvas, together with good gripping and



WALKER'S JUMP-JOINT FASTENER.

lasting qualities. It is manufactured from the best "long-staple" Egyptian cotton canvas, which is cut to the required breadth, dried, and covered on each side with balata, dried again, folded into the number of plies and breadth of the belt, covered on the outside with guttapercha, and finally, is stretched in long lengths in order to render it as inextensible as possible under working conditions.

Any required length, breadth, and thickness of belt can be made in this way, and the final single joint may be so neatly scarfed, glued, and pressed that it is not thicker than the ordinary belt and nearly as strong and supple. Or Walker's jump-joint fasteners may be employed, as illustrated by the accompanying figures.

Strength of, Working Tension in, and Horse Power transmitted by, Belts.—The ultimate strength of ordinary bark tanned single leather belting varies from 3,000 to 5,000 lbs. per square inch of

cross-section.* It is, however, sometimes considered more convenient to state the tenacity of a belt in lbs. per inch of width. The thickness of single belting varies from $\frac{3}{16}$ to $\frac{5}{16}$ inch, and from $\frac{3}{8}$ to $\frac{5}{8}$ inch for double belting. Taking the mean thicknesses, we may say that the breaking stresses are from :—

750 to 1,250 lbs. per inch of width for single belts,
and 1,500 to 2,500 „ „ double „

It is found, however, that the strength of the joints is sometimes only about one-third of the strength of the solid leather. Hence, the *final* strength of an endless belt should not be reckoned at more than *one-third* of the above values. Further, the *safe working tension* should never exceed *one-fifth* of this final strength (of the joint), in order to provide for deterioration and sudden changes of load. From this point of view, we thus arrive at approximate values for the safe working tensions by taking one-fifteenth (*i.e.*, one-third for joint and one-fifth for factor of safety) of the breaking stress *per inch of width*, viz. :—

For single belting, 50 to 80 lbs.
„ double „ 100 „ 160 „

In *practice*, however, the rule is not to put on a greater tension than 50 lbs.† for single, and 80 lbs. for double belts per inch of width, since it is found that under these mild conditions, leather belts run for many years with a minimum of lubrication on the bearings, and with far less chance of heating the journals or stressing the shafts.

The theoretical safe working tension per inch of belt width to transmit a certain power, at a certain speed, without slipping, may be arrived at from another point of view, viz. :—By ascertaining the coefficient of friction between a belt and its pulley, and substituting the same in the formulæ given in Lects. VII. and VIII., Vol. I., from which it will be seen, that the power which may be transmitted by a belt is determined by the difference between the tensions of the driving and following sides, and the speed of the belt, and that the logarithm of the ratio of these ten-

* Messrs. Tullis & Sons, of Glasgow, claim the following from tests carried out at Lloyd's Proving House :—

For *their* orange tanned leather, a breaking stress of 8,244 lbs. per sq. in., with 1·3 per cent. elongation.

For the best oak tanned leather, a breaking stress of 5,746 lbs. per sq. in., with 3·8 per cent. elongation.

† Mr. John H. Cooper, in his *Treatise on the Use of Belting*, pp. 383, 384, tabulates the results of fifty-three rules for the H.P. and stress transmitted by belts, and also arrives at the above mean result of 50 lbs. per inch of width.

sions is proportional to the coefficient of friction, and to the arc of the pulley embraced by the belt.*

EXAMPLE I.—Referring to the following table, we see that one H.P. can be transmitted per inch of width of belt, when travelling at 700 feet per minute. Let the arc embraced by the belt be 180° and the coefficient of friction ·25. Find the tensions on the driving and slack sides of the belt, and their difference.

Let T_d = Tension on driving side in lbs.
„ T_s = Tension on slack side in lbs.
„ V = Velocity of belt in feet per minute = 700.
„ μ = Coefficient of friction = ·25.
„ θ = Ratio of length of arc of contact to radius of pulley.

Then, $$\text{H.P.} = \frac{(T_d - T_s)\,V}{33{,}000}.$$

$$\therefore \quad (T_d - T_s) = \frac{\text{H.P.} \times 33{,}000}{V}.$$

Or, $$T_d - T_s = \frac{1 \times 33{,}000}{700} = 47{\cdot}2 \text{ lbs.} \quad . \quad . \quad (1)$$

From Lecture VII., equation (XI.), we get:—

$$\log_e \frac{T_d}{T_s} = \mu\,\theta.$$

But, $$\theta = \frac{\text{arc}}{\text{radius}} = \frac{\frac{1}{2}(2\,\pi\,r)}{r} = \pi.$$

$$\therefore \log_e \frac{T_d}{T_s} = \log \frac{T_d}{T_s} \times {\cdot}4343 = {\cdot}25 \times 3{\cdot}1416 \times {\cdot}4343 = {\cdot}3411.$$

$$\therefore \quad \frac{T_d}{T_s} = 2{\cdot}2. \quad \text{Or, } T_d = 2{\cdot}2\,T_s \quad . \quad . \quad (2)$$

Inserting this value in equation (1), we get:—

$$2{\cdot}2\,T_s - T_s = 47{\cdot}2. \quad \therefore T_s = 39{\cdot}3 \text{ lbs.}$$

And, $$T_d = 47{\cdot}2 + T_s = 86{\cdot}5 \text{ lbs.}$$

These are the minimum working stresses per inch width of belt which must be applied to prevent slipping under the above-mentioned conditions.

* For a description of Morin's experiments on the tension of belts made at Metz in 1834, and his use of this formula, see Cooper on "Use of Belting," Chap. VII.

HORSE-POWER THAT DIFFERENT LEATHER BELTS WILL TRANSMIT PER INCH IN WIDTH AT VARIOUS SPEEDS.

(By A. G. Brown, M.E., for Musgrave & Co.)

Velocity of Belt per Minute.	KIND OF BELTS.								
	Best Oak-tanned Belts.			Best Link or Chain Belts.					
	Single belts.	Light double belts.	Heavy double belts.	½"	¼"	⅜"	½"	⅝"	1"
Feet.	HORSE-POWER THEY WILL TRANSMIT.								
100	·15	·21	·27	·13	·15	·17	·20	·24	·27
200	·30	·42	·55	·25	·29	·35	·40	·47	·55
300	·45	·64	·82	·38	·44	·52	·60	·71	·82
400	·61	·85	1·09	·51	·58	·69	·80	·95	1·09
500	·76	1·06	1·36	·64	·73	·86	1·00	1·18	1·36
600	·91	1·27	1·64	·76	·87	1·04	1·20	1·42	1·64
700	1·06	1·49	1·91	·89	1·02	1·21	1·40	1·65	1·91
800	1·21	1·70	2·18	·92	1·16	1·38	1·60	1·89	2·18
900	1·36	1·91	2·45	1·05	1·31	1·55	1·80	2·13	2·45
1000	1·51	2·12	2·73	1·27	1·45	1·73	2·00	2·36	2·73
1100	1·67	2·33	3·00	1·40	1·60	1·90	2·20	2·60	3·00
1200	1·82	2·55	3·27	1·53	1·75	2·07	2·40	2·84	3·27
1300	1·97	2·76	3·55	1·65	1·89	2·25	2·60	3·07	3·55
1400	2·12	2·97	3·82	1·78	2·04	2·42	2·80	3·31	3·82
1500	2·27	3·18	4·09	1·91	2·18	2·59	3·00	3·55	4·09
1600	2·42	3·39	4·36	2·04	2·33	2·76	3·20	3·78	4·36
1700	2·58	3·61	4·64	2·16	2·47	2·94	3·40	4·02	4·64
1800	2·73	3·82	4·91	2·29	2·62	3·11	3·60	4·25	4·91
1900	2·88	4·03	5·18	2·42	2·76	3·28	3·80	4·49	5·18
2000	3·03	4·24	5·45	2·55	2·91	3·45	4·00	4·73	5·45
2100	3·18	4·45	5·73	2·67	3·05	3·63	4·20	4·96	5·73
2200	3·33	4·67	6·00	2·80	3·20	3·80	4·40	5·20	6·00
2300	3·49	4·88	6·27	2·93	3·35	3·97	4·60	5·44	6·27
2400	3·64	5·09	6·55	3·05	3·49	4·15	4·80	5·67	6·55
2500	3·79	5·30	6·82	3·18	3·64	4·32	5·00	5·91	6·82
2600	3·94	5·52	7·09	3·24	3·78	4·49	5·20	6·15	7·09
2700	4·09	5·73	7·36	3·28	3·85	4·66	5·40	6·38	7·36
2800	4·24	5·94	7·64	3·31	3·86	4·73	5·60	6·62	7·64
2900	4·39	6·15	7·91	3·32	3·87	4·78	5·80	6·85	7·91
3000	4·50	6·36	8·18	3·31	3·86	4·75	5·97	7·09	8·18
3100	4·60	6·58	8·45	3·30	3·85	4·73	5·96	7·33	8·45
3200	4·69	6·79	8·70	3·28	3·82	4·71	5·94	7·37	8·73
3300	4·77	7·00	8·86	3·24	3·77	4·70	5·92	7·35	8·88
3400	4·84	7·21	8·96	3·19	3·71	4·64	5·87	7·32	8·86
3500	4·90	7·31	9·06	3·13	3·61	4·50	5·78	7·26	8·80
3600	4·95	7·40	9·16	3·05	3·50	4·37	5·67	7·16	8·73
3700	4·99	7·48	9·24	2·96	3·39	4·26	5·55	7·01	8·58
3800	5·03	7·54	9·29	2·84	3·28	4·15	5·41	6·87	8·41
3900	5·03	7·60	9·34	2·72	3·13	4·02	5·20	6·70	8·27
4000	5·08	7·64	9·37	2·58	2·95	3·84	5·01	6·48	8·04

General Requirements for Belting.—The ultimate strength of a belt is, however, a secondary consideration, since it is so very much greater than the normal working stress. The properties of greatest importance are—(1) straightness, (2) stretchlessness, (3) pliability, (4) homogeneity, (5) uniformity in thickness, (6) good flexible joints, (7) ease of repair, (8) endurance or longevity. These can only be determined by practical experience. Further, the very best belting cannot be expected to work well upon pulleys which are unbalanced or out of truth and line, or which are too close together.* The pulleys should be as far apart as possible and their diameters as large as possible. For example, at Messrs. Clark & Co.'s thread works at Paisley there are pulleys as far apart as 90 feet. For high speeds the pulleys should have little or no crowning; for, when a speed of about 3,000 feet per minute is reached, the sides of the belt will rise up from the face of a heavily crowned pulley due to centrifugal force, and thus greatly diminish the area of contact, inducing slipping, wearing of the belt, and unsatisfactory driving.

Rope Gearing.—The transmission of power is frequently accomplished by means of ropes instead of belts. In addition to their other advantages mentioned at the beginning of this Lecture, ropes seldom give way without due warning by slackening or fraying in one or more of the strands, thus reducing the risk of accident and stoppage of the works to a minimum. The working stress in the ropes being but a small fraction of their breaking strength, any signs of weakness in an individual rope would allow it to be removed and the engine run with the remaining ropes until a convenient opportunity is offered for the replacement of the weak member. Further, their comparative slackness between the pulleys facilitates their cancelling any small irregularity in the motive power.

These ropes are made of manilla-hemp, cotton, leather, and raw ox hide. Hemp ropes are preferred to cotton ropes for main drives with large pulleys since they are cheaper, stronger, and last nearly as long if spun with a soft greased core. In Messrs. J. & P. Coats' great thread works, Paisley, cotton ropes are, however, universally employed. They are made "hawser laid," from the best "long staple" Egyptian, white, untarred cotton. By "hawser laid" is meant that the fibres of the material are first spun into yarns having a *right-handed twist*. These yarns are

* Mr. Henry A. Mavor in his reply to the discussion on his paper "Belting for Machinery," read before the Institution of Engineers and Shipbuilders in Scotland, states that (where possible) the pulleys should be kept apart not less than three times, and not more than four times the diameter of the larger pulley.

next twisted *left-handedly* into strands. Lastly, three such strands are twisted together *right-handedly* so as to complete the rope. It will be noticed that when the strand is twisted it untwists *each* of the threads, and that when the three strands are twisted together into rope this action untwists the strands, but at the same time re-twists the threads. It is this opposite twist that serves to keep the rope in its proper form.* Cotton ropes, being softer and more pliable than manilla ropes, can be used with smaller pulleys without undue injury to the fibres. This is also considerably aided by the natural wax in the structure of the long staple variety which acts as a lubricant, and permits of greater freedom of motion between the several fibres. The life of a good cotton rope is usually about thirteen years if properly adjusted and well treated.

Sizes of Ropes and Pulleys.—The size and number of ropes, as well as the least diameter of pulley, for any given power, are points of importance and should be considered for each case.

The ropes commonly used for the transmission of power in factories or mills vary from 3 to 5 inches in circumference. No matter what the diameter of the pulley may be, ropes of $1\frac{3}{4}$ inches diameter should not be exceeded for main drives, and $1\frac{1}{4}$ inches diameter for secondary drives. The diameter of the smallest pulley should not be less than thirty times the diameter of the rope, as the larger the pulley the less will be the internal friction, and consequent injury to the rope from bending and unbending (see the following table).

Strength of Cotton and Hemp Ropes.—The ultimate strength of white untarred cotton ropes may be taken at 9,000 lbs. per square inch of the nett sectional area, which is about 90 per cent. of the area of the circumscribing circle. The normal working stress should not exceed $\frac{1}{30}$ of the ultimate breaking stress, or say 300 lbs. per square inch, although ropes are frequently worked at even a less tension. Messrs. Musgrave & Sons, of Bolton, allow a working stress of about 300 lbs. per square inch of sectional area. Of this about 20 per cent. or 60 lbs. is absorbed in overcoming back tension, wedging of rope, &c., leaving 240 lbs. for centrifugal force and transmission of power.

The following table from *The Practical Engineer* gives useful data regarding these points.

The ultimate strength of *new manilla* ropes is about 11,000 lbs. per square inch of the nett sectional area, which in a three stranded hawser is only about 80 per cent. of the area of its circumscribing circle. The necessary lubrication, however, reduces the strength by

* See *The Practical Engineer* of June 22, 1894, p. 486, &c., for "Notes on the Manufacture of Ropes," by W. C. Popplewell, M.Sc.

20 to 30 per cent., but the lubrication of the fibres is of much greater importance than the actual breaking stress. The greater freedom of movement amongst the fibres permits a heavier working stress to be carried, and ensures a much longer life; for a properly lubricated manilla rope will outlast from two to four similar dry laid ropes working under the same conditions. There are many ways of lessening the internal friction; one of the best being that of coating the several yarns with a mixture of black lead and tallow prior to twisting the same into strands. Under favourable conditions, when thus treated, it is practically waterproof, and will last for about eight years.

COTTON DRIVING ROPES AND PULLEYS.

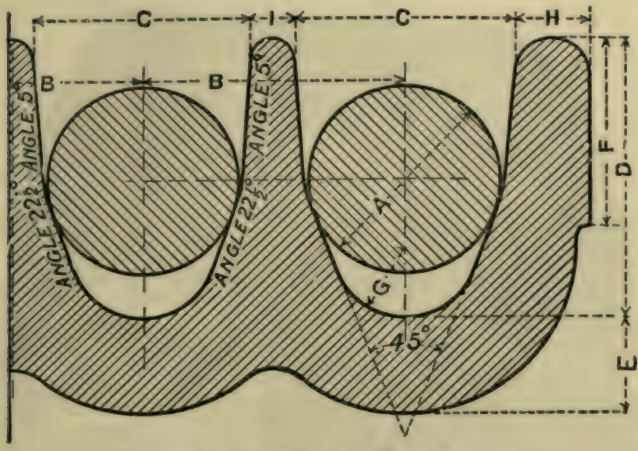
Diameters.	Areas.	Weight per Foot.	Ordinary Working Tension including Centrifugal Force.	VELOCITY OF ROPES, 4,700 FT. PER MINUTE (THEIR MOST EFFECTIVE SPEED).			ROPE PULLEYS.	
				Stress due to Centrifugal Force.	Effective Tension.	Power each Rope will Transmit.	Centre to centre of ropes	Diam. of smallest pulley.
Ins.	Ins.	Lbs.	Lbs.	Lbs.	Lbs.	H.P.	Ins.	Ins.
$\frac{1}{2}$	·1963	·081	47	16	31	4·43	$\frac{7}{8}$	15
$\frac{5}{8}$	·3067	·125	72	24	48	6·84	1	18
$\frac{3}{4}$	·4417	·184	106	35	71	10·07	$1\frac{1}{8}$	22
$\frac{7}{8}$	·6013	·25	144	48	96	13 67	$1\frac{5}{16}$	26
1	·7854	·33	190	63	127	18·05	$1\frac{1}{2}$	30
$1\frac{1}{4}$	1·2272	·51	294	98	196	27·90	$1\frac{13}{16}$	37
$1\frac{1}{2}$	1·7671	·74	426	142	284	40·48	$2\frac{1}{8}$	45
$1\frac{3}{4}$	2·4053	1 00	576	192	384	54·70	$2\frac{1}{2}$	52
2	3·1416	1·30	750	250	500	71·10	$2\frac{3}{4}$	60

Splicing Ropes.—The splicing of ropes is a matter of great importance, and should never be left to unskilled hands. In order to secure a sufficiently strong joint its length requires to be from forty to fifty times the diameter of the rope, or say from 6 to 7 feet for main, and from 4 to 5 feet for secondary drives. The different strands should be neatly interlocked, so that the thickness of the rope is not increased. The strength of a good splice is only about 70 per cent. of that of the rope.

Rope Pulleys.—The pulleys used for rope gearing are made with V-shaped grooves around their rims, as shown by the accompanying figures.

The sides of the grooves usually make an angle of 45° with each other. The ropes must never be allowed to rest on the bottom of the grooves, but only on the sides, as shown. They are thus

wedged into the grooves, and the resistance to slipping is thereby greatly increased. This has the effect, however, of producing greater wear and tear of the ropes. In the case of guide pulleys, the rope should always rest on the bottom of the groove.



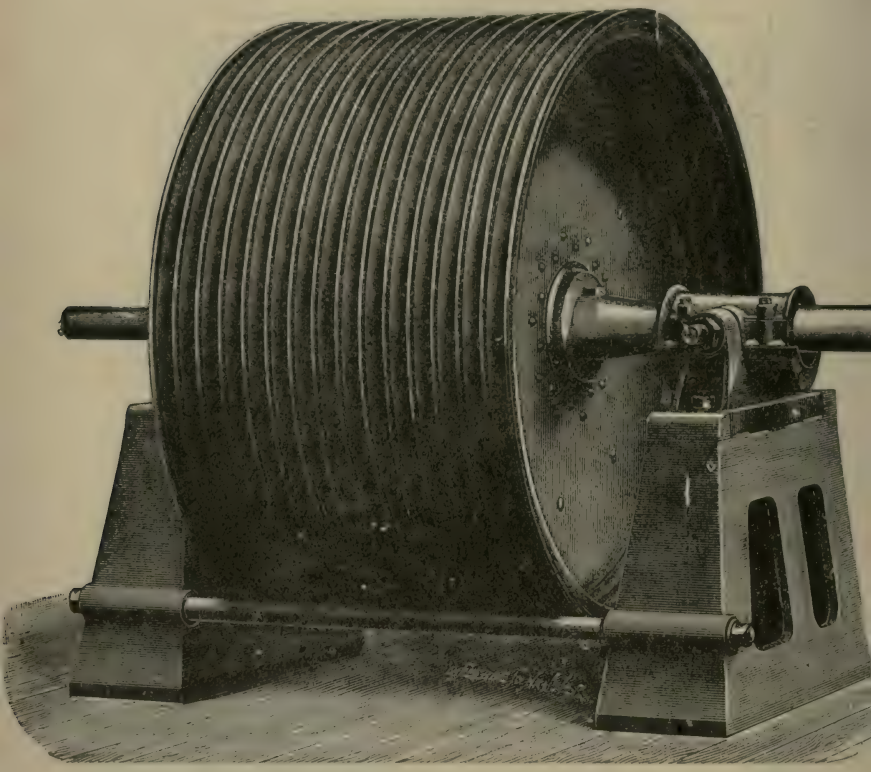
SECTION OF GROOVES FOR ROPE PULLEYS.

The following table gives the proportions of grooves of the form shown in the above figure, for ropes from $\frac{1}{2}$ inch diameter to $2\frac{1}{8}$ inches diameter :—

PROPORTIONS OF GROOVES FOR ROPE PULLEYS.

Diameter of Rope. A	B	C	D	E	F	G	H	I
Ins.	Ins.	Ins.	Ins.	In.	Ins.	In.	In.	In.
½, . . .	1	¾	⅛	1/16	¼	3/16	1/16	¼
⅝, . . .	1¼	⅞	1	⅛	⅜	3/16	⅛	¼
¾, . . .	1 5/16	1	1 3/16	1/16	⅜	3/16	⅛	¼
⅞, . . .	1 7/16	1¼	1⅜	⅛	⅜	1/16	⅛	1/16
1, . . .	1⅝	1¼	1 5/16	1/16	1	⅛	⅛	1/16
1¼, . . .	1⅞	1 5/16	1⅝	⅛	1¼	⅛	⅛	⅛
1⅜, . . .	2 3/16	1⅝	2¼	⅛	1½	1/16	1/16	⅛
1½, . . .	2 7/16	1⅞	2½	⅛	1⅝	⅛	⅛	⅛
1⅝, . . .	2¾	2 1/16	2⅝	⅛	1⅝	⅛	⅛	⅛
2, . . .	2⅞	2 5/16	3	1	2	⅛	⅛	1/16
2⅛, . . .	2⅞	2 7/16	3 3/16	1⅛	2⅛	⅛	⅛	⅛

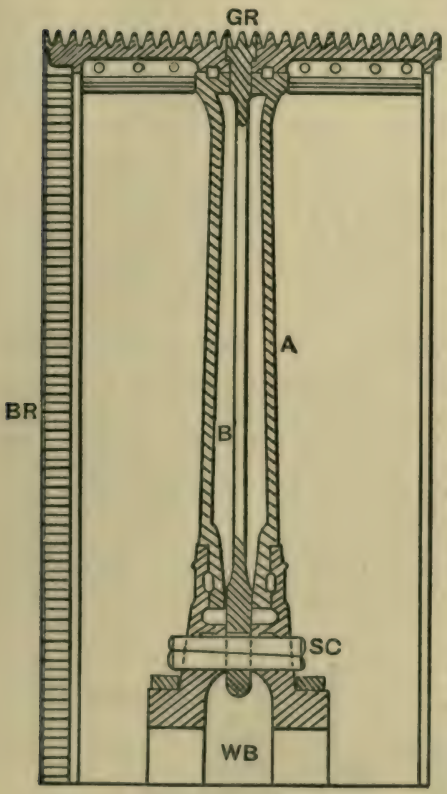
In the case of belts running on flat-rimmed pulleys, we have seen that sufficient adhesion to prevent slipping between the belt and pulleys is obtained by stretching the belt over the pulleys with an initial tension. But with ropes, it is found that smoother



Special Form of Rope Pulley by Watson, Laidlaw & Co.

working and less wear and tear take place when the rope is put on the pulleys with as little initial stress as possible. Hence the reason for increasing the frictional resistance between the rope and its pulleys, by allowing the rope to wedge itself into the grooves in the pulley rims. The necessary pressure between the rope and

the sides of the grooves to ensure sufficient frictional adhesion at the commencement of motion, is simply that due to the weight of the hanging parts of the ropes between the pulleys. The pulleys should, therefore, be large, and be placed at a sufficient horizontal distance apart, so as to have the arcs of contact between the rope and the pulleys as great as possible.



SECTION OF LARGE ROPE PULLEY.

The accompanying figure shows the construction of a large rope flywheel, in which the wrought-iron bolt, B, connects the grooved rim, G R, with the wheel boss, W B, and thus receives the tensile stress due to centrifugal action on G R.

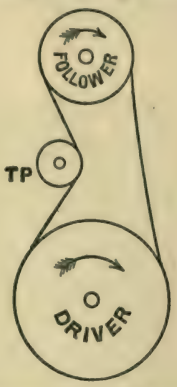
In horizontal drives, the tight side of the rope should always be in contact with the lower parts of the pulleys, and the slack side above, so as to obtain a maximum arc of contact between the rope and the pulleys, as shown by the following right-hand figure.

INDEX TO PARTS.

W B for Wheel boss.	A for Arm.
S C ,, Steel cotters.	G R ,, Grooved rim.
B ,, Wrought-iron bolt.	B R ,, Barring rack.

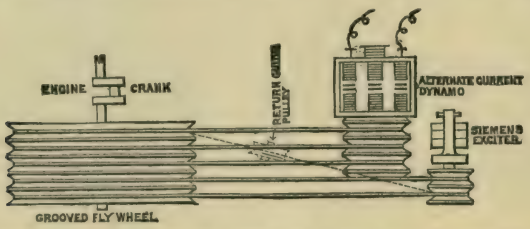
If the drive be vertical, then the rope may require a tightening pulley, in order to insure sufficient frictional resistance between it and the under side of the lower pulley, as shown by T P in the right-hand figure. A tightening pulley naturally shortens the life of the rope.

HORIZONTAL ROPE DRIVING.



VERTICAL ROPE DRIVING.

Multigroove Rope Drives.—Where great power has to be transmitted it is neither convenient nor advisable to use very thick ropes. The usual practice is to have a number of ropes running in parallel grooves on one large pulley or wheel. The grooves in each pulley must be of the same size and depth, and all the ropes of the same thickness. They should also be stretched as equally as possible between the pulleys. These conditions are necessary to prevent some of the ropes being more severely strained than others. Since ropes stretch, it is advisable to put them all on at



MULTIGROOVE ROPE DRIVE WITH ONE ROPE AND GUIDE PULLEY.

the same time when they are intended to work on the same pair of pulleys. In certain cases, such as when driving an alternator and its exciter, a single rope is used with a guide pulley and facilities for sliding the machines towards or away from the driving pulley, as shown by the figure, or the guide pulley may also be used independently for tightening the rope.

Horse-Power that Good Cotton Driving Ropes will Transmit at Various Speeds.*

(By A. G. Brown, M.E., for Musgrave & Co.)

Velocity in Feet per Minute.	Diameter of Ropes in Inches.								
	½"	⅝"	¾"	⅞"	1"	1¼"	1½"	1¾"	2"
	Horse-Power they will Transmit.								
600	·84	1·30	1·91	2·60	3 43	5·30	7·69	10·40	13·52
700	·98	1·52	2·23	3·03	4·00	6·18	8·96	12·12	15·75
800	1·12	1·73	2·54	3·45	4·56	7·05	10·22	13·82	17·96
900	1·26	1·94	2·86	3·88	5·12	7·92	11·48	15·52	20·17
1,000	1·39	2·15	3·16	4·30	5·67	8·76	12·72	17·18	22·34
1,100	1·53	2·35	3·47	4·71	6·22	9·61	13·94	18·83	24·48
1,200	1·66	2·56	3·77	5·12	6·76	10·44	15·15	20·47	26·61
1,300	1·79	2·76	4·07	5·53	7·29	11·27	16·35	22·10	28·73
1,400	1·92	2·96	4·36	5·93	7·83	12·10	17·55	23·72	30·83
1,500	2·05	3·16	4·65	6·32	8·34	12·89	18·70	25·27	32·86
1,600	2·18	3·36	4·94	6·74	8·86	13·70	19·88	26·86	34·92
1,700	2·30	3·55	5·22	7·10	9·37	14·48	21·01	28·39	36·90
1,800	2·42	3·74	5·50	7·47	9·86	15·25	22·12	29·89	38·85
1,900	2 54	3·92	5·76	7·83	10·34	15·97	23·18	31·32	40·71
2,000	2·66	4·10	6·03	8·20	10·82	16·72	24·26	32·79	42·62
2,100	2·77	4·27	6·29	8·54	11·28	17·43	25·29	34·17	44·42
2,200	2·88	4·45	6·55	8·90	11·75	18·16	26·35	35·60	46·29
2,300	2·99	4·62	6·80	9·24	12·19	18·84	27·34	36·94	48·03
2,400	3·10	4·78	7·04	9·56	12·62	19·51	28·31	38·26	49·73
2,500	3·20	4·94	7·28	9·89	13·05	20·17	29·26	39·55	51·41
2,600	3·30	5·09	7·50	10·18	13·44	20·77	30·14	40·73	52·96
2,700	3·39	5·24	7·71	10·48	13·83	21·37	31·00	41·90	54·47
2,800	3·48	5·38	7·92	10·75	14·20	21·94	31·84	43·02	55·93
2,900	3·57	5·51	8·12	11·03	14·56	22·50	32·64	44·11	57·35
3,000	3·66	5·65	8·31	11·30	14·91	23·04	33·44	45·18	58·74
3,100	3·74	5·78	8·50	11·56	15·25	23·57	34·20	46·22	60·08
3,200	3·83	5·90	8·69	11·81	15·59	24·09	34·95	47·23	61·40
3,300	3·90	6·01	8·85	12·02	15·87	24·53	35·59	48·10	62·53
3,400	3·96	6·12	9·01	12·23	16·15	24·96	36·21	48·94	63·62
3,500	4·03	6·22	9·15	12·44	16·42	25·37	36·81	49·75	64·67
3,600	4·09	6·31	9·29	12·63	16·67	25·76	37·38	50·51	65·66
3,700	4·15	6·41	9·43	12·81	16·91	26·13	37·92	51·24	66·62
3,800	4·20	6·48	9·53	12·95	17·10	26·43	38·35	51·82	67·36
3,900	4·25	6·56	9·65	13·12	17·32	26·76	38·83	52·48	68·22
4,000	4·29	6·62	9·75	13·24	17·48	27·01	39·20	52·97	68·86
4,100	4·33	6·68	9·83	13·36	17·63	27·25	39·53	53·42	69·44
4,200	4·36	6·73	9·91	13·46	17·77	27·46	39·84	53·84	69·99
4,300	4·39	6·78	9·98	13·55	17·89	27·65	40·11	54·21	70·47
4,400	4·41	6·80	10·01	13·60	17·95	27·75	40·26	54·40	70·72
4,500	4·42	6·82	10·04	13 64	18·00	27·82	40·36	54·55	70·91
4,600	4·43	6·83	10·06	13·66	18·03	27·87	40·44	54·64	71·04
4,700	4·43	6·84	10·07	13·67	18·05	27·90	40·48	54·70	71·10
4,800	4·43	6·84	10·07	13·67	18·05	27·90	40·48	54·70	71·10
4,900	4·43	6·83	10·06	13·66	18·03	27·87	40·44	54·64	71·04
5,000	4·41	6·80	10·01	13·60	17·95	27·74	40·25	54·40	70·70

* In practice, Messrs. Clark & Co. of Paisley find these powers to be rather high for the larger ropes. For example, they only allow 35 to 40 H.P. for a 1¾-inch cotton rope at

Speed of, and Horse-Power Transmitted by, Ropes.—The speed of ropes is generally very high, being from 3,000 to 5,000 feet per minute. The usual or average speed may be stated to be about 4,500 feet per minute, although some engineers have used as high as 5,600 with advantage. The preceding table shows the power which good cotton ropes will transmit at various speeds.

Power Absorbed by Rope Driving.—It is stated by some engineers that rope gearing absorbs 10 per cent. less power than toothed gearing. I am assured, however, that this is an error, for well designed and well applied tooth gearing consumes little more than 4 per cent., belts from 5 to 5½ per cent., and ropes about 7 per cent.

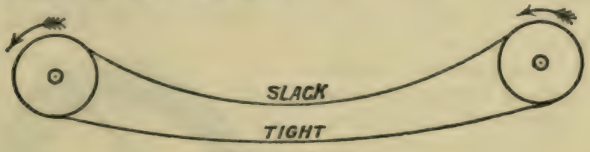
Telodynamic Transmission.*—The successful transmission of power by round endless wire ropes commences where a belt, or cotton and hemp ropes, would be too long to be used profitably (*i.e.*, say about 50 feet between the driver and follower for belting and 100 feet for ordinary ropes), and ends economically at distances of from 10,000 to 13,000 feet. For, the efficiency rapidly decreases as the distance increases, being about 95 per cent. for 100 yards, 90 per cent. for 500 yards, and only 60 per cent. for 5,000 yards under the most favourable conditions.

This system has been much more extensively employed on the Continent than in this country, although the author has seen numerous instances of its adoption in Scotland, and in Orkney, for driving ordinary thrashing mills where the water power was down in a hollow and removed from the steadings about 200 to 400 yards. Messrs. Rochling and Trenton, N.J., state, that in point of economy, this system costs only about $\frac{1}{15}$ of an equivalent amount of belting and $\frac{1}{25}$ of shafting. This is not to be wondered at, since steel wire ropes are cheap and strong, and can be run at very high speeds so that great power may be transmitted by them with comparatively light gearing. The range in the size of the cables used is, however, small, for the employment of a large wire rope means self destruction and loss of power due to its bending and unbending over the pulleys; and further loss of power due to moving it at the required velocity over great distances. For example, a rope of $\frac{3}{8}$ inch diameter will transmit 20 H.P. or less, and a 1-inch rope 300 H.P., whereas a 1¼-inch one would not

* See Chapter VIII. of the fourth edition of a Treatise *On the Use of Belting*, by John H. Cooper (Edward Meeks, Walnut Street, Philadelphia, or E. & F. Spon, London, 1891); *Elements of Machine Design*, by Prof. W. C. Unwin (Longmans, Green & Co., London); also the *Howard Lectures*, by Prof. Unwin, *Society of Arts Journal*, 1893, where an interesting account is given of the rise and progress of Telodynamic Transmission as well as details of the latest practice.

successfully transmit any more power, with the same speed and size of pulleys and the same extreme distance, owing to its greater stiffness and weight.

The first of the following figures shows a single span with the slack part of the rope uppermost :—

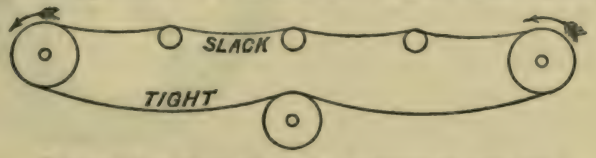


SINGLE SPAN.

The second also represents a single span but with the slack part below and supported in the middle by a guide pulley.

SPAN WITH SINGLE SUPPORTING PULLEY.

When the length of a span is great, and the height of the pulleys not sufficient to prevent the rope trailing on the ground, it may be supported in the manner shown in the third figure.



SINGLE SPAN WITH MULTIPLE SUPPORTING PULLEYS.

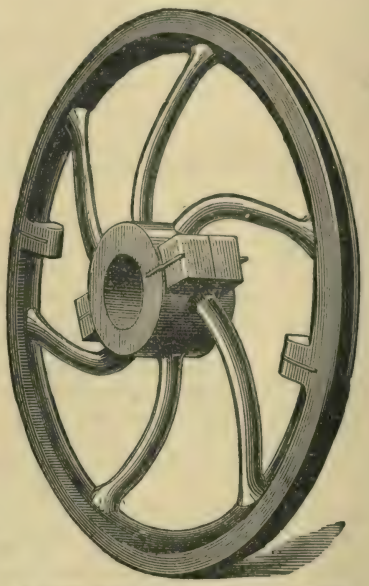
When the power has to be conveyed over a very great distance, it is advisable to split up the length into intermediate stations or "relays," each relay being worked by a separate rope, as shown by the following figure. The pulleys at the relays are double grooved,

INDEPENDENT SPANS.

so that the two ropes run together on the same pulley. The length of a relay may be from 400 to 500 feet, with guide pulleys

every 50 or 60 feet. A good example of this is to be found at Schaffhausen on the Rhine, where about 650 horse-power are transmitted through a distance of 2,700 feet to twenty-three customers, where a small rope of 1-inch diameter, moving at 62 feet per second, transmits 280 horse-power.

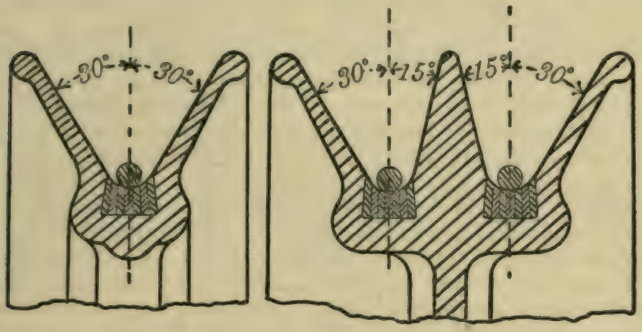
Pulleys.—Since bending is more injurious to wire ropes than those made of hemp or cotton, it is necessary to use very large pulleys with the former. This not only diminishes the damage done to the cable, but also the power expended in bending and unbending the rope. Further, the arcs of contact should be as large as possible, in order to secure sufficient frictional resistance between the rope and the pulleys. Wire ropes, unlike hemp or cotton ones, must not be pressed laterally against the sides of the V-shaped grooves, but allowed to rest on the bottom. The grooves for wire ropes are much wider than those for hemp or cotton ropes, and it is necessary to line the bottom of the grooves with some material softer than iron, such as wood, guttapercha, old rope, tarred oakum, or leather, so as to protect the rope, and increase its resistance to slipping. The last material is most extensively used for this purpose. The leather is cut into sections of the dove-tailed shape shown shaded dark in the figures on the next page, and set in on end around the rim. Scrap leather, cut from old shoes or pieces of belting, does very well, but, being very thin, it takes at least a thousand of them for a 7-foot wheel. When many are wanted, it is worth while to make a die to cut them out accurately and quickly. This is the most durable filling that can be made, but it is reported that even leather does not last more than six months' continuous work.* The guide or supporting pulleys do not require to be lined in this way.



LARGE PULLEY FOR WIRE ROPES.

* It occurs to the author that compressed brown paper might make a good lining for the bottoms of the grooves of these wire-rope pulleys.

The ropes used for telodynamic transmission are generally made of steel wires. There are usually six strands in each rope. Each strand is composed of six wires twisted around a hempen core, and these six strands are then laid around a central hempen core. As with hemp or cotton ropes, the strands are twisted in the opposite direction to the wires composing them. Finally,



SINGLE GROOVE. DOUBLE GROOVE FOR RELAYS.

the rope is protected from oxidation by a coating of boiled linseed oil. Considerable trouble is caused by the stretching of these cables, but this may, to a large extent, be prevented by passing them (before use) between grooved compression rollers, which kills this tendency to stretch, although, at the same time, it of necessity slightly diminishes their diameter.

The splicing of these ropes must be done by a practised hand, in order that the splices may not be distinguishable in size, strength, and appearance from the factory made cable. The splices should be of such a length—say 20 to 30 feet for 1-inch ropes—that the friction between the interlaid wires may easily withstand the tension.

These wire ropes, when at work, are subjected to three different stresses—(1) the longitudinal tension due to the power transmitted and their own weight, (2) the bending stress when passing over the pulleys, and (3) the centrifugal stress. As a rule, the longitudinal tension on the tight side is made twice that on the slack side, and the diameter of the pulleys is so chosen that the bending stress is about equal to the maximum longitudinal stress. The centrifugal stress is usually neglected, unless the velocity of the ropes is exceptionally great. The working tension is seldom greater than 15,000 lbs. per square inch of section, although the steel wires composing the same withstand from 70 to 100 tons per

square inch.* The life of these ropes does not appear to be much more than one year when working continuously; consequently, it is found advisable to keep a spare rope spliced and ready for action in case of accident.

Wire Rope Haulage and Transport.— Although this large subject naturally follows what has been said on telodynamic transmission, there is neither space nor time for a full treatment of the question in this book. Consequently, we must refer students who may desire to pursue this subject to those books and papers wherein the different systems are described and discussed.† The various Rope Tramways to be met with in San Francisco, Chicago, New York, Melbourne, London, and Edinburgh are excellent examples of fast speed rope haulage. These instances are, however, excelled as far as speed and distance are concerned by the District Subway of Glasgow. Here, there are two parallel circular iron tunnels, each 6½ miles long, with thirteen stations. Through these run seven trains of two cars each, at a speed of between 18 and 20 miles per hour. These trains are worked by an endless steel wire rope kept in continuous motion by stationary engines coupled to grooved drums and the necessary accessories. When about to leave a station, all that the train driver has to do is to bring the cable-grip into action with the moving rope, and when arriving at one he has simply to disengage it and apply the brake, as may be seen from the accompanying figures.‡

* See Paper on "Wire Ropes," by A. S. Biggart. *Proceedings of the Institute of Civil Engineers*, vol. ci., p. 231, 1889-90.

† See Paper on "The Monte Penna Wire Ropeway," by W. P. Churchward. *Proc. Inst. Civil Engineers*, vol. lxiii., p. 273.

Paper on "Three Systems of Wire Rope Transport," by W. T. H. Carrington. *Proc. Inst. Civil Engineers*, vol. lxv., p. 299.

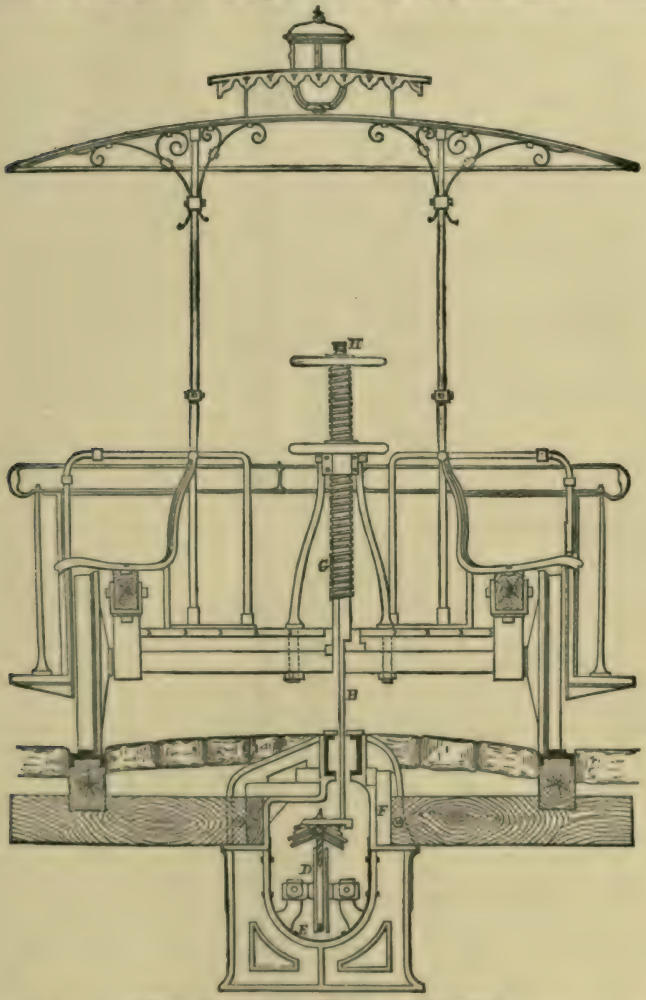
Paper on "Wire Rope Street Railways of San Francisco and Chicago," by W. Morris. *Proc. Inst. Civil Engineers*, vol. lxxii., p. 210.

Paper on "The Temple Street Cable Railway, Los Angelos, California," by F. W. Wood and H. Hawgood. *Proc. Inst. Civil Engineers*, vol. cvii., p. 323.

Street Railways: Their Construction, Operation, and Maintenance, by C. B. Fairchild (The Street Railway Publishing Co., World Buildings, New York).

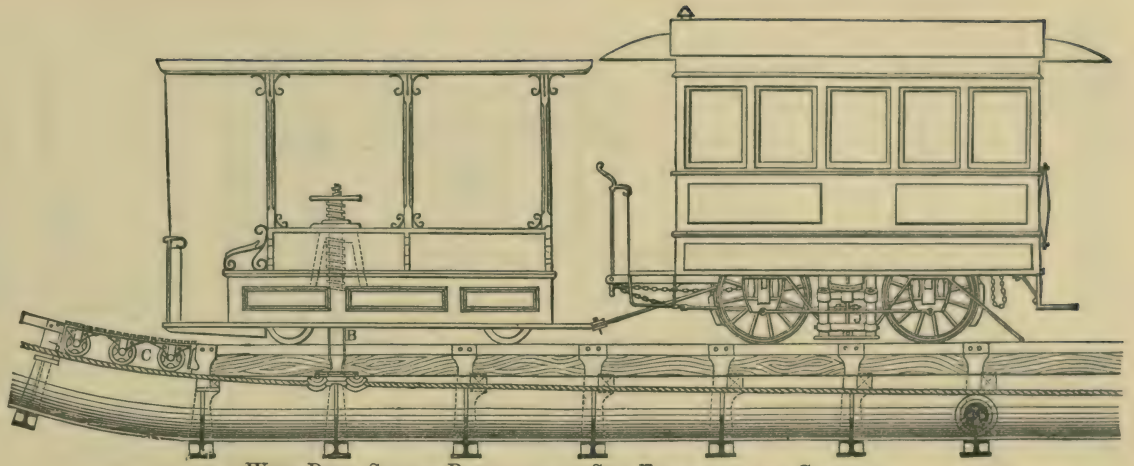
‡ These two figures, and the one of a "Multigroove Rope Drive" for one rope and guide pulley, were kindly provided by the Institution of Civil Engineers with the approval of the Council.

The gripper attachment is fixed to the end of the shank, B, which is a flat bar of iron 5½ inches wide by ⅝ inch thick, working



Cross Section of Car and Gripper.
Wire-Rope Street Railroads in San Francisco and Chicago.

through a longitudinal slot. This gripper, A, consists of two pairs of small sheaves about 3½ inches in diameter, and placed at



WIRE-ROPE STREET RAILROADS IN SAN FRANCISCO AND CHICAGO.

an angle, each pair being about 10 inches apart and secured to sliding frames, in which there are also placed jaws, for the purpose of taking a firm hold of the wire rope. The small sheaves act as guides, in the first instance, for the rope to pass between the jaws. When starting, these sheaves are gradually tightened on the rope by the screw, G, and this allows the front, or hauling, car to gradually acquire the speed of the rope. The gripping jaws are then brought into action by means of the small screw, H, and take a firm hold of the rope.

The second car, besides the usual brakes to the wheels, has between each pair of wheels a brake, J, which presses vertically on the rails. It is 28 inches long, and is strong enough to lift the car off the track when empty.

Lecture XV.—Questions.

1. Under what respective circumstances would you use belt, rope, and chain gearing, and state the advantages and disadvantages of each?

2. Explain how the best leather belts are made and spliced.

3. What is meant by "compound" belting? For what reasons is it preferred to "double" or "treble" belting?

4. Give a short description of "chain," "Victoria," and "composite guttapercha" belting. State their respective advantages and disadvantages.

5. The tension on the slack side of a belt is half that on the tight side. The limiting tension is 40 lbs. for each inch in width of the belt. Find the breadth of the belt to transmit 40 horse-power from a pulley 3 feet in diameter making 100 revolutions per minute. *Ans.* 70 inches.

6. A belt transmits 35 horse-power when moving at 3,300 feet per minute. Find the net driving tension. If the coefficient of friction be ·3, and the belt embraces half the circumference of the pulley, find the tensions on the driving and slack sides respectively. What width of belt would you use? *Ans.* 350 lbs.; 575 lbs.; 225 lbs.; 11·5 inches.

7. Explain the construction of cotton and hemp ropes for driving machinery, and give their respective strengths and advantages.

8. Sketch a section of the rim of a rope pulley for a 1-inch rope, marking all the dimensions.

9. What is "Telodynamic Transmission of Power?" How is it applied for short and long distances? Give sections and description of the ropes, and of the single and double grooved pulleys used in this system.

10. Describe the machinery employed in the manufacture of wire ropes, and give a detailed account of the process of constructing a wire rope. With the aid of sketches, describe how the ends of such a rope are secured so that its full strength may be utilised (Sc. & A. Hons. Mach. Cons. Exam., 1895). (See Mr. Smith's paper in the *Proc. Mech. Eng.* 1862, and Unwin's *Machine Design*, part I.)

11. Give a sketch and short description of tramway rope haulage.

12. Answer only one of the following, A, B, C, or D:—A. A bevil-wheel, revolving at 93 revolutions per minute, 40 cogs, 2-inch pitch, 5-inch face, drives a mortise wheel at 120 revolutions; sketch the wheels in section, roughly correct as to the scales of the various parts. B. Describe how the above gearing may be replaced by rope gearing. C. Describe an escapement of a clock or watch. State exactly how it gives the impulse to the pendulum or balance. D. Describe four well-known mechanisms based on the slider-crank-chain. (S. & A. Adv. Exam., 1897.)

Ans. $19\frac{8}{11}$ inches and $25\frac{8}{11}$ inches.

13. Make a section of a flywheel rim suitable for rope driving. If 22 ropes are used to transmit 450 H.P., the diameter of the flywheel being 23 feet, and the revolutions of the crank shaft 73 per minute, find the effective pull in each rope. (C. & G., 1900, O., Sec. A.)

14. The difference of tensions on the tight and slack side of a belt which drives a 30-inch pulley being 750 lbs., what H.P. is being transmitted to the shaft on which this pulley runs, if it makes 120 revolutions a minute?

(C. & G. 1901, O., Sec. A.)

15. The lap of a belt on a pulley is θ radians; the coefficient of friction is μ; the belt has to transmit the horse-power H.P.; how do we find the section of belt necessary? Prove your rule. (B. of E., S. 3, 1908.)

LECTURE XV.—A.M.INST.C.E. EXAM. QUESTIONS.

1. A main shaft carries a pulley 4 feet in diameter, and drives on to a 2-foot pulley on a countershaft by means of a leather belt 6 inches wide. Assuming the belt to work at a maximum tension of 75 lbs. per inch of width, calculate the horse-power which can be transmitted when the main shaft makes 200 revolutions per minute. Assume the ratio of the maximum to the minimum tension to be 1·8. (I.C.E., *Oct.*, 1906.)

2. Give a sketch of what you would consider a suitable groove for a hemp rope $1\frac{1}{2}$ inches in diameter. Show that, if μ is the coefficient of friction for a rope on a plain pulley, and α is the angle of inclination of the sides of the groove, the resistance to slipping in the groove is that appropriate to a plain pulley for which the coefficient of friction is μ cosec $\frac{\alpha}{2}$. A rope pulley carrying 20 ropes is 8 feet in diameter and has 0·5 of its circumference embraced by each rope; it is revolving at 150 revolutions per minute. Find the horse-power transmitted under the following conditions, disregarding the effect of centrifugal force.

$$\text{Coefficient of friction } \mu = 0{\cdot}28\,;\ \text{cosec}\,\frac{\alpha}{2} = 2{\cdot}6.$$

$$\text{Common log}\,\frac{\text{maximum tension}}{\text{tension on slack side}} = 2{\cdot}7\,\mu\,\text{cosec}\,\frac{\alpha}{2} \times \left\{\begin{array}{l}\text{fraction of circumfer-}\\ \text{ence embraced.}\end{array}\right.$$

Maximum tension = 200 lbs. per rope. (I.C.E., *Oct.*, 1907.)

3. A belt is strained over a pulley, and is on the point of slipping. The angle embraced by the belt is $\alpha°$, The coefficient of friction is μ, and the tensions in the belt are T_1 and T_2, the latter being the greater. Find an expression connecting these four quantities. (I.C.E., *Feb.*, 1908.)

4. Describe some form of chain gearing with which you are acquainted, suitable for transmitting power to the driving wheels of a motor vehicle, giving full details. (I.C.E., *Feb.*, 1908.)

NOTE.—See Prof. Unwin's *Machine Design*, vol. i., latest Edition. Also, Prof. Cotterill's *Applied Mechanics.*

LECTURE XVI.

CONTENTS.—Velocity-Ratio with Belt and Rope Transmission—Example I—Velocity-Ratio in a Compound System of Belt Gearing—Example II.—Length of a Crossed Belt—Examples III. and IV.—Length of an Open Belt—Examples V. and VI.—Frictional Resistance between a Belt or Rope and its Pulley—Frictional Resistance between a Rope and a Grooved Pulley—"Slip" or "Creep" of Belts due to Elasticity—Horse-power Transmitted by Belt and Rope Gearing—Examples VII. and VIII.—Influence of Centrifugal Tension on the Strength of High-Speed Belts and Ropes—Example IX.—Questions.

Velocity-Ratio with Belt and Rope Transmission.—It is shown in our elementary treatise that when two pulleys of diameters, D_1, D_2, are connected by a belt or rope, their angular velocity-ratios are inversely as their diameters—*i.e.*, if N_1 and N_2 be their respective number of revolutions in a given time, then :—

$$N_1 : N_2 = D_2 : D_1 . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (I)$$

This equation is only true on the supposition that there is no slipping between the belt or rope and the pulleys ; and also, that the thickness of the belt or rope is so small in comparison with the diameters of the pulleys, that it may be neglected.

Let δ = Diameter of the rope or thickness of the belt.

Then, the working diameter of the pulleys will be $D_1 + \delta$, and $D_2 + \delta$, respectively. Consequently :—

$$N_1 : N_2 = D_2 + \delta : D_1 + \delta . \quad . \quad . \quad . \quad . \quad . \quad (II)$$

EXAMPLE I.—Compare the angular velocities of two pulleys of diameters 24 and 10 inches respectively when the thickness of the belt is $\frac{3}{8}$ inch.

(1) Neglecting the thickness of the belt, we get :—

$$\frac{N_1}{N_2} = \frac{D_2}{D_1} = \frac{24}{10} = \frac{2{\cdot}4}{1}.$$

(2) Taking the thickness of the belt into account, we get :—

$$\frac{N_1}{N_2} = \frac{D_2 + \delta}{D_1 + \delta} = \frac{24 + \frac{3}{8}}{10 + \frac{3}{8}} = \frac{195}{83} = \frac{2{\cdot}35}{1}.$$

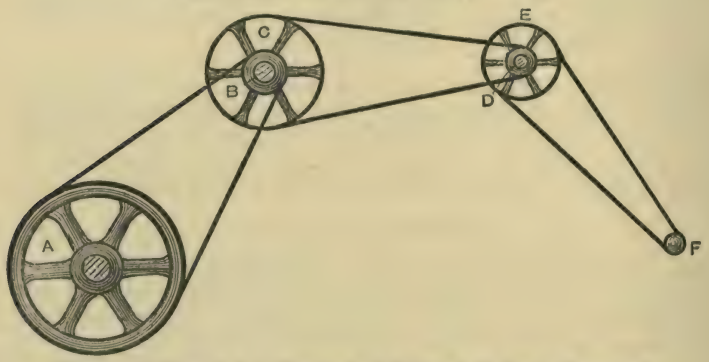
We thus see that by taking the thickness of the belt into account the velocity-ratio is diminished, and in this example in the proportion of 240 to 235, or by 2·1 per cent.

Velocity-Ratio in a Compound System of Belt Gearing.—When there are a number of pulleys, A, B, C, D, E, F, connected together by belting, as illustrated by the above figure, let D_A, D_B, &c., denote their respective diameters, and N_A, N_B, &c., their revolutions per minute.

Then, $$\frac{N_B}{N_A} = \frac{D_A}{D_B}, \text{ and } \frac{N_D}{N_C} = \frac{D_C}{D_D}, \text{ also } \frac{N_F}{N_E} = \frac{D_E}{D_F}.$$

By multiplying together the corresponding members of these equations, we get:—

$$\frac{N_B}{N_A} \times \frac{N_D}{N_C} \times \frac{N_F}{N_E} = \frac{D_A}{D_B} \times \frac{D_C}{D_D} \times \frac{D_E}{D_F}.$$



Velocity-Ratio in a Compound System of Belt Gearing.

Since the pairs of pulleys, B and C, and D and E, are fixed to their respective shafts, $N_B = N_C$ and $N_D = N_E$. By cancelling these equal values, we get:—

$$\frac{N_F}{N_A} = \frac{D_A \times D_C \times D_E}{D_B \times D_D \times D_F} \quad . \quad . \quad . \quad . \quad . \quad \text{(III)}$$

If we call pulleys A, C, and E the *drivers* and B, D, and F the *followers*, we obtain the following rule :—

$$\frac{\text{Revolutions of last follower}}{\text{Revolutions of first driver}} = \frac{\text{Product of diameters of all the drivers.}}{\text{Product of diameters of all the followers.}}$$

Or, *Speed of first driver* × *product of the diameters of all the drivers.*

= *Speed of last follower* × *product of the diameters of all the followers.*

The radii or circumferences of the pulleys may be substituted for the diameters.

If the thickness of the belts be taken into account, then the pulley diameters must be increased by the thickness of the belts.

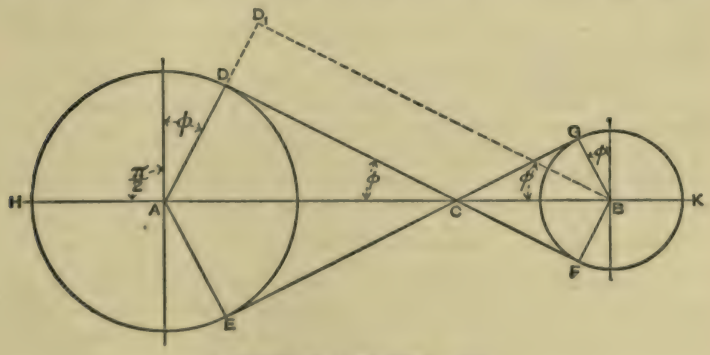
EXAMPLE II.—An engine is employed to drive a fan by means of pulleys and bands, the first driving band passing over the flywheel of the engine. The train consists of (1) the flywheel, A; (2) two pulleys, B and C, on one axis; (3) two other pulleys, D and E, on another axis; and (4) a pulley, F, to the axis of which the fan is attached. The diameters of A, B, C, D, E, F are as 12, 3, 8, 2, 5, 1 respectively, and A makes 20 revolutions per minute. How many revolutions does F make per minute?

Here $D_A = 12$; $D_B = 3$; $D_C = 8$; $D_D = 2$; $D_E = 5$; $D_F = 1$; $N_A = 20$.

Then, $$\frac{N_F}{N_A} = \frac{D_A \times D_C \times D_E}{D_B \times D_D \times D_F} = \frac{12 \times 8 \times 5}{3 \times 2 \times 1} = 80.$$

$\therefore$ $N_F = N_A \times 80 = 20 \times 80 = 1{,}600$ revs. per min.

Length of a Crossed Belt.—An endless belt stretched over two pulleys may be either crossed or open, according as the pulleys are required to rotate in the same or in the opposite direction. We shall now prove by aid of the following figure



LENGTH OF A CROSSED BELT.

that a driving belt when crossed will serve for any pair of pulleys, so long as the distance between the centres of the pulleys is the same, and the sum of the diameters is constant.

Let R, D = Radius and diameter of pulley, A,
" r, d = Radius and diameter of pulley, B,
" a = Distance, A B,
" L = Length of belt,
" Σ = Sum of diameters = D + d,
" Δ = Difference of diameters = D − d.

Then, $L = 2\{\text{arc H D} + \text{D F} + \text{arc F K}\}$.

Or, $$L = 2\left\{R\left(\frac{\pi}{2} + \phi\right) + a\cos\phi + r\left(\frac{\pi}{2} + \phi\right)\right\}$$

$$\therefore \quad L = \left(\frac{\pi}{2} + \phi\right)(D + d) + 2a\cos\phi. \quad . \quad . \quad . \quad (IV)$$

If (D + d) has always the same value, then ϕ will also remain constant; and, therefore, L will be a constant length.

From the figure we see that:—

$$\left.\begin{aligned} \sin\phi = \frac{A\,D_1}{A\,B} = \frac{R + r}{a} = \frac{D + d}{2a} = \frac{\Sigma}{2a} \\ \text{But, } \cos\phi = \sqrt{1 - \sin^2\phi} = \frac{\sqrt{4a^2 - \Sigma^2}}{2a} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (V)$$

$$\therefore \quad L = \left(\frac{\pi}{2} + \phi\right)\Sigma + \sqrt{4a^2 - \Sigma^2}. \quad . \quad . \quad . \quad . \quad . \quad (IV_a)$$

This is one equation from which the length of a crossed belt may be calculated.

In using equation (IV_a) it must be observed, that ϕ is expressed in *circular* measure. After obtaining sin ϕ from equation (V), the angle ϕ can be found in circular measure or in degrees by referring to Trigonometrical Tables of Natural Sines. If ϕ be expressed in degrees, let it be denoted by ϕ°, then the circular measure of ϕ is found by multiplying ϕ° by 0·0175.*

Thus, $$\phi = \phi^\circ \times 0{\cdot}0175.$$

EXAMPLE III.—A shaft making 100 revolutions per minute carries a driving pulley 2½ feet in diameter and communicates motion by means of a belt to a parallel shaft at a distance of 6 feet, carrying a pulley 1 foot in diameter. Find the speed of the belt and an expression for its length when crossed. Find also the number of revolutions per minute of the driven shaft, allowing a slip of 1½ per cent. (S. & A. Adv. Exam., 1893).

* How this number is obtained is explained further on in this Lecture.

ANSWER.—(1) *Speed of belt = Circumferential speed of driving pulley,*

$$\text{,, ,, } = \pi D N,$$

$$\text{,, ,, } = 3{\cdot}1416 \times 2\tfrac{1}{2} \times 100 = 785{\cdot}4 \text{ feet per minute.}$$

(2) The length of a crossed belt has been shown in the text to be :—

$$L = \left(\frac{\pi}{2} + \phi\right)\Sigma + \sqrt{4a^2 - \Sigma^2}.$$

From the data, we get :—$\Sigma = D + d = 2{\cdot}5 + 1 = 3{\cdot}5$ feet, $a = 6$ feet, and $\sin\theta = \dfrac{\Sigma}{2a} = \dfrac{3{\cdot}5}{12} = {\cdot}2917$.

Referring to a table of Natural Sines, we find the angle whose sine is = ·2917 to be about 17°.

Hence, $\phi = 17 \times 0{\cdot}0175 = {\cdot}2975$ radians.

$$\therefore \quad L = \left(\frac{3{\cdot}1416}{2} + {\cdot}2975\right) 3{\cdot}5 + \sqrt{4 \times 6^2 - 3{\cdot}5^2}.$$

i.e., $L = 6{\cdot}54 + 11{\cdot}48 = 18{\cdot}02$ feet.*

(3) If there were no slip, the speed of the driven pulley would be :—

$$n = N \times \frac{D}{d} = 100 \times \frac{2{\cdot}5}{1} = 250 \text{ revolutions per minute.}$$

* The student should notice that $\sin\phi$ and ϕ agree to the second decimal figure, and this is true for all angles up to about 21°. The smaller the angle the more nearly do $\sin\phi$ and ϕ agree in numerical value. Now, in the examination room, no tables of Natural Sines are allowed, and the student is not expected to calculate ϕ from the value of its sine. But in engineering problems of this kind it is considered sufficiently accurate to take numbers to two or three decimal figures; hence the student may assume that—

$$\phi = \sin\phi = {\cdot}292.$$

Then, $$L = \left(\frac{3{\cdot}1416}{2} + {\cdot}292\right) 3{\cdot}5 + \sqrt{4 \times 6^2 - 3{\cdot}5^2},$$

i.e., $L = 6{\cdot}52 + 11{\cdot}48 = 18$ feet.

This only differs by about ·1 per cent. from the correct answer. The error introduced when we take $\phi = \sin\phi$ is ·2975 − ·292 = ·0055, or ·55 per cent. only.

But since there is $1\frac{1}{2}$ per cent. slip the actual speed is only $98\frac{1}{2}$ per cent. of this.

$$\therefore \quad n_1 = \frac{98{\cdot}5}{100} \times 250 = 246{\cdot}25 \text{ revolutions per minute.}$$

The length of the belt could also be obtained by drawing the figure accurately to scale. The student should, therefore, obtain an answer in this way to the second part of the above question, and compare his results with those which we have just found.

EXAMPLE IV.—A shaft, having a stepped speed-cone with four steps, revolves at a constant speed of 180 revolutions per minute, and is connected, by a crossed belt, to another shaft having a similar stepped cone. The diameter of the largest step of the cone on the driving shaft is 16 inches. The driven shaft is required to run at speeds 480, 300, 160, and 90 revolutions per minute respectively. Determine the diameters of the remaining steps of the two cones.

ANSWER.—Let D_1, D_2, D_3, D_4 denote the diameters of the four steps of the cone on the driving shaft; D_1 being the diameter of the largest step.

Let d_1, d_2, d_3, d_4 denote the diameters of the four steps of the cone on the driven shaft; d_1 being the diameter of the smallest step.

Then, $\quad D_1 + d_1 = D_2 + d_2 = D_3 + d_3 = D_4 + d_4 = \Sigma.$

Let N denote the speed of the driving shaft, and let N_1, N_2, N_3, N_4 denote the four different speeds of the driven shaft, so that

$$N = 180;\ N_1 = 480;\ N_2 = 300;\ N_3 = 160;\ N_4 = 90.$$

We have first to determine d_1, and thereby Σ.

Here, $\quad \dfrac{d_1}{D_1} = \dfrac{N}{N_1}.$

$$\therefore \quad d_1 = \frac{180}{480} \times 16 = 6 \text{ inches.}$$

$$\therefore \quad \Sigma = 16 + 6 = 22 \text{ inches.}$$

For steps D_2, d_2, we get :—

$$\frac{D_2}{d_2} = \frac{N_2}{N}; \quad \therefore \frac{D_2}{D_2 + d_2} = \frac{N_2}{N_2 + N}.$$

Hence, $\quad D_2 = \dfrac{300}{300 + 180} \times 22 = 13{\cdot}75$ inches.

And, $\quad d_2 = 22 - 13{\cdot}75 = 8{\cdot}25$ inches.

Similarly, $D_3 = \frac{N_3}{N_3 + N} \Sigma = \frac{160}{160 + 180} \times 22 = 10{\cdot}35$ ins.

And, $d_3 = 22 - 10{\cdot}35 = 11{\cdot}65$ inches.

Again, $D_4 = \frac{N_4}{N_4 + N} \Sigma = \frac{90}{90 + 180} \times 22 = 7{\cdot}\dot{3}$ inches.

And, $d_4 = 22 - 7{\cdot}\dot{3} = 14{\cdot}\dot{6}$ inches.

Hence, we have:—

Driving Cone.	Driven Cone.
$D_1 = 16$ inches.	$d_1 = 6$ inches.
$D_2 = 13{\cdot}75$ „	$d_2 = 8{\cdot}25$ „
$D_3 = 10{\cdot}35$ „	$d_3 = 11{\cdot}65$ „
$D_4 = 7{\cdot}\dot{3}$ „	$d_4 = 14{\cdot}\dot{6}$ „

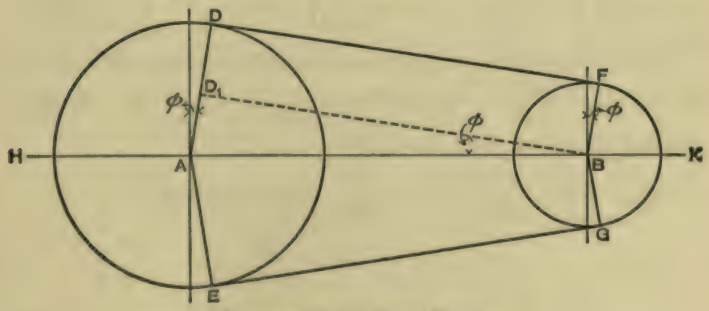
Length of an Open Belt.—By referring to the next figure and proceeding as before, we can determine the length of an open belt connecting any pair of pulleys.

Here, $L = 2\{\text{arc HD} + \text{DF} + \text{arc FK}\}$.

Or, $L = 2\left\{R\left(\frac{\pi}{2} + \varphi\right) + a\cos\varphi + r\left(\frac{\pi}{2} - \varphi\right)\right\}$.

Or, $L = \frac{\pi}{2}(D + d) + \varphi(D - d) + 2a\cos\varphi$.

$\therefore$ $L = \frac{\pi}{2}\Sigma + \varphi\Delta + 2a\cos\varphi$. . . . . (VI)



Length of an Open Belt.

Here Σ and Δ cannot both be constant quantities for two or more pairs of pulleys. Hence, if the sum of the diameters of each pair of pulleys be the same, an *open* belt of constant length

cannot be made to work equally tight on the various pairs of pulleys.

We shall now obtain an approximate and more convenient formula for the length of an open belt than that given by equation (VI).

Since, in this case, φ is always very small, we may write :—

$$\varphi = \sin \varphi.$$

$$\left.\begin{aligned} \text{But,} \quad \sin \varphi &= \frac{A D_1}{A B} = \frac{\Delta}{2a}. \\ \therefore \quad \varphi &= \frac{\Delta}{2a}, \text{ approximately.} \end{aligned}\right\} \quad \ldots\ldots \text{(VII)}$$

$$\therefore \quad \varphi \Delta = \frac{\Delta^2}{2a}, \qquad \text{,,}$$

$$\text{Also,} \quad \cos \varphi = \sqrt{1 - \sin^2 \varphi} = \frac{\sqrt{4a^2 - \Delta^2}}{2a}$$

Substituting these values of $\varphi \Delta$ and $\cos \varphi$ in equation (VI),

$$\text{We get,} \quad L = \frac{\pi}{2}\Sigma + \frac{\Delta^2}{2a} + \sqrt{4a^2 - \Delta^2}.$$

$$\text{Or,} \quad L = \frac{\pi}{2}\Sigma + 2a\left\{\frac{\Delta^2}{4a^2} + \left(1 - \frac{\Delta^2}{4a^2}\right)^{\frac{1}{2}}\right\}$$

By the binomial theorem, we can expand the last expression in this equation to any required number of terms.

$$\text{Thus,} \quad \left(1 - \frac{\Delta^2}{4a^2}\right)^{\frac{1}{2}} = 1 - \tfrac{1}{2}\left(\frac{\Delta^2}{4a^2}\right) + \frac{\frac{1}{2}(\frac{1}{2} - 1)}{1 \cdot 2} \cdot \left(\frac{\Delta^2}{4a^2}\right)^2, \text{ \&c.}$$

The third and following terms on the right-hand side are very small, since $\frac{\Delta^2}{4a^2}$ is of itself very small. We may, therefore, neglect all terms after the second.

$$\text{Thus,} \quad \left(1 - \frac{\Delta^2}{4a^2}\right)^{\frac{1}{2}} = 1 - \frac{\Delta^2}{8a^2}, \text{ approximately.}$$

$$\text{Hence,} \quad L = \frac{\pi}{2}\Sigma + 2a\left\{\frac{\Delta^2}{4a^2} + 1 - \frac{\Delta^2}{8a^2}\right\}.$$

$$\textit{i.e.,} \quad L = \frac{\pi}{2}\Sigma + 2a\left\{1 + \frac{\Delta^2}{8a^2}\right\}. \quad \ldots \quad (\text{VI}_a)$$

This is evidently a more convenient expression for the length of an open belt than equation (VI).

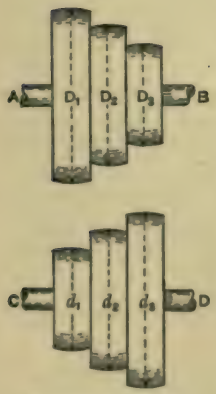
In the case of a crossed belt it has been shown that the belt will work equally tight and well on any pair of pulleys on the same shafts so long as the sum of the diameters of the two pulleys is constant. Thus, if a crossed belt be used for connecting two stepped speed cones, the sum of the diameters of alternate pulleys must be constant. If, however, an open belt be used the sum of the diameters of each pair of working pulleys will not be constant. Consider two pairs of pulleys on the stepped cones.

Let D_1, d_1 = Diameters of first pair.
„ D_2, d_2 = Diameters of second pair.
„ $\Sigma_1 = D_1 + d_1$; $\Sigma_2 = D_2 + d_2$,
„ $\Delta_1 = D_1 - d_1$; $\Delta_2 = D_2 - d_2$.

Then, since the length of the belt is constant, we get :—

$$\frac{\pi}{2}\Sigma_2 + 2a\left\{1 + \frac{\Delta_2^2}{8a^2}\right\} = \frac{\pi}{2}\Sigma_1 + 2a\left\{1 + \frac{\Delta_1^2}{8a^2}\right\}.$$

$$\therefore \quad \Sigma_2 = \Sigma_1 + \frac{\Delta_1^2 - \Delta_2^2}{2\pi a} \quad . \quad . \quad . \text{(VIII)}$$



STEPPED CONES.

This formula gives us the sum of the diameters of the second pair of pulleys in terms of the sum and difference of the diameters of the first pair. The difference, Δ_2, of the diameters of the second pair of pulleys also enters into this equation, and this must be found before Σ_2 can be obtained.

Let A B be the main shaft running constantly at N revolutions per minute. Let N_1, N_2 be the speeds of the shaft C D when the belt is on the pulleys 1 and 2 respectively.

Now, calculate Δ_2 on the assumption that the belt is crossed. This is not exactly, but very approximately, correct. Hence, neglecting the thickness of the belt, we get :—

$$\frac{D_2}{d_2} = \frac{N_2}{N}; \qquad \therefore \frac{D_2 - d_2}{D_2 + d_2} = \frac{N_2 - N}{N_2 + N}.$$

i.e., $$\frac{\Delta_2}{\Sigma_2} = \frac{N_2 - N}{N_2 + N}; \quad \therefore \Delta_2 = \frac{N_2 - N}{N_2 + N}\Sigma_2. \quad . \quad . \quad \text{(IX)}$$

But, since we are to *assume* that the belt is crossed in this calculation, we must have $\Sigma_2 = \Sigma_1$.

$$\therefore \quad \Delta_2 = \frac{N_2 - N}{N_2 + N} \Sigma_1, \text{ approximately.} \quad . \quad . \quad . \quad . \quad (IX_a)$$

Substituting this approximate value for Δ_2 in equation (VIII), we get Σ_2. Having now calculated Σ_2, we can easily find D_2 and d_2. Thus:—

$$\frac{D_2}{d_2} = \frac{N_2}{N}; \quad \therefore \frac{D_2}{D_2 + d_2} = \frac{N_2}{N_2 + N},$$

$$\therefore \quad D_2 = \frac{N_2}{N_2 + N} \Sigma_2, \text{ and } d_2 = \frac{N}{N_2 + N} \Sigma_2. \quad . \quad . \quad (X)$$

In the very same way, the diameters of all the other pairs of pulleys in the stepped cone can be found.

Hence, the following practical rule for designing a set of stepped speed-cones, worked by an open belt:—

Let N = Constant speed of driving or main shaft, A B.

N_1, N_2, N_3, &c., = Required speeds of driven shaft, C D.

(*a*) Fix on convenient diameters D_1, d_1 to give the required velocity-ratio with any one pair of pulleys. This will give Σ_1 and Δ_1.

(*b*) Next calculate Δ_2, Δ_3, &c., for the other pulleys *on the assumption that the belt is crossed;* or by formula (IX_a).

(*c*) Insert these values successively in equation (VIII), from which Σ_2, Σ_3, &c., can be found.

(*d*) The diameters can then be found from equations (X).

EXAMPLE V.—The centres of two pulleys, 4 and 2 feet in diameter respectively, are 8 feet apart. The pulleys are connected by an open belt; find its length.

ANSWER.—Here $\Sigma = 6$ feet; $\Delta = 2$ feet; $a = 8$ feet.

From equation (VI_a) we get:—

$$L = \frac{\pi}{2} \Sigma + 2a \left\{ 1 + \frac{\Delta^2}{8a^2} \right\}$$

$$\text{,,} = \frac{3{\cdot}1416}{2} \times 6 + 2 \times 8 \left\{ 1 + \frac{4}{8 \times 64} \right\} \text{ feet.}$$

$$\text{,,} = 9{\cdot}4248 + 16{\cdot}125 = 25{\cdot}55 \text{ feet.}$$

EXAMPLE VI.—If the speed cones in Example IV. are connected by means of an open belt instead of a crossed one, and

the distance between the centres of the two shafts is 8 feet, determine the diameters of the various steps.

ANSWER.—Using the same notation as in Example IV., we get, as before, $D_1 = 16$ inches; $d_1 = 6$ inches; $a = 8 \times 12 = 96$ inches; $\Sigma_1 = 22$ inches; and $\Delta_1 = 10$ inches.

(1) *To find* D_2, *and* d_2.

First, calculate D_2 and d_2 on the assumption that the belt is crossed.

From Example IV. we get :—

$$D_2 = 13{\cdot}75 \text{ inches, and } d_2 = 8{\cdot}25 \text{ inches.}$$

Hence, as a first approximation, we may write :—

$$\Sigma_2 = 22 \text{ inches, and } \Delta_2 = 5{\cdot}5 \text{ inches.}$$

Next, recalculate Σ_2 from equation (VIII) in the text, viz.:—

$$\Sigma_2 = \Sigma_1 + \frac{\Delta_1^2 - \Delta_2^2}{2\pi a} = 22 + \frac{10^2 - 5{\cdot}5^2}{2 \times \frac{22}{7} \times 96} = 22{\cdot}1155 \text{ inches.}$$

Substituting this new value of Σ_2 in equation (X), we get :—

$$D_2 = \frac{N_2}{N_2 + N}\Sigma_2 = \frac{300}{300 + 180} \times 22{\cdot}1155 = 13{\cdot}822 \text{ inches.}$$

$$d_2 = 22{\cdot}1155 - 13{\cdot}822 = 8{\cdot}293 \text{ inches.}$$

These are the diameters of the second step as obtained for *one correction*. If a closer approximation be required, then we must again recalculate Σ_2 from the results just obtained. Thus :—

$$\Delta_2 = 13{\cdot}822 - 8{\cdot}293 = 5{\cdot}529 \text{ inches.}$$

$$\therefore \Sigma_2 = \Sigma_1 + \frac{\Delta_1^2 - \Delta_2^2}{2\pi a} = 22 + \frac{10^2 - 5{\cdot}529^2}{2 \times \frac{22}{7} \times 96} = 22{\cdot}1148 \text{ inches.}$$

Substituting this new value for Σ_2 in equation (X), we get:—

$$D_2 = \frac{300}{300 + 180} \times 22{\cdot}1148 = 13{\cdot}8217 \text{ inches.}$$

$$\therefore d_2 = 22{\cdot}1148 - 13{\cdot}8217 = 8{\cdot}2931 \text{ inches.}$$

which are the values of D_2 and d_2 *corrected twice.*

It will be at once seen that these last values differ very slightly from those obtained by one correction. Hence, it will be sufficiently accurate to make one correction only.

(2) *To find* D_3 *and* d_3.

As before, calculate D_3 and d_3 on the assumption that the belt is crossed. From Example IV., we get :—

$$D_3 = 10{\cdot}35 \text{ inches, and } d_3 = 11{\cdot}65 \text{ inches.}$$

$$\therefore \quad \Sigma_3 = 22 \text{ inches, and } \Delta_3 = 1{\cdot}3 \text{ inches.}$$

Next recalculate Σ_3 from equation (VIII).

$$\therefore \quad \Sigma_3 = \Sigma_1 + \frac{\Delta_1^2 - \Delta_3^2}{2\pi a} = 22 + \frac{10^2 - 1{\cdot}3^2}{2 \times \frac{22}{7} \times 96} = 22{\cdot}163 \text{ inches.}$$

Substituting this value in equation (X), we get :—

$$D_3 = \frac{N_3}{N_3 + N} \Sigma_3 = \frac{160}{160 + 180} \times 22{\cdot}163 = 10{\cdot}43 \text{ inches.}$$

$$\therefore d_3 = 22{\cdot}163 - 10{\cdot}43 = 11{\cdot}733 \text{ inches.}$$

(3) In a similar way, we get :—

$$D_4 = 7{\cdot}359 \text{ inches.}$$

And, $$d_4 = 14{\cdot}718 \text{ inches.}$$

Hence, for an open belt we get the following sizes for the steps of the cones :—

Driving Cone.	Driven Cone.
$D_1 = 16$ inches.	$d_1 = 6$ inches.
$D_2 = 13{\cdot}822$ „	$d_2 = 8{\cdot}293$ „
$D_3 = 10{\cdot}43$ „	$d_3 = 11{\cdot}733$ „
$D_4 = 7{\cdot}359$ „	$d_4 = 14{\cdot}718$ „

Frictional Resistance between a Belt or Rope and its Pulley.—In Lect. VII., Vol. I., we deduced a formula for the ratio of the tensions of the two parts of a belt or rope when stretched round a pulley or post, viz. :—

$$\log_e \frac{T_d}{T_s} = \mu \theta.$$

Where T_d = Tension on driving or tight side of belt or rope.

„ T_s = Tension on driven or slack side of belt or rope.

„ μ = Coefficient of friction between the belt or rope and the pulley.

„ θ = Circular measure of angle subtended at the centre of the pulley by the belt or rope.

The following values of μ have been obtained by experiment:—

For leather belts on iron pulleys, $\mu = 0{\cdot}3$ to $0{\cdot}4$.

For hemp or cotton ropes on iron pulleys, $\mu = 0{\cdot}35$ (*pulleys dry*). $\mu = 0{\cdot}15$ (*pulleys greased*).

For wire ropes on iron pulleys, $\mu = 0{\cdot}15$.

For wire ropes on iron pulleys, the grooves being lined with leather or guttapercha, $\mu = 0{\cdot}25$.

Slipping of a belt generally occurs at the smaller pulley, the angle of contact, θ, being smaller on this pulley than on the larger one. Hence, in calculations relating to the slipping of a belt it is usual to consider the small pulley only. The average length of arc embraced by the belt on the small pulley may be taken at $\frac{4}{10}$ of the whole circumference—*i.e.*, $\theta = {\cdot}4 \times 2\pi = 2{\cdot}5133$ radians. If, then, we take $\mu = 0{\cdot}3$, we can easily find the ratio between T_d and T_s in the case of a leather belt.

Thus, $$\log_\epsilon \frac{T_d}{T_s} = 0{\cdot}3 \times 2{\cdot}5133.$$

Or, changing this into common logarithms, by multiplying by $0{\cdot}4343$, we get:—

$$\text{Log } \frac{T_d}{T_s} = 0{\cdot}3 \times 2{\cdot}5133 \times 0{\cdot}4343 = {\cdot}327457$$

$$\text{,, } \quad \text{,,} = \log 2{\cdot}125, \text{ nearly.}$$

$$\therefore \quad T_d = 2\,T_s \text{ approximately.}$$

If the angle of contact between the belt and the pulley be expressed in degrees, then the equation requires to be modified.

Let θ° = angle of contact expressed in degrees. Then, we know that:—

$$\frac{\theta}{\theta^\circ} = \frac{\text{Circular measure of two right angles}}{\text{Number of degrees in two right angles}} = \frac{\pi}{180^\circ}$$

$$\therefore \quad \theta = \frac{3{\cdot}1416}{180} \times \theta^\circ = 0{\cdot}0175\,\theta^\circ.$$

Substituting this value for θ, we get:—

$$\log_\epsilon \frac{T_d}{T_s} = 0{\cdot}0175\,\mu\,\theta^\circ. \quad \ldots\ldots\ldots\ldots \quad \text{(XI)}$$

Or, changing into common logarithms, by multiplying by $0{\cdot}4343$, we get:—

$$\log \frac{T_d}{T_s} = 0{\cdot}4343 \times 0{\cdot}0175\,\mu\,\theta^\circ = 0{\cdot}0076\,\mu\,\theta^\circ. \quad \ldots \quad (\text{XI}_a)$$

GREATEST VALUE OF THE RATIO OF TENSIONS ON TIGHT AND SLACK SIDES OF BELTING.*

Angle embraced by Belt.			Ratio of Tensions $= \frac{T_d}{T_s}$.			
In Degrees. $\theta°$.	In Circular Measure. θ.	In Fraction of Circumference. $\theta°/360° = \theta/2\pi$.	$\mu = 0{\cdot}2$.	$\mu = 0{\cdot}3$.	$\mu = 0{\cdot}4$.	$\mu = 0{\cdot}5$.
30	·524	·083	1·110	1·170	1·233	1·299
45	·785	·125	1·170	1·266	1·369	1·481
60	1·047	·167	1·233	1·369	1·521	1·689
75	1·309	·208	1·299	1·481	1·689	1·924
90	1·571	·250	1·369	1·602	1·874	2·193
105	1·833	·319	1·443	1·733	2·082	2·500
120	2·094	·334	1·521	1·875	2·312	2·851
135	2·356	·375	1·602	2·027	2·565	3·247
150	2·618	·417	1·689	2·194	2·849	3·702
165	2·880	·458	1·778	2·372	3·163	4·219
180	3·142	·500	1·875	2·566	3·514	4·808
195	3·403	·541	1·975	2·776	3·901	5·483
210	3·665	·583	2·082	3·003	4·333	6·252
240	4·188	·666	2·311	3·514	5·340	8·119
270	4·712	·750	2·566	4·112	6·589	10·550
300	5·236	·833	2·849	4·808	8·117	13·700

Frictional Resistance between a Rope and a Grooved Pulley.—In rope transmission the pulleys over which the rope passes are provided with grooves, and we saw in our last Lecture that for hemp or cotton ropes the grooves are so constructed that the ropes get wedged into them and bear on their sides. This, as we have seen, is the method adopted in order to increase the resistance to slipping. In Lecture VII. we found that the ratio between T_d and T_s in this case is given by the equation :—

$$\text{Log}_\epsilon \frac{T_d}{T_s} = \mu \text{ cosec } \frac{\alpha}{2} \theta.$$

Where μ = Coefficient of friction between rope and sides of groove.

„ α = Angle which sides of groove make with each other.

„ θ = Angle embraced by rope on pulley.

Comparing this equation with that for flat pulleys we see that the logarithm of the ratio of the tensions is increased in this case in the proportion :—cosec $\frac{\alpha}{2}$: 1.

* From Unwin's *Machine Design*, Part I., p. 377 (13th Edition).

Generally the groove angle, α, is about 45°, and then we get :—

$$\text{cosec } 22\tfrac{1}{2}^\circ = 2{\cdot}6.$$

Hence,
$$\text{Log}_s \frac{T_d}{T_s} = 2{\cdot}6\ \mu\ \theta.$$

Or, converting into common logarithms, we get :—

$$\text{Log} \frac{T_d}{T_s} = 0{\cdot}4343 \times 2{\cdot}6\ \mu\ \theta = 1{\cdot}13\ \mu\ \theta.$$

We have already stated that $\mu = 0{\cdot}15$ to $0{\cdot}35$ for hemp or cotton ropes, the smaller value being taken when the pulleys are greased. Substituting these values in the last equation, we get :—

$$\text{Log} \frac{T_d}{T_s} = 0{\cdot}17\ \theta \text{ (pulleys greased).}$$

Also,
$$\text{Log} \frac{T_d}{T_s} = 0{\cdot}39\ \theta \text{ (pulleys dry).}$$

If for μ cosec $\frac{\alpha}{2}$ we write μ_1 the equation becomes :—

$$\log_e \frac{T_d}{T_s} = \mu_1 \theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(XII)}$$

Where μ_1 can be looked upon as the coefficient of friction for grooved pulleys.

Converting into common logarithms, by multiplying by 0·4343, and substituting θ° for θ, equation (XII) becomes :—

$$\text{Log} \frac{T_d}{T_s} = 0{\cdot}4343 \times 0{\cdot}0175\ \mu_1\ \theta^\circ = 0{\cdot}0076\ \mu_1\ \theta^\circ. \quad . \quad . \quad . \quad (\text{XII}_a)$$

VALUES OF μ_1 CORRESPONDING TO DIFFERENT VALUES OF μ AND α.

Angle of Groove in Degrees α.	Values of μ_1 when				
	$\mu = 0{\cdot}15$.	$\mu = 0{\cdot}20$.	$\mu = 0{\cdot}25$.	$\mu = 0{\cdot}30$.	$\mu = 0{\cdot}35$.
30	·58	·77	·97	1·16	1·35
35	·50	·66	·83	1·00	1·16
40	·44	·58	·73	·88	1·02
45	·39	·52	·65	·78	·91

GREATEST VALUE OF THE RATIO OF TENSIONS ON TIGHT AND SLACK SIDES OF A ROPE WORKING IN GROOVED PULLEYS.

Angle Embraced by Rope.			Ratio of Tensions $= \frac{T_d}{T_s}$.					
In Degrees. θ°.	In Circular Measure. θ.	In Fraction of Circumference. $\frac{\theta^\circ}{360^\circ} = \frac{\theta}{2\pi}$.	$\mu_1 = 0{\cdot}4$.	$\mu_1 = 0{\cdot}5$.	$\mu_1 = 0{\cdot}6$.	$\mu_1 = 0{\cdot}7$.	$\mu_1 = 0{\cdot}8$.	$\mu_1 = 0{\cdot}9$.
60	1·047	·167	1·52	1·69	1·87	2·08	2·31	2·57
90	1·571	·250	1·87	2·19	2·57	3·00	3·51	4·11
120	2·094	·334	2·31	2·85	3·51	4·33	5·34	6·59
150	2·618	·417	2·85	3·70	4·81	6·25	8·12	10·55
180	3·142	·500	3·51	4·81	6·59	9·02	12·35	16·90
210	3·665	·583	4·33	6·25	9·02	13·01	18·77	27·08
240	4·188	·666	5·34	8·12	12·35	18·77	28·53	43·38

"Slip" or "Creep" of Belts due to Elasticity.—Although there may be no actual slipping as a whole between a belt and its pulleys, yet the unequal stretching of the two parts of the belt, due to the unequal tensions, T_d and T_s, causes a difference in angular velocity-ratio, which becomes very serious when this requires to be kept uniform

It will be seen in a subsequent Lecture that any elastic material subjected to tension becomes elongated, and that the elongation depends directly on the stretching force. Hence, the belt will be stretched to a greater extent on the driving side than on the slack side.

Let l = Length of belt which would leave either pulley in unit time, if the belt were unstretchable or inelastic.

„ l_d = Length of belt running on to driving pulley in unit time.

„ l_s = Length of belt running on to driven pulley in unit time.

„ e = Elongation produced by a tension of 1 lb. on a length of 1 foot of belting.

Then, $$l_d = l + e\,T_d\,l = (1 + e\,T_d)\,l.$$

Similarly, $$l_s = (1 + e\,T_s)\,l.$$

Since we have supposed that there is no actual slipping between the belt and pulley as a whole, it is clear that the

length of belting coming on to a pulley in unit time will be equal to the length of the arc described by a point on the circumference of the pulley in the same time. In other words, the length of belting coming on to the pulley in unit time, is equal to the circumferential speed of pulley. Hence, for the driving pulley, $\pi D N = l_d$; and for the driven pulley, $\pi d n = l_s$.

$$\therefore \quad \frac{d n}{D N} = \frac{l_s}{l_d} = \frac{(1 + e T_s) l}{(1 + e T_d) l}.$$

Or,
$$\frac{n}{N} = \left(\frac{1 + e T_s}{1 + e T_d}\right) \frac{D}{d}. \quad . \quad . \quad . \quad . \quad (XIII)$$

If we knew the value of e in all cases, and then calculated T_d and T_s by the previous formulæ, we might obtain the actual velocity-ratio for such cases. From experiments by M. Kretz, it appears that :—

$$\frac{1 + e T_s}{1 + e T_d} = 0{\cdot}975 \textit{ for new, and } = 0{\cdot}978 \textit{ for old, leather belts.}$$

Taking these results, we get :—

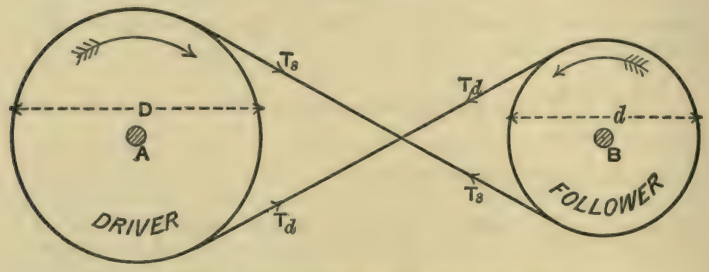
$$\left.\begin{aligned} \frac{n}{N} &= 0{\cdot}975 \times \frac{D}{d} \textit{ for new leather belts.} \\ \text{And,} \quad \frac{n}{N} &= 0{\cdot}978 \times \frac{D}{d} \textit{ for old leather belts.} \end{aligned}\right\} . \quad . \quad (XIII_a)$$

It therefore appears that the velocity of the driven pulley is about 2 per cent. less than it would be if the belt were inelastic. This loss of velocity is termed the "slip," or "creep," due to elasticity. Consequently, when motion has to be transmitted through several belts, this loss becomes serious, and renders belt gearing unsuitable for exact velocity-ratios.

Horse Power transmitted by Belt and Rope Gearing.—When one pulley, A, drives another pulley, B, by means of a belt or rope, there is necessarily a difference in tension in the two parts of the belt or rope. First of all, there is a resistance to motion offered by the follower, B. Hence, when the driver, A, is made to rotate, the friction between the belt or rope and the pulleys prevents slipping, with the result that the tension in that part of the belt or rope going on to the driver *increases*, while the tension in the other part *decreases*. This increase of tension in the driving side, and the decrease in the slack side, will go on until the resultant moment of the tensions in the two parts

of the belt or rope acting on the follower overcomes the resisting couple acting on that pulley. When this takes place :—

Let T_d = Tension in *driving* side of belt or rope.
„ T_s = Tension in *slack* side of belt or rope.
„ M = Moment of couple resisting rotation of follower.
„ V = Velocity of belt or rope in feet per minute.
„ D = Diameter of driver in feet.
„ d = Diameter of follower in feet.
„ N = Speed of driver in revolutions per minute.
„ n = Speed of follower in revolutions per minute.
„ H.P. = Horse-power transmitted.



Power Transmitted by a Belt or Rope.

Then, $$M = (T_d - T_s)\frac{d}{2}.$$

$\therefore$ *Work transmitted per minute* $= M \times 2\pi n = (T_d - T_s) \times \pi d n.$

$$\therefore \quad H.P. = \frac{(T_d - T_s)\pi d n}{33{,}000}$$

But, $$V = \pi D N = \pi d n.$$

$$\therefore \quad H.P. = \frac{(T_d - T_s)V}{33{,}000} = \frac{PV}{33{,}000} \quad \ldots \text{(XIV)}$$

Where $P = T_d - T_s$, and is called the driving force or tension.

$$\therefore \quad P = \frac{33{,}000 \text{ H.P.}}{V}$$

Let T = Initial or average tension; or tension in *both* parts of belt when at rest.

The average tension during motion must be the same as the tension before motion commenced, since the lengthening of the belt on the driving side must be equal to the shortening of the belt on the slack side.

$$\therefore \qquad T = \frac{T_d + T_s}{2}.$$

This must be the tension with which the belt should be initially stretched over the pulleys.

If, for these reasons, we suppose $T_d = 2\,T_s$, we get the following practical rules for belt gearing :—

$$\textit{Driving Tension} = P = \frac{33{,}000 \text{ H.P.}}{V}$$

$$\textit{Greatest Tension} = T_d = 2\,P.$$

$$\textit{Initial Tension} = T = 1\tfrac{1}{2}\,P.$$

$$\textit{Breadth of Belt} = \beta = \frac{T_d}{f} = \frac{2\,P}{f}.$$

Where, f is the safe working tension per inch of width or breadth β.

The following table gives the safe working tension, f, in lbs. per inch of width for leather belts, when the safe stress in lbs. per square inch cross sectional area is known :—

WORKING TENSION OF LEATHER BELTS.

Thickness of Belt in Inches.	Safe working tension, f, in lbs. per inch of width when the safe stress in lbs. per square inch of cross sectional area is :—		
	250	300	350
$\frac{3}{16}$	47	56	66
$\frac{7}{32}$	55	66	77
$\frac{1}{4}$	62	75	87
$\frac{5}{16}$	78	94	109

In the case of hemp or cotton rope gearing, the tension T_d may be taken at 140 lbs. per square inch of *gross sectional area*, or about $\frac{1}{10}$ of the working strength of the rope.

The following table gives the working strength and driving force for hemp or cotton ropes:—

WORKING STRENGTH, &c., FOR DRIVING ROPES.

Diameter of Rope in inches. $= \delta$	Girth of Rope in inches. $= \gamma$	Working strength in lbs.	Driving Force in lbs. $= P$
1	$3\frac{1}{8}$	842	76
$1\frac{5}{16}$	$4\frac{1}{4}$	1,548	140
$1\frac{1}{2}$	$4\frac{3}{4}$	1,940	176
$1\frac{11}{16}$	$5\frac{1}{4}$	2,602	236
$2\frac{1}{16}$	$6\frac{1}{2}$	3,633	330

Taking the greatest stress in a hemp or cotton rope at 140 lbs. per square inch gross sectional area, and assuming that $T_d = 4\ T_s$, so that the driving tension $P = T_d - T_s$ is 105 lbs. per square inch gross sectional area, we get the following table showing the horse-power transmitted by a single rope for given speeds:—*

HORSE-POWER TRANSMITTED BY A SINGLE ROPE AT HIGH SPEEDS.

Diameter of Rope in Inches.	Girth of Rope in Inches.	Horse-power Transmitted by one Rope when the Speed in Feet per Minute is:—						
δ	γ	3,000	3,500	4,000	4,500	5,000	5,500	6,000
1	3·14	7·50	8·75	10·00	11·25	12·50	13·75	15·00
$1\frac{1}{8}$	3·53	9·48	11·07	12·66	14·24	15·82	17·40	18·98
$1\frac{1}{4}$	3·93	11·72	13·67	15·62	17·58	19·53	21·48	23·44
$1\frac{3}{8}$	4·32	14·18	16·54	18·91	21·27	23·63	26·00	28·36
$1\frac{1}{2}$	4·71	16·87	19·69	22·50	25·31	28·12	30·94	33·75
$1\frac{5}{8}$	5·10	19·80	23·11	26·41	29·71	33·01	36·31	39·61
$1\frac{3}{4}$	5·50	22·97	26·80	30·62	34·45	38·28	42·11	45·94
$1\frac{7}{8}$	5·89	26·37	30·76	35·15	39·55	43·95	48·34	52·73
2	6·28	30·00	35·00	40·00	45·00	50·00	55·00	60·00

The following table of particulars regarding power transmitted, &c., by wire ropes, as calculated by Roebling, is given in Unwin's *Machine Design*, Part I., p. 434. The ropes have each 42 wires.

* From Low & Bevis's *Machine Drawing and Design*, p. 161.

POWER TRANSMITTED BY WIRE ROPES.

Diameter of Rope in Inches.	Diameter of Pulley in Feet.	Number of Revolutions of Pulley per Minute.	Velocity of Belt in Feet per Second.	Breaking Strength of Rope in lbs.	Horse-power Transmitted.
$\frac{7}{16}$	5	100	26	4,260	8·6
$\frac{15}{32}$	6	100	31	5,660	13·4
$\frac{1}{2}$	7	100	36	8,200	21·1

EXAMPLE VII.—The belt pulley of an engine is 6 feet in diameter, and the engine makes 80 revolutions per minute. What width of belt would be required to transmit 30 H.P. from the pulley, supposing the belt to be $\frac{3}{8}$ of an inch in thickness, and the tension on the slack side to be $\frac{4}{10}$ of that on the tight side of the belt? The stress in the belt is not to exceed 300 lbs. per square inch of section. (S. & A. Adv. Exam., 1892.)

ANSWER.—Here D = 6 ft.; N = 80; H.P. = 30.

Thickness of belt = $\delta = \frac{3}{8}$ inch.

Greatest stress allowed = f = 300 lbs. per sq. inch.

Let β = required width of belt in inches.

Then, *Total stress on driving side of belt* $= T_d = \beta \delta f$,

$= \beta \times \frac{3}{8} \times 300 = 112{\cdot}5\,\beta$ lbs.

Again, $T_s = \frac{4}{10} T_d = 45\,\beta$ lbs.

$\therefore$ *Driving force* $= P = T_d - T_s = 67{\cdot}5\,\beta$ lbs.

$\therefore$ $$H.P. = \frac{P \times \pi D N}{33{,}000}.$$

Or, $$30 = \frac{67{\cdot}5\,\beta \times \frac{22}{7} \times 6 \times 80}{33{,}000},$$

$\therefore$ $$\beta = \frac{33{,}000 \times 30 \times 7}{67{\cdot}5 \times 22 \times 6 \times 80} = 9{\cdot}72 \text{ ins.}$$

EXAMPLE VIII.—Two pulleys on parallel shafts 10 feet apart, and having diameters of 6 feet and 1 foot respectively, are connected by an open belt 8 inches broad. The larger pulley makes 120 revolutions per minute. Find the greatest power which can be transmitted, when the maximum stress in the belt

has not to exceed 80 lbs. per inch of width, and taking the coefficient of friction between belt and pulleys at 0·4.

ANSWER.—The first thing to be determined here is the ratio of the tensions in the two parts of the belt. This is obtained from the equation :—

$$\text{Log}\,\frac{T_d}{T_s} = \cdot 4343\,\mu\,\theta.$$

Where, θ is the circular measure of the angle subtended at the centre of the *smaller pulley* by the part of the belt in contact with that pulley.

To obtain this angle we must first find the angle, ϕ, used in equations (VI) and (VII) in this Lecture, for finding the length of an open belt. Referring to these equations and the figure at that part of the text, we clearly see that :—

$$\theta = 180^\circ - 2\,\phi^\circ = (\pi - 2\,\phi) \text{ radians.}$$

From equation (VII),

$$\phi = \sin\,\phi \text{ (approx.)} = \frac{D - d}{2\,a} = \frac{6 - 1}{2 \times 10} = \cdot 25.$$

$$\therefore \quad \theta = 3 \cdot 1416 - 2 \times \cdot 25 = 2 \cdot 64 \text{ radians, approx.}$$

$$\text{Hence, } \text{Log}\,\frac{T_d}{T_s} = \cdot 4343 \times \cdot 4 \times 2 \cdot 64 = \cdot 458621.$$

Referring to a table of common logarithms, we find that :—

$$0 \cdot 458621 = \text{Log}\ 2 \cdot 87,$$

$$\therefore \quad \frac{T_d}{T_s} = 2 \cdot 87.$$

Since, from the question, the greatest stress in the belt must not exceed 80 lbs. per inch of width, and the width is 8 inches, we get :—

$$T_d = 8 \times 80 = 640 \text{ lbs.}$$

$$\therefore \quad T_s = \frac{640}{2 \cdot 87} = 223 \text{ lbs.}$$

$$\therefore \quad P = T_d - T_s = 417 \text{ lbs.}$$

$$\text{And,} \quad V = \pi\,D\,N = \frac{22}{7} \times 6 \times 120 \text{ ft. per minute.}$$

$$\therefore \quad \text{H.P.} = \frac{P\,V}{33{,}000} = \frac{417 \times \frac{22}{7} \times 6 \times 120}{33{,}000} = 28 \cdot 6 \text{ nearly.}$$

This is the greatest power which can be transmitted. If the power to be transmitted be greater than this, then slipping will take place between the belt and the smaller pulley.

Influence of Centrifugal Tension on the Strength of High-Speed Belts and Ropes.—When belts or ropes are run at high speeds the tensions in the two parts of the belt or rope between the pulleys are greater than that calculated from the horse-power transmitted. This increase of tension is due to the centrifugal force set up in those parts of the belt which are in contact with the pulleys. In addition to this increase in tension the centrifugal action has also the effect of diminishing the normal pressure between the belt and the pulleys, and, therefore, of diminishing the resistance to slipping.

Let w = Weight of belt or rope, in *lbs. per linear foot.*

„ v = Velocity of belt or rope, in *feet per second.*

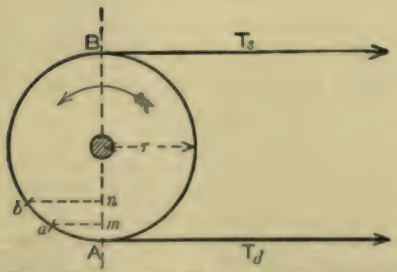
„ r = Radius of the pulley in *feet.*

„ T_d, T_s = Tensions in driving and slack sides of belt or rope *as calculated from power transmitted.*

„ T^1_d, T^1_s = Tensions in driving and slack sides of belt or rope *corrected for centrifugal action.*

For simplicity, suppose the belt embraces half the circumference of the pulley considered.

Then, *Centrifugal force of belt per linear foot* $= \frac{w\,v^2}{g\,r}$ lbs.



Belt Embracing Half Circumference of Pulley.

In the figure, let $a\,b$ represent a part of the belt 1 foot in length. Let $m\,n$ be the projection of $a\,b$ on the diameter A B. It will be shown in a subsequent Lecture, that the horizontal component of the centrifugal force on the part, $a\,b$, of the belt is:—

$$\frac{w\,v^2}{g\,r} \times m\,n.$$

Hence, for the whole arc of contact, A C B, we get:—

$$\left.\begin{matrix}\textit{Total horizontal component}\\ \textit{of centrifugal force}\end{matrix}\right\} = \frac{w\,v^2}{g\,r} \times \text{A B} = \frac{2\,w\,v^2}{g} \text{ lbs.}$$

One-half of this centrifugal tension is felt at A, and the other half at B. Therefore, the actual tensions in the two parts of the belt are :—

$$\left.\begin{aligned} T_d^1 &= T_d + \frac{w\,v^2}{g} \\ T_s^1 &= T_s + \frac{w\,v^2}{g} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (XV)$$

From these equations it will be seen :—

(1) That the diameters of the pulleys have nothing to do with the results, and

(2) That the driving force or tension, $P = T_d^1 - T_s^1 = T_d - T_s$ and is, therefore, the same as before.

If the speed be great, the above value for T_d^1 must be taken when calculating the size of a belt or rope to transmit a given power.

The values of w may be calculated from the following approximate formulæ :—

Let β = Breadth of leather belt in *inches*.

„ δ = Diameter of rope, or thickness of belt in *inches*.

„ γ = Girth of rope in inches.

Then, $w = 0{\cdot}43\ \beta\ \delta$ *lbs., nearly,* for *leather belts.*

$w = 0{\cdot}281\ \delta^2$ „ } „ *dry hemp or cotton ropes.*
$w = 0{\cdot}0285\ \gamma^2$ „ }

$w = 0{\cdot}3376\ \delta^2$ „ } „ *wet or tarred hemp or cotton*
$w = 0{\cdot}0342\ \gamma^2$ „ } *ropes.*

$w = 1{\cdot}341\ \delta^2$ „ „ *wire ropes.*

Example IX.—Determine the horse-power which may be transmitted by a leather belt, 5 inches wide and ¼ inch thick, running at a speed of 50 feet per second, the tension in the loose side being $\frac{4}{10}$ of that on the tight side of the belt, and the stress allowed being 275 lbs. per square inch (S. & A. Hons. Mach. Const. Exam., 1886). Again, taking the weight of a cubic foot of the leather at 60 lbs., determine the effects due to centrifugal action.

Answer.—Here, V = 3,000 ft. per minute; $\beta = 5$ inches; $\delta = \frac{1}{4}$ inch; $f = 275$ lbs. per square inch; $T_s = \frac{4}{10}\ T_d$.

(1) *Neglecting centrifugal action.*

$$P = T_d - T_s = \cdot 6\, T_d.$$

But, $T_d = \beta\, \delta f = 5 \times \frac{1}{4} \times 275 = 343{\cdot}75$ lbs.

Hence, $P = \cdot 6 \times 343{\cdot}75 = 206{\cdot}25$ lbs.

$$\therefore \quad \text{H.P.} = \frac{P\,V}{33{,}000} = \frac{206{\cdot}25 \times 3{,}000}{33{,}000} = 18{\cdot}75.$$

(2) *Taking centrifugal action into account.*

Let W = Weight of a *cubic* foot of leather in lbs.

„ w = Weight of a *linear* „ „

„ A = Cross sectional area of belt in square inches.

Then, clearly, $w : W = A : 144.$

$$\therefore \quad w = \frac{W\,A}{144}.$$

Substituting the values given in the question, we get:—

$$w = \frac{60 \times 5 \times \frac{1}{4}}{144} = \cdot 521 \text{ lbs.}$$

Substituting this in equation (XV), in the text, and observing that $v = 50$ ft. per second, we get:—

$$\left.\begin{array}{l}\textit{Increase of tension due}\\ \textit{to centrifugal force}\end{array}\right\} = \frac{w\,v^2}{g} = \frac{\cdot 521 \times 50 \times 50}{32} = 40{\cdot}7 \text{ lbs.}$$

But the maximum tension allowed in the belt is 343·75 lbs., hence:—

Maximum effective tension = 343·75 − 40·7 = 303 lbs. nearly.

$$\therefore \quad P = \cdot 6 \times 303 = 181{\cdot}8 \text{ lbs.}$$

$$\therefore \quad \text{H.P.} = \frac{P\,V}{33{,}000} = \frac{181{\cdot}8 \times 3{,}000}{33{,}000} = 16{\cdot}53$$

This example shows that the power is reduced by about 12 per cent. when centrifugal action is taken into account.

LECTURE XVI.—QUESTIONS.

1. Find an expression for the length of a crossed belt, and show that the same driving belt will serve for any pair of pulleys, so long as the belt is crossed and the distance apart of their centres and the sum of their diameters remains constant.

2. The centres of two pulleys, 4 and $2\frac{1}{2}$ feet in diameter respectively, are 12 feet apart. Find length of crossed belt required. *Ans.* 35·1 feet.

3. A crossed belt is employed to connect two equal coned drums, having their axes parallel, and their vertices lying in opposite directions. Prove that the belt will be equally tight in all positions when shifted along the cones. Would the same be true if the belt were not crossed?

4. By aid of a *graphical construction* determine the length of a crossed belt required to embrace either of two pairs of pulleys which are mounted on parallel shafts 3 feet 6 inches apart. The smaller pulley on each shaft is 8 inches diameter, and the velocity-ratio at the higher speed is required to be four times that at the lower speed. *Ans.* 10·4 feet.

5. The diameters of the pulleys of a stepped speed-cone for a machine are $13\frac{1}{2}$, $11\frac{1}{2}$, $9\frac{1}{2}$, and $7\frac{1}{2}$ inches respectively, and the diameter of the smallest pulley of the stepped driver is $8\frac{1}{2}$ inches. The connection being made by means of a crossed belt, what should be the diameters of the other pulleys of the stepped driver? If the driving shaft makes 120 revolutions per minute, find the revolutions per minute of the machine pulley for all positions of the belt.

Ans. (1) $10\frac{1}{2}$, $12\frac{1}{2}$, and $14\frac{1}{2}$ inches; (2) 75·5, 109·6, 157·9, and 232.

6. A shaft having a stepped speed-cone, with four steps, revolves at a constant speed of 150 revolutions per minute, and is connected by means of a crossed belt to another shaft having a similar stepped cone. The diameter of the largest step of the cone on the driving shaft is $13\frac{1}{2}$ inches. The driven shaft is required to run at speeds 250, 200, 120, and 60 revolutions per minute. Determine the diameters of all the remaining steps of the two cones, and the length of belt required, the distance between the two shafts being 8 feet. *Ans.* (1) $D_2 = 12$ inches, $D_3 = 9{\cdot}333$ inches, $D_4 = 6$ inches; $d_1 = 7{\cdot}875$ inches, $d_2 = 9$ inches, $d_3 = 11{\cdot}667$ inches, $d_4 = 15$ inches; (2) 18·94 feet.

7. Find an expression for the length of an open belt. The centres of two pulleys, 5 and $2\frac{3}{4}$ feet in diameter respectively, are 15 feet apart. Find length of open belt required. *Ans.* 42·26 feet.

8. Two pulleys, whose diameters are 4 feet 8 inches and 2 feet 3 inches respectively, their centres being 10 feet apart, are connected by an open belt, determine, by a graphical construction, the length of belt required. *Ans.* 31 feet.

9. Explain how you would design a set of speed cones to be worked by an open belt, the angular velocity-ratios being given you.

10. A countershaft revolves at a constant speed of 250 revolutions per minute, and carries a stepped speed-cone with four steps, and drives a similar cone on another shaft by means of an open belt. The driven shaft is required to run at speeds 520, 300, 245, and 180 revolutions per minute respectively. Given the diameter of the largest step on the driving cone 22 inches, and the distance between the shafts $8\frac{1}{4}$ feet; find the remaining

sizes of the steps on both cones. *Ans.* $D_2 = 17{\cdot}87$ inches; $D_3 = 16{\cdot}22$ inches $D_4 = 13{\cdot}70$ inches; $d_1 = 10{\cdot}58$ inches; $d_2 = 14{\cdot}90$ inches; $d_3 = 16{\cdot}56$ inches; $d_4 = 19{\cdot}04$ inches.

11. Deduce a formula for the greatest ratio of the tensions in the two parts of a belt stretched over a pulley when slipping is just about to take place. Two pulleys, whose diameters are $5\frac{3}{4}$ and 2 feet respectively, are 15 feet apart. Find the maximum ratio of tensions in tight and slack sides of belt (1) when the belt is crossed, (2) when the belt is open, given $\mu = 0{\cdot}3$. *Ans.* (1) 3 : 1; (2) 2·38 : 1.

12. Deduce a formula for the greatest ratio of the tensions in the two parts of a rope stretched over a grooved pulley, the rope being wedged into the grooves slipping being just about to occur. Find the greatest ratio of the tensions in the two parts of a cotton rope running over grooved pulleys, the arc of contact on the smaller pulley being $\frac{5}{12}$ of the whole circumference, angle of groove 40°, coefficient of friction, $\mu = 0{\cdot}25$. *Ans.* 6·77 : 1.

13. Explain the following paradox in connection with belt gearing:—For every foot of belt length that goes on to the driving pulley, less than a foot comes off and goes on to the driven pulley. Would the statement still be true if we substituted unit weight of belt instead of unit length of belt? If not, why not?

14. Explain how the formula for obtaining the power transmitted by a stretched belt running over pulleys is arrived at. What horse-power may be transmitted by a belt 10 inches wide, and $\frac{1}{4}$ inch thick, running at a speed of 42 feet per second; the tension on the slack side of the belt being 0·4 of that on the driving side? The stress allowed is 300 lbs. per square inch of belt section. *Ans.* 68·72 H.P.

15. A belt is required to transmit 4 horse-power from a shaft running at 120 revolutions to one at 160 revolutions per minute. Find the stresses in the belt, the small pulley being 2 feet in diameter, and the ratio of the tensions on the belt being as 7 is to 4. Find also the width of belt that would be required in the above case, if the stress is taken at 100 lbs. per inch of width. *Ans.* 306·25 lbs.; 175 lbs.; 3·06 inches.

16. A pulley, 3 feet 6 inches in diameter, and making 150 revolutions per minute, drives by means of a belt a machine which absorbs 7 horse-power. What must be the width of the belt so that its greatest tension shall be 70 lbs. per inch of width, it being assumed that the tension in the driving side is twice that in the slack side? Take $\pi = 3\frac{1}{7}$. *Ans.* 4 inches.

17. In the modern system of transmitting power through long distances by a slender wire rope moving at a high velocity, the following example occurs:—The pulley which drives the rope is 15 feet in diameter, making 100 revolutions per minute, and the energy to be transmitted is measured by 250 horse-power. Find the tension of the wire rope, which in this case is $\frac{3}{4}$ inch in diameter. *Ans.* 1,750 lbs., or 3,960 lbs. per square inch nearly.

18. A leather belt is required to transmit 2 H.P. from a shaft running at 80 revolutions per minute to a shaft running at 160. Find the stresses in the belt, assuming that the smaller pulley is 12 inches in diameter, and that the ratio of the tensions in the tight and slack sides of the belt is $2\frac{1}{2}$: 1. Hence, find the width of belt, taking the working stress at 100 lbs. per inch of width. *Ans.* 236·3 lbs.; 105 lbs.; 2·36 inches.

19. Suppose the friction of two pulleys is such that the ratio of the tensions in the tight and slack sides of the belt is 1·75. Also, suppose the greatest safe working tension to be 120 lbs. per inch width of belt. Find the width of a belt to transmit 10 horse-power, the circumferential speed of the pulleys being 20 feet per second. *Ans.* 5·34 inches.

20. Assuming that the arc embraced by a belt on the smaller of two

pulleys over which it runs is $\frac{1}{16}$ of the circumference, and that μ is taken = 0·3, prove the following simple approximate rule for the breadth of a leather belt :—$\beta = \frac{208 \text{ H.P.}}{V\,\delta}$ inches ; where H.P. is the horse-power transmitted, V the velocity of belt in feet per minute, and δ the thickness of belt in inches. Greatest working stress, 300 lbs. per square inch.

21. It is required to transmit 10 H.P. from a pulley 5 feet in diameter, and making 200 revolutions per minute, to one 18 inches in diameter, by means of an open belt, the centres of the pulleys being 12 feet apart. Taking coefficient of friction between belt and pulley at 0·35; find (1) angle of contact on smaller pulley ; (2) the speed of smaller pulley ; and (3) the width of single belt $\frac{3}{16}$ inch thick which will be necessary. *Ans.* (1) 163°; (2) 52·36 feet per second; (3) 3 inches.

22. It is required to transmit 16 H.P. from a pulley 20 inches in diameter by means of a belt which embraces only $\frac{2}{3}$ of the circumference of the pulley. Find the tensions in the two parts of the belt when slipping is just prevented, and the width of belt required, the thickness of the belt being $\frac{3}{8}$ inch, speed of pulley 120 revolutions per minute, coefficient of friction $\mu = 0{\cdot}35$. *Ans.* 2174·5 lbs.; 1333·75 lbs.; 19·35 inches.

23. What circumstances affect the action of a belt when the speed is high ?

24. Find an expression for the increase in the tensions in the tight and slack sides of a belt, taking centrifugal action into account. In question 14 make the necessary corrections for centrifugal action, being given that the weight of a cubic foot of leather weighs 60 lbs. *Ans.* 63·45 H.P.

25. In question 17 make the necessary corrections for centrifugal action, given that the weight of a linear foot of wire rope $\frac{3}{4}$ inch in diameter is 3 lbs. *Ans.* 2,330 lbs.

26. Show that the fractional resistance between a belt and a flat pulley may be represented by the formula—

$$\log\left(\frac{T}{t}\right) = 0{\cdot}4343\,\mu\,\theta$$

when T and t represent the tensions respectively on the two sides of the belt, of which T is greater than t ; μ is the coefficient of friction between the belt and the rim of the pulley, and θ is the circular measure of the angle subtended at the centre of the pulley by the part of the belt which is in contact with the pulley. If $\mu = 0{\cdot}1$, what would be the greatest load that could be supported by the rope or chain which passes around the drum, 12 inches in diameter, of a treble purchase crab or winch, which is fitted with a strap friction brake worked by a lever, to the long arm of which a pressure of 60 lbs. is applied ? The diameter of the brake pulley is 30 inches, and the brake handle is 3 feet in length from its fulcrum : one end of the brake strap is immovable, being attached to the pin forming the fulcrum of the brake handle, while the other end of the strap or belt is attached to the shorter arm, 3 inches in length of the brake lever. The angle subtended by the strap at the centre of the brake pulley measures $\frac{3\pi}{2}$. The gearing of the crab is as follows :—On the shaft which carries the brake wheel is a pinion of 15 teeth, and this gears into a wheel of 50 teeth on the second shaft ; a pinion of 20 teeth on this latter shaft gears into a wheel with 60 teeth carried upon the drum or barrel shaft. Sketch the crab and show the construction of the brake. Given ·20466 = log 1·602. (S. & A. Hons. Exam., 1896.)

27. Prove the law for the slipping of a belt, and show how it enables us to calculate the size of a belt. A belt laps 150 degrees round a pulley of 3 feet diameter, making 130 revolutions per minute; the coefficient of friction is 0·35. What is the maximum pull on the belt when 20 horse-power is being transmitted and the belt is just on the point of slipping? (S. & A. Hons. Exam., 1897.)

28. Give the theory of the slipping of a belt, and deduce the rule employed in calculating the proper size of a belt. What experiment have you made or heard of to illustrate the law of slipping? (S. & A. Hons. Exam., 1898.)

29. How does the friction between a belt and pulley depend on the amount of lapping? Describe how you would determine this experimentally. With a certain amount of lapping suppose that slipping occurs when the tension on the tight side is twice what it is on the slack side; find the tension when the belt, moving at 15 feet per second, gives 20 horse-power to the pulley, slipping just not taking place.

Ans. $T_1 = 733{\cdot}3$ lbs.; $T_2 = 1{,}466{\cdot}6$ lbs. (B. of E., Adv., 1900.)

30. A leather belt a quarter of an inch thick has to transmit 10 H.P. from a pulley 4 feet in diameter to a second pulley 3 feet in diameter, the driving pulley making 120 revolutions per minute. Assuming that the tension in the tight side of the belt is twice that in the slack side, find the width of belt necessary, the safe working stress of belting being 320 lbs. per square inch. (C. & G., 1902, O., Sec. A.)

31. Power is transmitted from one shaft to another by means of belting. How is the amount of power capable of being transmitted affected by, (1) the angle of lapping, (2) the weight of the belting, (3) journal friction, (4) the stiffness of the belt, (5) elastic slip, (6) centrifugal tension in the belt? B. of E., H., Part I., 1903.)

32. A belt transmits 60 H.P. to a pulley 20 inches in diameter running at 260 revolutions per minute. What is the difference of the tensions on the tight and slack sides? If the material is such, that to prevent slipping it is necessary to keep one tension $2\frac{1}{2}$ times the other, what is the tension on the tight side in this belt? (B. of E., S. 2, 1904.)

33. What width of leather belt would be required to transmit 25 H.P. under the following conditions?—Diameter of pulley, 27 inches. Revolutions of pulley per minute, 130. Coefficient of friction between the belt and the pulley rim, 0·18. Angle of contact of belt with the pulley, 210°. Maximum tension in the belt per inch of its width, 120 lbs. Prove the formula which you employ. (B. of E., S. 3, 1905.)

34. The speed cone of a small lathe has three steps of diameters, $5\frac{3}{4}$ inches, $4\frac{1}{4}$ inches, and $2\frac{3}{4}$ inches, respectively. On the corresponding cone of the countershaft the diameter of the largest step is $7\frac{1}{2}$ inches. Find the diameters of the other two steps on this cone in order to obtain uniform tightness of the driving belt at all speeds, if the driving belt is crossed. (B. of E., S. 2, 1906.)

35. Find the maximum horse-power which can be transmitted by a hemp rope 1 inch in diameter at a speed of 70 feet per second, if the rope is broken with a pull of 5,700 lbs., and it is desired to have a factor of safety of 30. The angle of the groove in which the rope runs is 60°, and the coefficient of friction may be taken as 0·25, and it is in contact with the pulley for half the circumference. Find also the centrifugal tension in the rope if the flywheel is 10 feet in diameter, and the reduction in the horse-power transmitted due to this tension. Weight of rope for 1 foot of length = 0·28 lb. (B. of E., S. 3, 1906.)

36. A leather belt weighs 0·036 lb. per cubic inch. Determine the linear speed of the belt when the power transmitted is a maximum, if the tensile stress in the leather is not to exceed 500 lbs. per square inch.

(B. of E., S. 3, 1908.)

LECTURE XVI.—A.M.INST.C.E. EXAM. QUESTIONS.

1. When a band is slipping over a pulley, show how the ratio of the tensions on the tight and slack sides depends on the friction and on the angle of contact. Apply your result to explain why a rope exerting a great pull may be readily held by giving it two or three turns round a post. (I.C.E., *Oct.*, 1897.)

2. A stepped pulley carried on a driving shaft making 120 revolutions a minute, has the first step 20 inches in diameter. From this step another shaft 10 feet away has to be driven at 220 revolutions per minute by means of a crossed belt. From another step the speed of the driven shaft has to be 165 revolutions per minute Calculate the length of the belt and the sizes of the steps. (I.C.E., *Feb.*, 1899.)

3. Find the relation between the tensions in the two ends of a cord which makes one turn round a drum. (I.C.E., *Oct.*, 1899.)

4. The tension per inch width of a belt must not exceed 110 lbs., find the width of belt necessary to transmit 12 H.P. from a shaft running at 80 revolutions, the diameter of the driving pulley being 4 feet 6 inches, and the ratio of the tensions $1\frac{3}{4}$ to 1. (I.C.E., *Oct.*, 1900.)

5. A driving shaft runs at 100 revolutions per minute, and carries a pulley 22 inches in diameter, from which a belt communicates motion to a pulley 12 inches in diameter carried upon a countershaft. On the countershaft is a cone pulley having steps 8 inches, 6 inches, and 4 inches diameter, which gives motion to another cone pulley with corresponding steps on a lathe spindle. Sketch the arrangement, and find the greatest and least speeds at which the lathe spindle can revolve. (I.C.E., *Feb.*, 1902.)

6. Compare the use of a crossed with that of an open belt for connecting two rotating pulleys, pointing out the differences, advantages, and disadvantages in the action. Calculate the revolutions per minute of a dynamo driven by a belt $\frac{1}{4}$ inch thick, the diameter of pulley of dynamo being 6 inches, and the angular velocity of the driving pulley of 4 feet 6 inches diameter being 10 radians per second. Allow 2 per cent. slip for elasticity of belt. (I.C.E., *Feb.*, 1903.)

7. Upon what does the limiting ratio of the tensions in the tight and slack sides of a belt, when transmitting power, depend? A leather belt $\frac{1}{4}$ inch thick has to transmit 10 H.P. from a pulley 4 feet in diameter to a second pulley, the driving-pulley making 120 revolutions per minute. Assuming that the tension in the tight side is twice that in the slack side, find the width of belt necessary, the safe working-stress of beltlng being 320 lbs. per square inch. *Ans.* The limiting ratio of the tensions in the tight and slack sides of a belt depend upon the coefficient of friction and the angle of contact. Width of belt = $5\frac{1}{2}$ inches. (I.C.E., *Oct.*, 1903.)

8. Explain, with sketches, the arrangement of fast and loose pulleys and speed cone for driving any machine tool, explaining the objects of the arrangement. Point out on which side of the belt the shifting fork must press, giving your reasons. (I.C.E., *Feb.*, 1904.)

9. A belt, which weighs w lbs. per foot length, passes over a pulley whose circumferential velocity is v feet per second. Show that, due to motion, the pull in the belt is $\frac{w\,v^2}{g}$ lbs. (I.C.E., *Feb.*, 1905.)

10. Show that a crossed belt will be equally tight on any pair of pulleys mounted on parallel shafts in fixed bearings, if the sum of the diameters is the same for each pair. On one shaft there are three pulleys, of which

the diameter of the largest is 9 inches, and of the smallest 6 inches. On the other shaft, connected by a crossed belt, the smallest pulley has a diameter of 4 inches:—Determine the diameters of the other pulleys, it being required that the intermediate pair should give an angular velocity ratio, which is an arithmetical mean of those due to the extreme pairs. (I.C.E., *Feb.*, 1905.)

11. Explain what is meant by the centrifugal tension of a running belt. Show that it is equal to the momentum of the mass of belt which passes a fixed point in the unit of time. Also show that, when the belt is transmitting the maximum power, the centrifugal tension will amount to one-third of the tensile strength of the belt. (I.C.E., *Feb.*, 1905.)

12. Show that the necessary condition for the proper running of belt gearing is, that the point at which the belt is delivered from any pulley shall be in the plane of the next pulley in the direction in which the belt is running. Apply this rule to show under what conditions two shafts, which are not parallel, and do not intersect, can be connected by a continuous belt without guide pulleys. Show also under what conditions it becomes necessary to use guide pulleys, and describe how the belt should be twisted so that the same side will be in contact with all the pulleys. (I.C.E., *Oct.*, 1905.)

13. Define an "open belt" and a "crossed belt," giving explanatory hand sketch. If D and d are the diameters of the pulleys, and l is the distance between their centres, show how to find the length of the belt in the case of a crossed belt. (I.C.E., *Feb.*, 1906.)

14. A pulley on a driving shaft is 36 inches in diameter, and makes 200 revolutions per minute. The tensions in the belt are 350 lbs. and 100 lbs. weight on either side of the pulley. Calculate the horse-power transmitted by the belt. If the belt is 1 square inch in section and weighs 0·50 lb. per foot run, find the total tension on the driving side of the belt when centrifugal action is taken into account. (I.C.E., *Oct.*, 1906.)

15. 50 H.P. have to be transmitted by a leather belt from a driving shaft placed 10 feet above and at right angles to the driven shaft. The driven shaft is to run twice as fast as the driving shaft. Describe and give hand sketches of the arrangement of pulleys and belt you would propose, marking approximate dimensions, the belt being 5 inches wide.
(I.C.E., *Feb.*, 1908.)

16. What are the chief sources of loss in belt gearing? Give an expression connecting the tensions on the two sides of a belt-driven pulley, the angle of contact and the frictional coefficient, and show how the efficiency of the drive is affected by alterations in the tensions. What are the practical limitations to these tensions in both directions?
(I.C.E., *Oct.*, 1908.)

17. An engine running at 50 revolutions per minute drives a main shaft through belt or ropes at 300 revolutions per minute. If 300 H.P. have to be transmitted, sketch the arrangement of drive you would propose and calculate the dimensions of the belt or ropes required.
(I.C.E., *Oct.*, 1908

LECTURE XVII.

CONTENTS. — Instantaneous Axis — Rittershaus's, Klein, and Massau's Methods for Determining the Inertia Forces in the Moving Parts of a Reciprocating Engine—Action of the Crank—Tangential and Radial Forces—Diagrams of Twisting Moments with Uniform and with Variable Steam Pressure on Piston, neglecting as well as taking Account of the Obliquity of Connecting-rod—Effect of Inertia of Moving Parts—Case of a Horizontal Engine with Connecting-rod of Infinite Length—Example I. —Indicator Diagrams as modified by Inertia—Graphic Representation of the Inertia—Case of a Horizontal Engine with Connecting-rod of Finite Length—Example II.—Position of Instantaneous Axis of Connecting-rod—Crank Effort Diagrams of "The Thomas Russell Engine" and of a "Triple-Expansion Engine" —Crank Effort Diagrams of the Quadruple-Expansion Five-Crank Engines of S.S. "Inchdune"—Example III.—Questions.

Inertia of the Reciprocating Parts of an Engine.—In the first place, the student should clearly understand how to find the ratio of the velocity of the piston to that of the crank-pin at any point, since the rate of change of velocity of the piston determines the inertia force.

Construction.—Draw O K and D L ⊥ to O D. Produce D C to K, and O C to L. Then L D represents the velocity of the piston to the same scale that L C does that of the crank-pin, since L is the instantaneous centre or axis of the connecting-rod, D C.

Hence, $$\frac{\text{Speed of piston}}{\text{Speed of crank-pin}} = \frac{\text{L D}}{\text{L C}}.$$

But the triangles L D C and O K C are similar.

$$\therefore \quad \frac{\text{Speed of piston}}{\text{Speed of crank-pin}} = \frac{v}{\text{V}} = \frac{\text{L D}}{\text{L C}} = \frac{\text{O K}}{\text{O C}}.$$

Or, $$\left\{\begin{matrix}\text{The velocity of cross-}\\ \text{head or piston, } v,\end{matrix}\right\} = \frac{dx}{dt} = \text{V}\frac{\text{O K}}{\text{O C}} = \text{V}\frac{h}{r} = \omega x \tan \varphi. \quad \text{(I.)}$$

Here, O C being the length of crank, it is constant, and may therefore represent, to scale, the constant linear velocity of the

crank-pin, whilst O K will represent, to the same scale, the linear velocity of the crosshead D or the piston. Also, the angular velocity of the crank-pin C, about its centre O, *is equal to* the angular velocity of the connecting-rod end C about the instantaneous axis L (see also Lecture V., Fig. 9).

Thus, $\dfrac{\text{Angular velocity of crank-pin}}{\text{Angular velocity of connecting-rod}} = \dfrac{\omega}{\Omega} = \dfrac{v/r}{v/\text{L C}} = \dfrac{\text{L C}}{\text{O C}}$.

Or, $$\frac{\Omega}{\omega} = \frac{\text{O C}}{\text{L C}}.$$

From similar triangles O C K and L C D,

$$\frac{\text{O C}}{\text{L C}} = \frac{\text{K C}}{\text{C D}}.$$

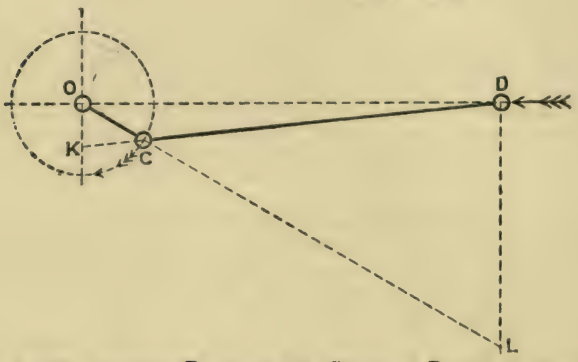


FIG. 1.—TO FIND THE RATIO OF THE SPEED OF PISTON TO THAT OF THE CRANK-PIN.

INDEX TO FIGS. 1, 2, AND 3.

Let O be Centre of crank-shaft.
,, C ,, Centre of crank-pin.
,, D ,, Centre of crosshead.
,, O C ,, Length of crank . . . . . $= r$.
,, D C ,, Length of connecting-rod . . . $= l$.
,, O D ,, Distance between centres O and D . $= x$.
,, O K ,, ⊥ to O D . . . . . $= h$.
,, K C ,, D C produced to K . . . . . $= k$.
,, D O C ,, at any position . . . . . $= \theta$.
,, O D C ,, at a relative position . . . . $= \phi$.
,, The linear velocity of crank-pin. . . . . $= \text{V} = \omega r$.
,, The linear velocity of piston . . . . . $= v$.
,, The angular velocity of crank-pin . . . . $= \omega = \dfrac{\text{V}}{r} = \dfrac{d\theta}{dt}$.
,, Instantaneous angular velocity of connecting-rod . $= \Omega$.

$$\therefore \frac{\left\{\begin{matrix}\text{Instantaneous angular}\\ \text{vel. of connecting-rod}\end{matrix}\right\}}{\text{Angular vel. of crank-pin}} = \frac{\Omega}{\omega} = \frac{OC}{LC} = \frac{KC}{CD} = \frac{k}{l}$$

Or,
$$\Omega = \frac{\omega k}{l}. \quad . \quad . \quad . \quad . \quad \text{(II.)}$$

But, *the acceleration at* D is $\alpha = \frac{dv}{dt} = \frac{d}{dt}(\omega x \tan \varphi)$.

Or,
$$\alpha = \omega\left\{\tan\varphi \frac{dx}{dt} + \frac{x}{\cos^2\varphi} \cdot \frac{d\varphi}{dt}\right\}. \quad \text{(III.)}$$

The angular velocity of the connecting-rod is—

$$\Omega = -\frac{d\varphi}{dt} = -\frac{\omega k}{l}. \quad . \quad \text{(IV.)}$$

Inserting this value of Ω and that of $\frac{dx}{dt}$ in (II.), we get—

$$\alpha = \omega^2\left\{x \tan^2\varphi - \frac{x}{\cos^2\varphi} \cdot \frac{k}{l}\right\}.$$

$$\therefore \left\{\begin{matrix}\text{The acceleration of cross-}\\ \text{head or piston, } \alpha,\end{matrix}\right\} = \omega^2\left\{\frac{h^2}{x} - \frac{x}{\cos^2\varphi} \cdot \frac{k}{l}\right\}. \quad . \quad \text{(V.)}$$

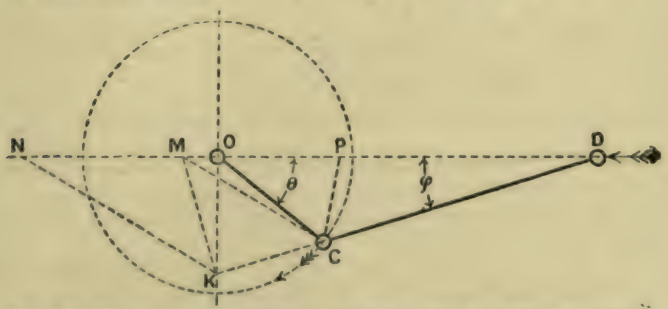


FIG. 2.—RITTERSHAUS'S INERTIA DIAGRAM FOR THE RECIPROCATING PARTS OF AN ENGINE.

Rittershaus's Graphic Construction of the Curve of Accelerations of the Reciprocating Parts of an Engine.—After having drawn the centre-line diagram of the crank and connecting-rod to scale, as in Fig. 1, then draw K M $\perp$ to K D, meeting the centre line of the engine in M (Fig. 2). Join C M, and draw K N parallel to C M, meeting the centre line of the engine in N.

If O C represents the radial acceleration $\omega^2 r = V^2/r$ of C, then it will be shown that M N – M O represents the acceleration of D to the same scale. For example, if r inches give length of crank as = 1 foot in length on the scale drawing, then r also represents the acceleration of ω^2 feet per second per second on the scale of acceleration.

Make M P = M N, and join P C.

Then O P C is a diagram of acceleration, just as O C K is a diagram of velocities, and the acceleration of D is—

$$a = OP = MN - MO.$$

By construction, $$\cos\varphi = \frac{KD}{MD} = \frac{OD}{KD} = \frac{x}{KD}.$$

$$\therefore \quad MD = \frac{KD}{\cos\varphi} = \frac{x}{\cos^2\varphi}.$$

Or, from similar triangles, $$\frac{NM}{MD} = \frac{KC}{CD} = \frac{k}{l}.$$

$$\therefore \quad MN = \frac{k}{l} \cdot MD = \frac{kx}{l\cos^2\varphi}.$$

But, $$\frac{MO}{OK} = \frac{OK}{OD}. \qquad \text{Or,} \quad MO = \frac{h^2}{x}.$$

$$\therefore \quad MN - MO = \frac{kx}{l\cos^2\varphi} - \frac{h^2}{x} = -\left(\frac{h^2}{x} - \frac{kx}{l\cos^2\varphi}\right) \text{(see Eq. V.)}$$

If M P = M N,

then, $$a = -\omega^2(MN - MO) = -\omega^2 \cdot OP.$$

So that O P represents the acceleration of D on the same scale on which O C represents the radial acceleration $\omega^2 r$ of C.

NOTE.—At the *two* dead centres, this construction obviously fails.

We shall now briefly describe Klein and Massau's method for ascertaining the inertia of the reciprocating parts of an engine,

Klein and Massau's Method—*Construction.*—With centre C and radius C K, describe a circle; and describe another on C D as diameter. Draw the common chord of these two circles, cutting O D in P.

As a matter of pure geometry, this point P is the same as P in Rittershaus's construction, and this construction does not, like the latter, fail when O C and C D lie in one straight line.

The student may prove this statement by drawing Figs. 2 and

3 having the respective lengths of the crank O C and connecting-rod D C the same in each case.

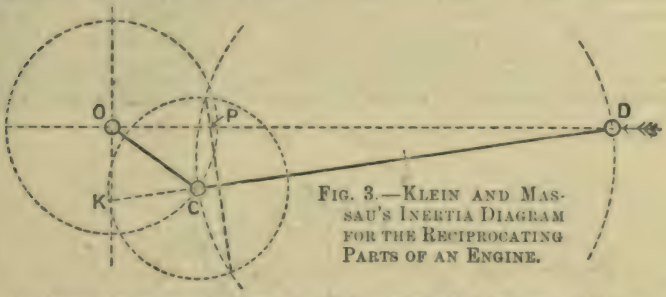


FIG. 3.—KLEIN AND MASSAU'S INERTIA DIAGRAM FOR THE RECIPROCATING PARTS OF AN ENGINE.

Action of the Crank—Tangential and Radial Forces.—In most steam engines the conversion of the reciprocating motion of the piston into circular motion is effected by means of the crank and connecting-rod.

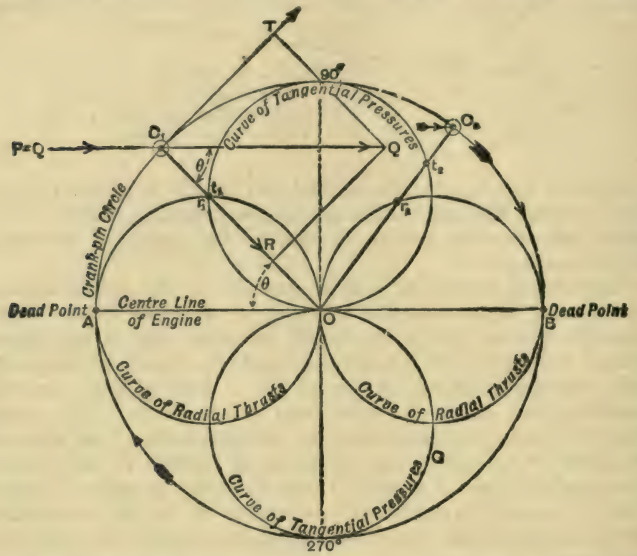
The turning or tangential force exerted by the connecting-rod on the crank varies with the position of the crank itself. Thus, when the centre line of the crank coincides with a line drawn through the centre of the cylinder and the centre of the crank shaft, the crank is said to be at the "dead points," and the connecting-rod exerts no rotational effort on it. The crank arrives in those positions twice in one revolution, just when on the point of reversing the direction of motion of the piston. These positions are O A and O B in the next diagram. Again, when the crank is at an angle of about 90° to the centre line through the cylinder and crank shaft, the tangential force is a maximum.

Diagram of Twisting Moments—Neglecting Length of Connecting-Rod.—The simplest case is that in which the pressure on the piston is uniform throughout the stroke, and the obliquity of the connecting-rod is neglected. Then the pressure or thrust, Q, on the connecting-rod is equal to the total pressure, P, on the piston (see next figure).

Suppose the crank to be in the position, O C_1, then by the parallelogram of forces the thrust, Q, on the connecting-rod may be resolved into two components, one, C_1 R, acting along the line of the crank and representing a radial pressure, R, on the crank-shaft bearing; the other, C_1 T, acting at right angles to O C_1, and representing the tangential pressure, T, acting on the crank pin. Of course, the *whole* thrust on crank-shaft bearing is equal to the *whole* pressure on piston.

Let the angle A O C_1 = θ, $\therefore$ $\angle$ Q C_1 O = θ, $\therefore$ C_1 Q is $\parallel$ to A O.
Then the radial pressure, R, or C_1 R = Q . cos θ.
And the tangential pressure, T, or C_1 T = Q . sin θ.

These components may be plotted out separately for every position of the crank by curves in the following manner:—Let OC_1 represent Q, to any convenient scale, and lay off to the same scale $Ot_1 = T = C_1T$, the tangential component of Q. Then t_1 is a point on the curve, and Ot_1 measures to scale the tangential pressure on the crank pin for the position, OC_1, of the crank. To find other points on this curve, take any other position of the crank and plot off along that line of the crank the tangential component of Q for that position. If we find a number of points and join them, they



POLAR CURVES OF TANGENTIAL FORCE (T) ON CRANK-PIN, AND RADIAL THRUST (R) THROUGH CRANK, WITH UNIFORM PRESSURE ON PISTON AND NEGLECTING OBLIQUITY OF CONNECTING-ROD.

will be found to lie on the circumference of two circles described with O to 90° and O to 270° as diameters. Similarly, if we lay off Or_1 on the position, OC_1, of the crank, equal to the radial component of Q, for that position of the crank, and do the same for a number of other positions, we have, by joining the points, two complete circles described with OA and OB as diameters, representing the radial thrust on the crank-shaft bearing for any position of the crank. In the position of the crank taken ($\theta = 45°$) the radial and tangential components are equal, and. therefore, r_1 coincides with t_1. These circles are known as "*Polar*" curves. For any other position, OC_2, of the crank, the tangential or turning force is given by Ot_2, whilst the radial thrust on the crank shaft is given by Or_2.

The TWISTING OR TURNING MOMENT OR TORQUE on the crank shaft at any position *is equal to the tangential pressure on the crank pin in that position multiplied by the length of the crank.*

Let r = radius of crank-pin circle or the length of the crank.

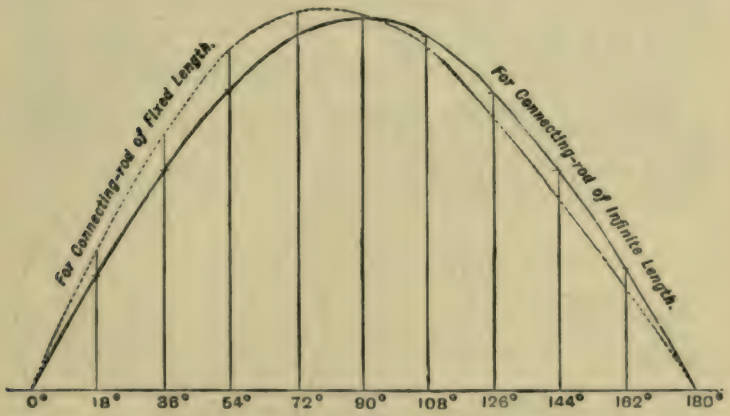
,, θ = the angle made by the crank with the line of dead points.

Then the twisting moment = $Q r \sin \theta$.

Or, ,, ,, = $P r \sin \theta$ (for in this case P = Q).

Since the polar curves, O to 90° and O to 270°, represent the tangential forces ($P \cdot \sin \theta$), their values must be multiplied by r, the length of the crank, in order to find the TWISTING MOMENT at any point; but, seeing that, r, is constant, the polar curves may be taken to represent the *relative* values of the twisting moments.

The twisting moments may also be represented by the following diagram, in which the horizontal line represents the path of the crank, and the height of each vertical ordinate gives the tangential force or the twisting moment for that point. To draw the diagram for one stroke of the piston, or one half revolution of the crank, lay off a horizontal line equal to the semi-circumference of the crank-pin circle, and divide it into 10 equal parts. Each division then represents a movement of 180 ÷ 10 = 18° of the crank.



DIAGRAMS OF TWISTING MOMENTS FOR ONE HALF REVOLUTION OF CRANK.
Both Curves are Drawn on the Assumption of Uniform Pressure on Piston.

Then calculate by the above formula, or plot out by the previous diagram of polar curves, tangential pressures for each of the 10 positions of the crank, and lay them off vertically at each division. Join these points, and we have the above full line curve which represents the twisting moments for one half revolution, neglecting the obliquity of the connecting-rod, and when the pressure on the piston is uniform throughout. A curve for the radial thrust through crank could be plotted out in the same way.

In the early days of the steam engine, it was imagined that the use of the crank and connecting-rod involved a considerable loss of the work developed on the piston. The fallacy of this idea may now be made clear.

The pressure on the crank-pin in the direction of rotation is = $P \sin \theta$.

therefore, in order to obtain the mean tangential pressure during a half revolution of the crank, we have only to find the mean value of sin θ, and multiply it by, P, the total mean pressure on the piston.

For an approximate result take the value of sin θ at every 10 degrees of the crank's movement and divide the total by 18, the number of divisions taken, thus—

Sin	10°	·173
,,	20°	·342
,,	30°	·500
,,	40°	·643
,,	50°	·766
,,	60°	·866
,,	70°	·939
,,	80°	·985
,,	90°	1·000
,,	100°	·985
,,	110°	·939
,,	120°	·866
,,	130°	·766
,,	140°	·643
,,	150°	·500
,,	160°	·324
,,	170°	·173
,,	180°	000
Total		11·428

$$\therefore P \times \frac{11{\cdot}428}{18} = P \times {\cdot}6349 = \text{(mean pressure)}.$$

Hence, if L = length of stroke = $2r$.

The work done on the crank in one revolution = Total mean pressure × distance passed through,

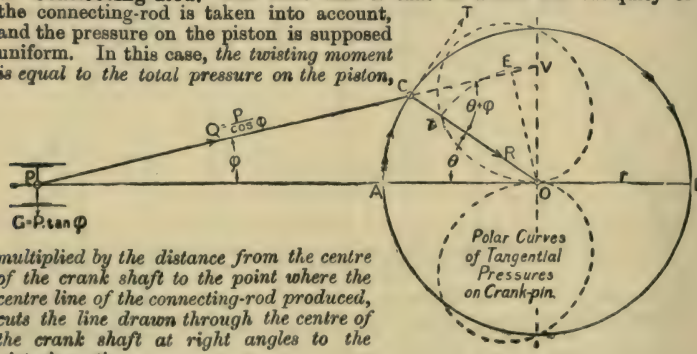
$= P \times {\cdot}6349 \times 2\pi r,$

$= P \times {\cdot}6349 \times 3{\cdot}1416 \times L = 1{\cdot}9946\ PL.$

Which is practically the same thing as 2 P L. But, the work done on the piston during one revolution is also equal to 2 P L. Consequently the employment of the crank and connecting-rod involves no loss of power if we neglect the power absorbed by friction due to bearing surfaces, &c.

NOTE.—No such combination of mechanism as the crank and connecting-rod can involve a loss of power (neglecting friction), as it would be contrary to the "principle of the conservation of energy."

Diagram of Twisting Moments—Taking Account of Length of Connecting-Rod.—The next case is that in which the obliquity of the connecting-rod is taken into account, and the pressure on the piston is supposed uniform. In this case, *the twisting moment is equal to the total pressure on the piston,*



multiplied by the distance from the centre of the crank shaft to the point where the centre line of the connecting-rod produced, cuts the line drawn through the centre of the crank shaft at right angles to the piston's motion.

DIAGRAM OF TWISTING MOMENTS—TAKING ACCOUNT OF LENGTH OF CONNECTING-ROD.

NOTE TO FIGURE.—The pressure along connecting-rod and on the crosshead guides may be found graphically for any position, thus—

Let P O = P, the pressure on piston to any convenient scale.

Then P V = Q, the direction and pressure along connecting-rod to the same scale.

And O V = G, the direction and pressure on the lower crosshead guide to same scale

$$\text{But } PV\cos\phi = PO \qquad \text{And } \frac{OV}{OP} = \frac{G}{P} = \tan\phi$$

$$\text{Or } \quad Q\cos\phi = P$$

$$\therefore Q = \frac{P}{\cos\phi} \qquad \therefore G = P\tan\phi$$

To prove this, let O be the centre of the crank shaft, and O P the centre line of the engine, passing through, O, and the centre of the cylinder. Let O C be the position of the crank, and P C the length of the connecting-rod. Produce P C to cut the vertical through O in the point V, and draw O E perpendicular to P V. Then $\angle$ V E O = $\angle$ P O V; also $\angle$ P V O is common:

$\therefore$ $\angle$ V O E = $\angle$ O P V = ϕ, the inclination of the connecting-rod to centre line of engine.

$$\text{Now the twisting moment} = Q \times OE = \frac{P}{\cos\phi} \times OV \cos\phi = P \times OV.$$

Knowing this, we can readily construct the polar curves. Suppose the crank in the position, O C, produce the centre line of the connecting-rod to cut the line O V in V. With centre, O, and radius, O V, describe the arc, V t, cutting O C in t. Then, t, is a point on the tangential pressures or twisting moment's curve, and the twisting moment for the position, O C, of the crank is thus P $\times$ O t. A similar construction for all the other positions of the crank gives all the other points, and the complete curve may then be described by joining them. We see that the curve is not a circle as in the last case, but differs therefrom in a marked degree. Now plot off the twisting moment at each of the 10 different points by this method on the rectangular diagram (page 371) as before, and we get the dotted line which shows the new diagram of twisting moments. It will be noticed that this curve rises more abruptly during the first quarter of a revolution, and falls flatter during the second quarter than when the obliquity of the connecting-rod is neglected, thus indicating a greater pressure on the crank pin during the first half of the stroke; also, the maximum pressure is reached before half stroke.

We can calculate the several twisting moments in this case without the aid of a scale diagram, thus—

$$\text{The pressure } Q = \frac{P}{\cos\phi}, \text{ also the angle } OCV = \theta + \phi,$$

Since O C P + O C V = 2 right angles, and O C P + θ + ϕ = 2 right angles.

$$\therefore \text{ tangential pressure on crank pin} = T = Q \cdot \sin(\theta + \phi) = \left(\frac{P}{\cos\phi}\right) \sin(\theta + \phi) = P \frac{\sin(\theta + \phi)}{\cos\phi},$$

$$\therefore \text{ the twisting moment} = Pr \frac{\sin(\theta + \phi)}{\cos\phi}.$$

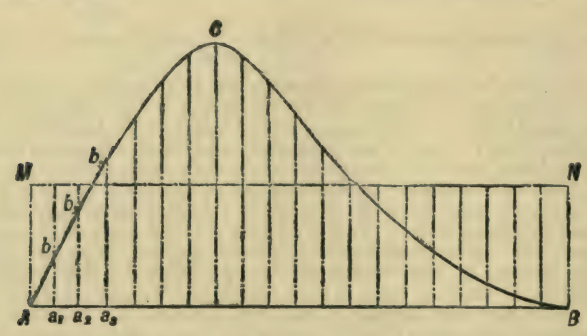
Where $r \sin\theta = l \sin\phi$; r being crank radius, and l the length of connecting-rod.

It is, however, more tedious to work out the results by this formula than by the previous graphic method.

The effect of shortening the connecting-rod is to increase the effort upon the crank pin at the beginning of the stroke, and to decrease it towards the end, thus causing greater irregularity in the tangential pressure on the crank, and greater stress on the crosshead guides. The pressure on the latter is = G = P $\cdot$ tan ϕ, as seen from the last figure and the footnote below it.

The actual state of things which takes place in practice is, however, not so easily represented, for the pressure on the piston is never uniform, but falls away from the point of cut-off. In order, therefore, to construct a truer diagram of the twisting moments, we must find the positions of the

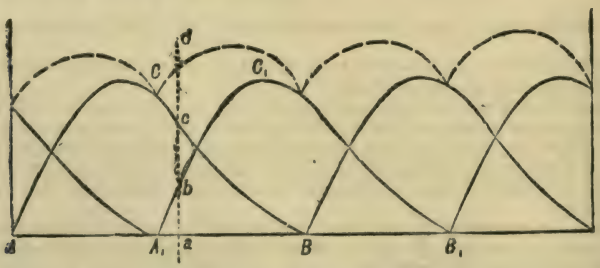
piston corresponding to the various positions of the crank by diagram (Lect. XIV. in my Advanced Steam Book), and mark these off on the indicator diagram. The steam pressure for the several positions of the crank can then be read off, and their values inserted for P in the equation $P \times O V$ or in $P r \sin(\theta + \phi) \div \cos \phi$. The curve of twisting moments on the crank due *merely* to steam pressure on the piston may then be constructed, as shown by the following diagram :—



CURVE OF TWISTING MOMENTS, TAKING ACCOUNT OF THE VARIATION OF THE STEAM PRESSURE IN THE CYLINDER, AND WITH A CONNECTING-ROD OF KNOWN LENGTH.

On comparing this curve with the two previous curves, it will be seen that between the points, C and B, it falls much lower ; this is due to the fall of steam pressure during expansion. The rectangle, A M N B, is of the same area as the figure, A C B, and, therefore, A M represents the *mean* twisting moment due to steam pressure on the piston.

When the engine has two cylinders having their pistons working on separate cranks, the curve of total twisting moments on the crank shaft can only be obtained by combining the curves of twisting moments for each crank. This is shown in the following figure, which represents the combined twisting moments on the crank shaft of an engine with two cranks at right angles to each other :—



CURVE OF COMBINED TWISTING MOMENTS FOR TWO CRANKS AT RIGHT ANGLES TO EACH OTHER.

A C B is the curve of twisting moments on one crank, and $A_1 C_1 B_1$ the curve of twisting moments on the other crank during one-half revolution, the remaining curves being for the other half revolution. To find the total twisting moment at any point, *a*, draw the vertical line *a d*, and make $a\,d = a\,c + a\,b$ (*i.e.*, the sum of the twisting moments on each crank). By finding a number of points in this way, the whole curve of total twisting moments may be plotted out.

Effect of Inertia of Moving Parts.—In finding the twisting moments by these methods, we have neglected a most important effect—viz., the variation of effort on the crank shaft due to the inertia of the moving parts. Since the piston is brought to rest at the end of each stroke, the inertia of the piston, piston-rod, crosshead, and connecting-rod, has to be overcome at the beginning of each stroke, in order to start the motion, and a portion of the energy of the steam is absorbed in doing this; therefore, the actual effort on the crank in the first half of the stroke is *less* than that given by the curves. The energy which is imparted to the moving parts is, however, given out on the crank during the latter part of the stroke, when these moving parts are being brought to rest; therefore, the effort on the crank during the second half of the stroke is *greater* than that shown by the curves. On the principle of the conservation of energy this alternate acceleration and retardation can neither add to, nor subtract from, the total power developed during the stroke. In ordinary cases, therefore, the inertia of the moving parts acts as a fly-wheel would do, and tends to equalise the effort on the crank. The effect, however, at different parts of the stroke is very interesting and instructive, especially when high-initial pressure and a large range of expansion are adopted, combined with heavy and quickly-moving parts. Allowance may be made for this inertia, if the weight of the moving parts and their velocity are known. We can make an alteration on the indicator diagram, reducing the effective pressure at the beginning of the stroke and increasing it at the end. The steam and the inertia stresses can, however, be combined, only so far as some of the effects are concerned. They are combined, of course, in their pressure on the crank pin, &c., but since the dynamical stresses are not taken up altogether by the engine framing, provision must be made for transmitting them to the engine foundation. These dynamical stresses, introduced by arresting the momentum of the moving parts, produce a much more serious effect in fast-running engines than is usually supposed.

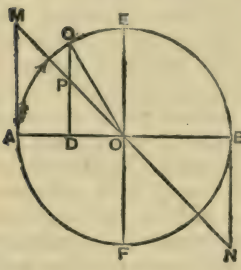
Effect of Inertia on Moving Parts of a Horizontal Engine with Connecting-Rod of Infinite Length.

The student should now try to follow the following investigations and practical examples.

All reference to trigonometry, co-ordinate geometry, and the differential calculus has been dispensed with; and any student with a knowledge of elementary geometry and mechanics will, by carefully following the argument, grasp this subject sufficiently for all practical purposes.

It may be well to state that we shall not take into account *all* the effects resulting from the conversion of reciprocating into circular motion by means of a crank and connecting-rod, but only those effects which directly influence the propelling force on the piston of the engine,* and unless otherwise stated, we shall suppose the engine to be horizontal.

The first case is that in which the motion of the piston is the same as if the engine worked with a connecting-rod of infinite length, such as, for example, in the common donkey engine arrangement where the crank-pin works in a slotted cross-head at right angles to the direction of the piston's motion.



In the figure, let O, be the centre of the crank shaft, O C, the crank, and, A B, the centre line of a horizontal engine of stroke, A B.

From any position of the crank-pin, C, draw the ordinate, C D, at right angles to A B.

Then when the crank-pin is at, C, the piston has evidently moved a distance, A D, and the speed of the piston is always that of the point, D.

When a point, such as D, moves on a diameter so as to always be at the foot of the ordinate drawn through a uniformly revolving point as, C, D, is said to move *harmonically* with, C.

We have to find what force must act on the piston at each moment to make it move thus harmonically.

Now a heavy point, C, moving uniformly in a circle, may be supposed to have its motion compounded of a harmonic motion in the direction, A B, and a harmonic motion in the direction, E F, at right angles to, A B. Each of the forces producing these motions taking effect in its own direction, irrespective of forces and motions at right angles to it.

That is, if, D, be a point of equal weight with, C, the force moving, C, horizontally at any moment is the same as that moving, D, because the horizontal velocity of, C, is always equal to that of, D. The force acting vertically on, C, at any moment makes no difference horizontally.

But the resultant of the vertical and horizontal forces on, C, is the force which compels it to move in a circle, that is, it is equal and opposite to what is commonly termed the *centrifugal force* of, C.

* See Paper by the late Prof. Fleeming Jenkin, published in the *Transactions of the Royal Society of Edinburgh*, vol. xxviii. (1879), p. 703, in which will be found a rigorous analysis of the forces in a reciprocating steam engine of practical proportions, accompanied by curves of crank-pin effort.

Therefore the force on the piston making it move harmonically on, A B, is *the horizontal component of the centrifugal force the piston would exert if its weight were concentrated at the centre of the crank-pin and revolved with it.*

If, C O, represents this centrifugal force, then, D O, represents the accelerating force when the piston is at, D.

At the beginning of the stroke, D O = A O. ∴ Erect the perpendicular, A M = A O, to represent the accelerating force when the piston is at, A.

At half stroke, D O = zero.

At any intermediate point, the ordinate, D P, intersected by the straight line, M O, equals, D O, the accelerating force for the position, D.

From half-stroke to the end, the acceleration is negative, and the ordinates must be measured below the line of abscissae. The triangle O B N, equal to, O A M, represents the retarding forces.

The algebraic sum of the two triangles, on the principle of the conservation of energy is zero, because the mass starts from rest and comes to rest again. So the inertia merely affects the *distribution* of power during the stroke, not its amount.

We see that the accelerating force is greatest at the ends of the stroke where the motion is slowest, and is *nil* at half stroke where the motion of the piston is fastest. It is not the velocity, but the *rate of change of velocity* which demands the accelerating force.

We have to find the numerical value of the centrifugal force.

This is given in all elementary treatises on mechanics as—

$$\frac{W v^2}{g r} = \frac{m v^2}{r}.$$

Where, v, is velocity in feet per second, r, is radius of circle in feet, and, m, mass in units of mass.

Engineers do not use this notation, for they speak of so many revolutions per minute in a circle of so many feet radius, and they measure mass by weight.

So $m = \frac{W}{g} = \frac{W}{32}$; W being in lbs., and g the acceleration of gravity.

Let N = number of revolutions per minute.

Then,
$$\frac{m v^2}{r} = \frac{W}{32 r} \cdot \left(\frac{2 \pi r N}{60}\right)^2$$

$$,, \quad = \cdot 000341 \; W r N^2.$$

This is a well-known formula for the centrifugal force of a body.

We are now in a position to correct the indicator diagram for any engine, and to say how much less the pressure on the crank-pin is, than that on the piston at the beginning, and how much greater at the end of the stroke.

EXAMPLE I.—The stroke of an engine with a slotted crosshead is 4 feet, the diameter of the cylinder is 20 inches. The effective pressure is 40 lbs. at the beginning and 20 lbs. at the end of the stroke, the weight of the reciprocating mass is 1,256 lbs. What are the pressures on the crank-pin at the beinning and end of the stroke, at 50 revolutions per minute?

Area of a cylinder 20 inches diameter = 314 square inches.

Accelerating force at ends of stroke from the previous formula.

$$\cdot 000341\ W\ r\ N^2 = \cdot 000341 \times 1{,}256 \times 2 \times 2{,}500 = 2{,}141 \text{ lbs.}$$

Pressure on piston at beginning of stroke = 314 × 40 = 12,560 lbs.
Pressure on crank-pin at beginning of stroke 12,560 − 2,141 = 10,419 lbs.
Pressure on piston at end of stroke = 314 × 20 = 6,280 lbs.
Pressure on crank-pin at end of stroke = 6,280 + 2,141 = 8,421 lbs.

The pressure on the crank-pin is thus much more equable than would be the case if the parts were not possessed of inertia.

Suppose it is required to make the pressure at the beginning exactly equal to that at the end of the stroke, the weight of the parts being unaltered, but the speed changed. What number of revolutions will make the accelerating force at beginning and end of stroke, equal to half the difference of the greatest and least pressures on the piston?

Greatest pressure, . . .	=	12,560 lbs.
Least, ...	=	6,280 lbs.
Half difference, . .	=	3,140 lbs.

Let, x, be the number of revolutions required.

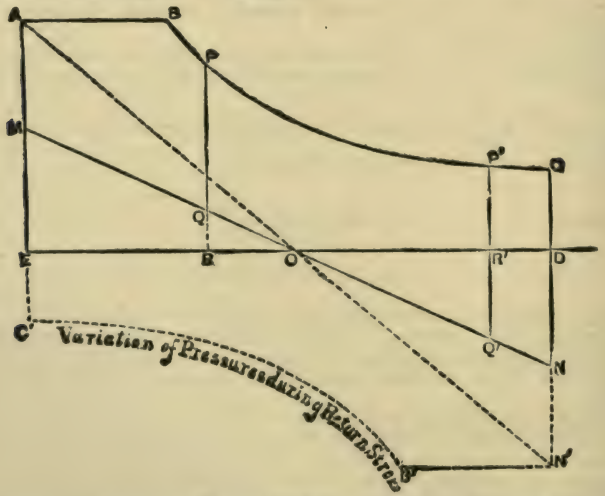
Then, $\cdot 000341 \times 1{,}256 \times 2 \times x^2 = 3{,}140$, whence $x = 60$ revs. nearly.

Suppose we are restricted to 50 revolutions, but may vary the weight of the reciprocating parts to obtain the same result.

Let, W, be the weight required.

Then, $\cdot 000341 \times W \times 2 \times 2{,}500 = 3{,}140$, $\therefore$ W = 1,840 lbs.

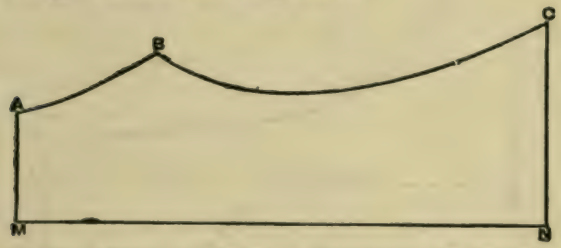
Indicator Diagrams as Modified by Inertia.— To find the pressure as modified by the inertia for *any* point of the stroke, take, A B C D E, as the indicator diagram of an engine.



THEORETICAL INDICATOR DIAGRAM CONVERTED INTO A STRESS DIAGRAM ON CRANK-PIN WITH CONNECTING-ROD OF INFINITE LENGTH.

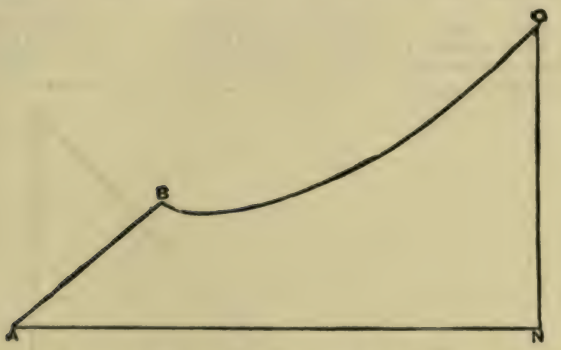
Then to the same scale to which, A E, represents initial steam pressure × area of piston, draw, E M and D N, to represent accelerating force at ends of stroke. Join M and N by the straight line M O N.

Then, M E O D N, is the inertia diagram, and, M O N, thus becomes the virtual base of the figure, from which pressures are to be measured instead of E D. With these heights measured from a horizontal base line the indicator diagram takes the following form :—



PREVIOUS INDICATOR DIAGRAM CORRECTED FOR INERTIA.

Suppose by increasing the speed of the engine or the weight of the reciprocating parts, the ordinate, E M, becomes equal to, A E, that is, the accelerating force is equal to the whole pressure of the steam on the piston : then there is no pressure on the crank-pin when the engine is on the centre, and as the piston advances the pressure will gradually increase, and become excessive towards the end, even with such a comparatively early cut-off as shown on the diagram in question. The corrected card is as follows :—



PREVIOUS INDICATOR DIAGRAM CORRECTED FOR INCREASED INERTIA.

With very high speeds or heavy parts the ordinate, E M, may be greater than, E A. The piston will then at the beginning of the stroke lag behind the crank, and be dragged until the acceleration ordinate and steam ordinate become equal. Indeed, with an early cut-off the piston may drag again at a later period of the stroke, as shown in the next figure.

Here the inertia line, M O N, cuts the steam line at K, L, and R. From, M to K, the pressure is negative, and power represented by area,

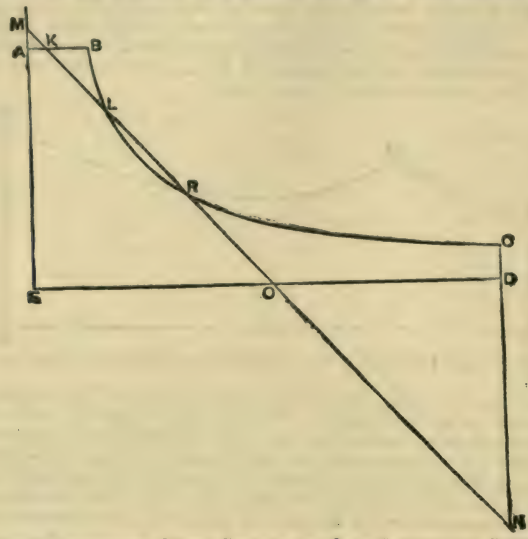
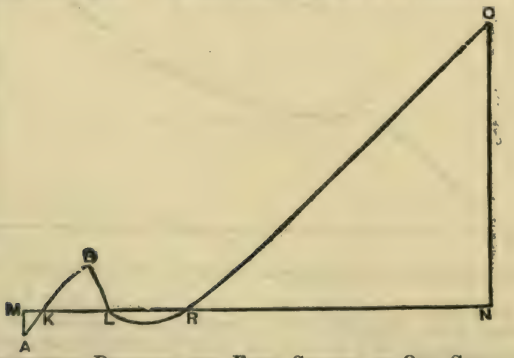


Diagram showing Four Shocks in One Stroke of Piston.

M A K, must be spent on the piston. From, K to L, the piston urges the engine, doing work on the crank equal to, K B L. From, L to R, the piston again drags and absorbs work equal to the loop, L R: and thereafter the piston drives the engine. The corrected diagram is as follows:—



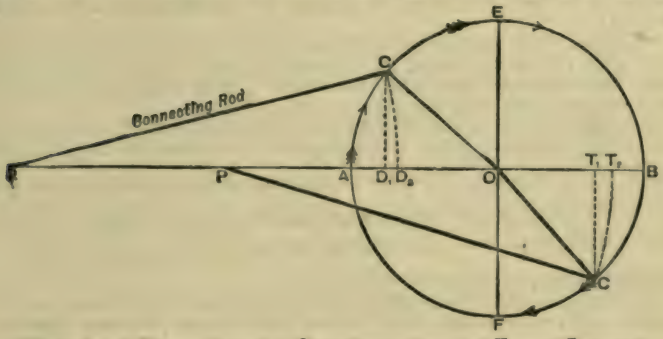
Corrected Diagram for Four Shocks in One Stroke.

It is needless to say that such a condition is not desirable in practice.

We now pass on to the more usual case of engines working with a connecting-rod of finite length.

Connecting-rod of Finite Length.—Let, P C, be the connecting-rod and, C O, the crank.

Then at the commencement of the out-stroke from. A to B, the motion of the piston is more rapid than in the pure harmonic motion, the travel being for a certain position of crank-pin, A D_2, instead of, A D_1. At the

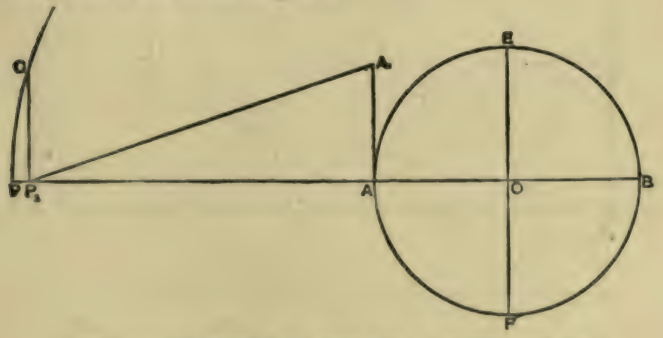


MOTIONS OF PISTON WITH A CONNECTING-ROD OF FINITE LENGTH.

beginning of the return stroke from, B to A, the speed is less than for harmonic motion, the travel corresponding to the position of crank-pin shown being, B T_2, instead of, B T_1.

It is plain that the mere raising or lowering the big end of the connecting-rod in a vertical line, will make, P, approach, O, with a certain acceleration. And the force necessary for this, must be added to that required for true harmonic motion when the engine is on the 'near' centre, A, and subtracted for the 'far' centre, B.

Now, what is this accelerating force?



ACCELERATING FORCE AT INNER DEAD CENTRE DUE TO A CONNECTING-ROD OF FINITE LENGTH.

In the above fig., P A, is the connecting-rod as before. We suppose the end, A, to ascend the vertical line, A A_1; then, P, will be drawn to, P_1, and, P_1 A_1, is a position of the connecting-rod.

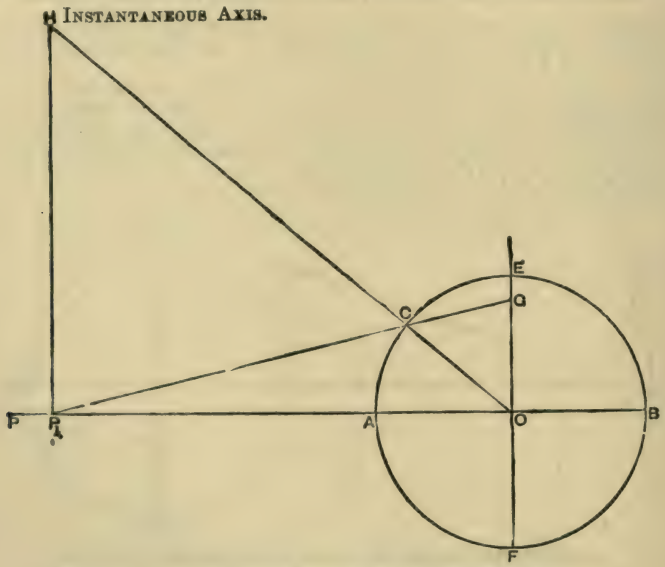
With centre, A, describe the arc, P G. Now we are only concerned with the velocity of, A, *at the moment it leaves the line, P O:* what its velocity is afterwards cannot affect the acceleration of, P, at that moment.

Suppose, therefore, the point, A, to ascend, A A_1, harmonically with the point, G, revolving uniformly in the circle, P G, with the velocity of the crank-pin. Then, obviously, P_1, moves also harmonically with, G, on line, P O.

Therefore, when the big end of the connecting-rod leaves the centre line, P O, with the said velocity, the accelerating force on, P, is the centrifugal force it would have when moving in a circle with radius equal to the connecting-rod, with the velocity of the crank-pin.

But with a given linear velocity, the centrifugal force is inversely as the radius: therefore, if the connecting-rod is, n, times the length of the crank, the accelerating force due to the connecting-rod will be, $\frac{1}{n}$*th*, that due to the crank, and the net accelerating force when the engine is on the near centre will be, $1+\frac{1}{n}$, and when on the far centre, $1-\frac{1}{n}$, times that of an engine with pure harmonic motion.

EXAMPLE II.—The reciprocating parts of an engine weigh one ton. The stroke is 3 feet 6 inches, and the revolutions 80 per minute. The con-



POSITION OF INSTANTANEOUS AXIS OF CONNECTING-ROD, &c.

necting-rod is 7 feet long. What is the accelerating force on the near and far centres?

Here $n = 4$. Therefore the accelerating force on near centre will be—
·000341 W r N².

$$= \cdot 000341 \times 2{,}240 \times 1{\cdot}75 \times 6{,}400 \times \frac{5}{4} = 10{,}700 \text{ lbs.}$$

And on far centre $10{,}700 \times \frac{3}{5} = 6{,}420$ lbs.

It is evident that the accelerating forces, when the engine is on the "dead points," are by far the most important; first, because they are there greatest, and secondly, because the motion of the piston changes its direction there.

We may, however, investigate one other point in the inertia diagram, viz.:—that at which the acceleration is *nil*, or the point where the line corresponding to, M O N, in the figures for an infinite connecting-rod, cuts the base line. (See fig. on previous page).

Obviously the acceleration is *nil* when the speed of the piston is greatest.

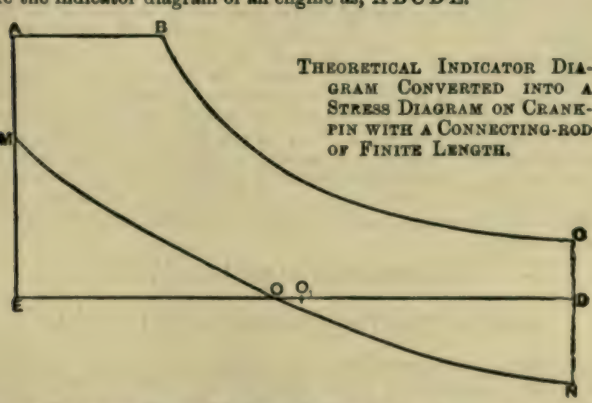
Let, P_1 C, be any position of the connecting-rod. Produce, P_1 C, to cut O E, in, G. Draw, P_1 H, at right angles to, P O, and produce, O C, to cut, P_1 H, in, H. (See fig. on previous page).

The connecting-rod at any moment is moving about an *instantaneous axis:* and every point in it is, of course, moving at right angles to the line joining it to this axis. The axis is, therefore, somewhere in the line, P_1 H, and also somewhere in the line, O C H, for, C, is moving at right angles to, O C.

Therefore, H, *is the instantaneous axis*, and the velocities of, P_1 and C, are in the ratio, H P_1 to H C. That is, by similar triangles, the ratio, O G to O C. O G, therefore represents *the velocity of the piston.*

By drawing the length of the connecting-rod on a piece of tracing paper and applying it to the diagram, keeping, P_1, in the line, P O, and, C, in the circumference, a position of, P_1, can be found for which, O G, is a maximum. To find this position of, P, by calculation requires the use of the higher mathematics, which is purposely avoided here.

Take the indicator diagram of an engine as, A B C D E.



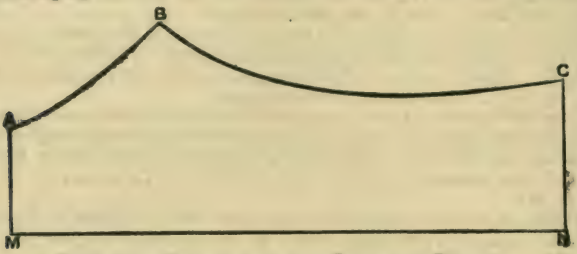
THEORETICAL INDICATOR DIAGRAM CONVERTED INTO A STRESS DIAGRAM ON CRANK-PIN WITH A CONNECTING-ROD OF FINITE LENGTH.

Suppose the connecting-rod is 4 cranks long, a very common proportion. Find, by trial, with a piece of tracing paper, as above, the position, O, of the piston when its velocity is greatest. With this proportion of

connecting-rod, O, will be found at a distance, OO_1, from mid-stroke a little more than $\frac{1}{10}$ of the half stroke, O_1E. Mark off to the proper scale, $EM = 1\frac{1}{4}$ times the centrifugal force of a weight equal to that of reciprocating parts revolving with the crank-pin, and, $DN = \frac{3}{4}$ of that force.

(If the connecting-rod were 5 cranks long these amounts would be respectively $1\frac{1}{4}$ and $\frac{3}{4}$ of the force).

Draw a fair curve through, M O N, bearing in mind that the triangles, M O E, O D N, must be equal in area, representing as they do the same, *vis viva*. Though not mathematically exact, this line, M O N, will very approximately represent the inertia diagram: and the pressures on the crank-pin are measured by vertical lengths, intercepted between, A B C and M O N. Placed on a horizontal base, the amended diagram becomes the following figure.



CORRECTED FIGURE FOR THE PREVIOUS DIAGRAM.

It will be seen that a considerable error in the line, M O N, will not greatly affect the shape of the figure, provided the points, M and N, are accurately determined by the method given above. The point, O, has been referred to, not because it is important, but in order to show the student that it is unimportant.

We have hitherto supposed all the reciprocating weights as having the same motion as the piston, and as being concentrated at, P or P_1, the centre of the crosshead. This supposition introduces no error into the estimation of accelerating forces caused by the motion of the crank alone: but in the case of the connecting-rod it will be seen that the motion of its *centre of gravity* caused by moving the "big end" off the centre line (see lower fig. on p. 381), is not the motion of, P_1, towards, A, but one-half that amount, supposing the centre of gravity to be at the middle of the rod. Therefore, strictly, if, W, be the weight of all purely reciprocating parts, such as piston, piston-rod, and crosshead, and, w, the weight of the connecting-rod, the accelerating force on the centre instead of being

$$\cdot000341\ (W + w)\ r\ N^2\left(1 \pm \frac{1}{n}\right)$$

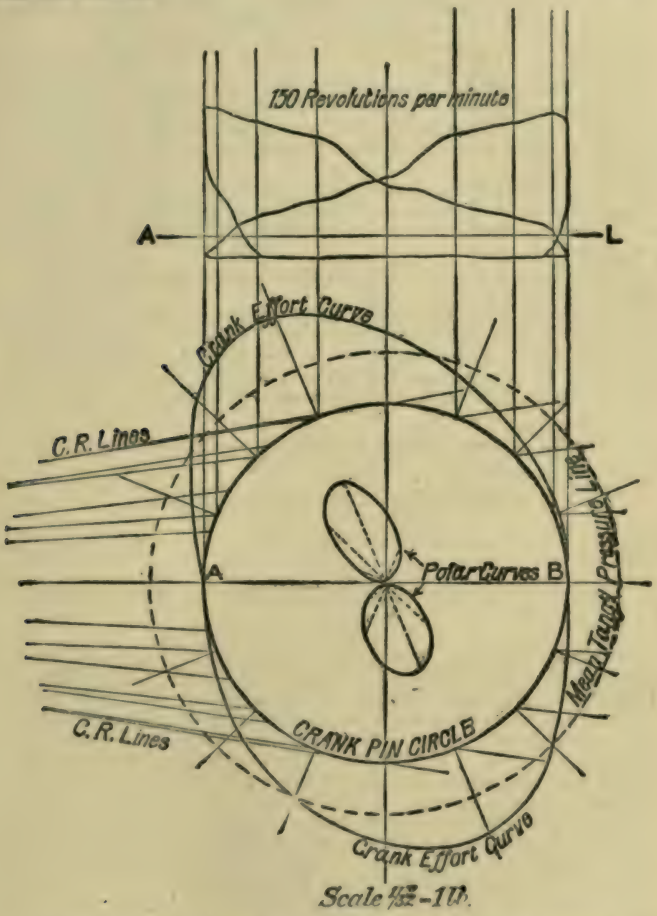
is,

$$\cdot000341\ W\ r\ N^2\left(1 \pm \frac{1}{n}\right) + \cdot000341\ w\ r\ N^2\left(1 \pm \frac{1}{2n}\right).$$

If instead of being horizontal the engine be vertical, the effect of inertia is just the same, but the pressure on the crank-pin will be further affected by the weight of the reciprocating parts, which must be added at every point of the down stroke, and subtracted for the up stroke.

In the case of a diagonal engine, the vertical component of the weight must be so added or subtracted.

Crank Effort Diagrams. — We shall now explain, by aid of the following figure, how to construct a crank effort diagram, when the obliquity of the connecting-rod and the varying pressures on the piston are taken into account.*



CRANK EFFORT CURVES OF "THE THOMAS RUSSELL ENGINE."

* The front and back indicator diagrams (from which the above crank effort curve has been drawn) were taken from "The Thomas Russell Experimental Steam Engine" in the author's laboratory. The diameter of the cylinder = 6 ins.; length of stroke = 12 ins.; length of connecting-rod = 36 ins.; revolutions per minute = 150.

(1) Transfer the indicator diagrams from the cards to a sheet of paper, taking care to make them at least 4 inches in length for clearness.

(2) Some distance below the redrawn diagrams put down a line, A B, parallel to the atmospheric line and equal in length to the diagrams.

(3) Upon A B, as a diameter, draw the crank-pin circle and divide the same into any convenient number of equal parts.

(4) Project each of these points of division vertically upwards, so as to cut the indicator diagrams.

(5) For the first point to be considered on the crank-pin circle, take that which is vertically over the centre of the crank. Measure the pressure on the piston (from the indicator diagram) corresponding to this point and plot it along the centre line of the connecting-rod as produced through this point.

(6) Resolve this pressure (as plotted along the centre line of the connecting-rod) into two forces at right angles to each other—viz., one along the crank centre line and the other at right angles to it—*i.e.*, tangentially to the crank-pin circle.

(7) Do precisely the same for each of the other positions into which the crank-pin circle is divided.

(8) Produce the several centre lines of the crank for each of these positions and plot off on each centre line, from the crank-pin circle, the corresponding tangential pressure to the same scale as the indicated diagrams. By joining these points the crank effort diagram is obtained.

(9) *To Find the Mean Tangential Pressure Line*—

Let P_m = Mean pressure on piston (from indicator diagrams).

„ r = Radius of crank.

„ P_t = Mean tangential pressure on crank-pin.

Then $$P_m \times 2r = P_t \times \pi r$$

$$\therefore \quad P_t = \frac{P_m \times 2r}{\pi r} = \frac{2P_m}{\pi} = \frac{P_m}{1{\cdot}57}$$

(10) In drawing the "crank effort curves" for compound, triple, or quadruple expansion engines, the several indicator diagrams have to be reduced to one scale.* Then the several steam pressures (for the several positions assumed on the crank-pin circle), *in the case of the intermediate and low-pressure cylinders*, must be multiplied by the ratios of the areas of those cylinders with respect to the area of the high-pressure cylinder *before* resolving their respective piston pressures along the centre lines of the several positions occupied by their connecting-rods. We have been fortunate in securing, for a practical example with which to illustrate this case, copies of the indicator diagrams taken on the trial trip of a steamer recently built by one of the best Clyde firms. There was nothing abnormal about the diagrams; consequently we need not reproduce them on the combined diagrams to one scale, from which we obtained the following set of "crank effort curves." The "total crank effort curve" is obtained by summing up the pressures on the three-crank effort curves along the respective radial ordinates to the tangential pressures on the crank-pin circle. The "mean crank effort curve" is found, as explained under (9). The chief data connected with this case is given immediately below the figure.

*See Lecture XVI. in the Author's *Text-Book on Steam and Steam Engines, including Turbines and Boilers.*

CRANK EFFORT DIAGRAM OF A TRIPLE EXPANSION ENGINE.

Ratio of expansion, 10·4

Length of connecting-rod = 9 feet. Stroke = 4·5 feet.

Mean efficiency of steam, or ratio of area of work in cylinder to full theoretical diagram, 55 per cent.

	H.P.	I.P.	L.P.
Cylinder's diameter, ...	28″	46″	77″
Area, ...	615·8	1661·9	4656·6
Ratio, ...	1	2·80	7·56
Mean pressures, lbs. per sq. in., .	67·6	28·2	9·7
Range of temperatures, Fah., .	64·3°	74·9°	80·6°

Steam, 164 lbs.; Vacuum, 26½ ins.; Receivers, 52 and 5 lbs.; Revolutions, 62½ per minute.

Cut-off H.P. = 33½ ins.	I.H.P. = 710 L.P.
,, I.P. = ,,	,, = 799 I.P.
,, L.P. = ,,	,, = 764 H.P.
	2273 total I.H.P.

Crank Effort Diagram of the Quadruple-Expansion Five-Crank Engines of S.S. "Inchdune."—The description of these engines is given in my Advanced Steam Book. The necessary calculations with explanations and the drawings from which the accompanying folding-plate has been directly reproduced by photography were kindly supplied to the author by his old student, Mr. William C. Borrowman, M.Inst.C.E., General Manager of the Central Marine Engineering Works, West Hartlepool, who designed the arrangement of engines and boilers.

The following calculations show clearly how the foregoing educational illustrations and descriptions are applied in practice. The results thus obtained were highly gratifying to the designer, not only in the even subdivision of the powers derived from the five cylinders, but in the mean crank effort diagram, comparative freedom from stresses and vibration; but also in the unprecedented economy in steam and coal of these engines and boilers. (*See the frontis folding-plate.*)

S.S. "Inchdune" Engines.

Preliminary Data.—

Diameter of cylinders, . . 17, 24, 34, 42, 42 inches.
Length of stroke, ... 42 „
Diameter of piston-rods, ... 4½ „
Length of connecting-rod, ... 7½ feet.
Boiler pressure = 265 lbs. by gauge, or 280 lbs. absolute.
Vacuum by gauge, ... = 27 inches.

Method of Calculating Inertia and Gravitating Effect of the Reciprocating Parts—viz., piston, piston-rod and ½ connecting-rod. (The crank-pin, webs, and ½ connecting-rod are supposed to balance each other.)

$$\text{Inertia effect for an infinite connecting-rod} = \frac{W v^2}{g r} = \frac{v^2}{g} = \frac{(2\pi \times N)^2}{(60)^2 \times 32{\cdot}2}$$

$$\text{„ „} = \frac{(2\pi N)^2}{32{\cdot}2 \times 60^2} = \frac{4 \times 22 \times 22 \times 100^2}{7 \times 7 \times 32{\cdot}2 \times 60^2}$$

$$\text{„ „} = 3{\cdot}4056.$$

When W = 1 ton (weight of reciprocating parts).
r = 1 foot = radius of crank-pin circle.
N = 100 revolutions per minute.
v = velocity of crank-pin in feet per second.

Correction of inertia effect for finite rod $\Big\}$ = $\Big\{$ Inertia effect for an infinite connecting-rod × constant.

$$\text{,,} \qquad \text{,,} \qquad = 3{\cdot}4056 \times \cos\theta + \frac{n^2 \cos 2\theta + \sin^4\theta}{(n^2 - \sin^2\theta)^{\frac{3}{2}}} = \pm f.$$

When θ = angle made by crank with line of stroke.

l = length of connecting feet in feet.

r = length of crank in feet.

$$n = \frac{\text{length of connecting-rod}}{\text{length of crank}} = \frac{l}{r}.$$

But, as inertia effect increases directly as W, r, and N^2,

$$\therefore \qquad \pm f\left(W \times r \times \frac{N^2}{(100)^2}\right) = \pm f \times \text{a constant.}$$

In this case, $n = \dfrac{\text{length of connecting-rod}}{\text{length of crank}} = \dfrac{l}{r} = \dfrac{7{\cdot}5}{1{\cdot}75} = 4{\cdot}285.$

N = revolutions per minute = 70.

l = length of connecting-rod = 7·5 feet.

r = radius of crank = 1·75 feet.

Then, $$\text{Inertia effect} = \pm f\left(Wr\,\frac{N^2}{(100)^2}\right)$$

$$\text{,,} \qquad \text{,,} \qquad = \pm f\left(W \times 1{\cdot}75 \times \frac{70 \times 70}{100 \times 100}\right).$$

Hence, Total effect of inertia and gravitation, F $\Big\}$ $$= \pm f\left(W \times 1{\cdot}75 \times \frac{70 \times 70}{100 \times 100}\right) + W.$$

(For down stroke.)

$$\text{,,} \qquad \text{,,} \qquad \text{,,} \qquad = \mp f\left(W \times 1{\cdot}75 \times \frac{70 \times 70}{100 \times 100}\right) - W.$$

(For up stroke.)

Cylinder Number.	Weight of Reciprocating Parts in Tons.	Total Inertia and Gravitation Effect.
1	·9569	$\pm (f \times \cdot 8205) \pm \cdot 957$
2	1·108	$\pm (f \times \cdot 9501) \pm 1{\cdot}108$
3	1·437	$\pm (f \times 1{\cdot}2327) \pm 1{\cdot}437$
4F	1·4634	$\pm (f \times 1{\cdot}2549) \pm 1{\cdot}463$
4A	1·59	$\pm (f \times 1{\cdot}3634) \pm 1{\cdot}590$

Crank Effort Diagrams for No. 1 Cylinder of S.S. "Inchdune" during the Down Stroke.

Crank Angle in Degs.	Crank Leverage in Inches.	Steam Pressure in Lbs. per Sq. Inch Absolute.	Total Pressure on Piston in Tons.	f		Constant.		Inertia in Tons.		Weight of Reciprocating Parts, W, in Tons.		Gravity and Inertia Effect, F, due to Reciprocating Parts, in Tons.*	Turning Moment, TM, in Inch-Tons, due to Gravity and Inertia only.	Turning Moment, TM, in Inch-Tons, due to Steam only.	Total Turning Moment, TM, in Inch-Tons, due to Gravity, Inertia and Steam.
0	0	147	14·890	−4·200	×	·8205	=	−3·446	+	·957	=	−2·489	0	0	0
10	4·48	146	14·790	−4·102	×	·8205	=	−3·366	+	·957	=	−2·409	−10·792	66·258	55·466
20	8·74	140	14·182	−3·815	×	·8205	=	−3·130	+	·957	=	−2·173	−18·992	123·951	104·959
30	12·61	134	13·574	−3·358	×	·8205	=	−2·755	+	·957	=	−1·798	−22·672	171·171	148·499
40	15·94	127	12·865	−2·759	×	·8205	=	−2·264	+	·957	=	−1·307	−20·833	205·069	184·236
50	18·52	122	12·359	−2·060	×	·8205	=	−1·690	+	·957	=	−·733	−13·575	228·881	215·306
60	20·34	118	11·953	−1·299	×	·8205	=	−1·066	+	·957	=	−·109	−2·217	243·132	240·915
70	21·33	116	11·751	−·546	×	·8205	=	−·448	+	·957	=	+·509	+10·857	250·645	261·502
80	21·52	111	11·244	+·175	×	·8205	=	+·144	+	·957	=	+1·101	+23·693	241·977	265·670
90	21·00	103	10·434	+·817	×	·8205	=	+·670	+	·957	=	+1·627	+34·167	219·112	253·279
100	19·80	80	8·104	+1·357	×	·8205	=	+1·113	+	·957	=	+2·070	+40·986	160·459	201·445
110	18·11	61	6·179	+1·784	×	·8205	=	+1·464	+	·957	=	+2·421	+43·844	111·907	155·751
120	16·01	37	3·748	+2·107	×	·8205	=	+1·729	+	·957	=	+2·686	+43·003	60·007	103·010
130	13·82	−8	−·810	+2·318	×	·8205	=	+1·902	+	·957	=	+2·859	+39·511	−11·200	28·311
140	11·05	−48	−4·862	+2·458	×	·8205	=	+2·017	+	·957	=	+2·974	+32·862	−53·729	−20·867
150	8·36	−90	−9·117	+2·541	×	·8205	=	+2·085	+	·957	=	+3·042	+25·431	−76·218	−50·787
160	5·52	−120	−12·156	+2·585	×	·8205	=	+2·121	+	·957	=	+3·078	+16·990	−67·101	−50·111
170	2·92	−131	−13·270	+2·605	×	·8205	=	+2·137	+	·957	=	+3·094	+9·035	−38·749	−29·714
180	0	−140	−14·182	+2·611	×	·8205	=	+2·142	+	·957	=	+3·099	0	0	0
															2,066·87

* It will be seen for this column that, for the same position of the piston, the combined inertia and gravitation effect for the *up stroke* is the same as for the *down stroke*, but with the signs changed.

CRANK EFFORT DIAGRAMS FOR NO. 1 CYLINDER OF S.S. "INCHDUNE" DURING THE UP STROKE.

Crank Angle in Degs.	Crank Leverage in Inches.	Steam Pressure in Lbs. per Sq. Inch Absolute.	Total Pressure on Piston in Tons.	f	Constant.	Inertia in Tons.	Weight of Reciprocating Parts, W, in Tons.	Gravity and Inertia Effect, F, due to Reciprocating Parts, in Tons.*	Turning Moment, T M, in Inch-Tons, due to Gravity and Inertia only.	Turning Moment, T M, in Inch-Tons, due to Steam only.	Total Turning Moment, T M, in Inch-Tons, due to Gravity, Inertia and Steam.
180	0	140	13·188	−2·611	× ·8205	= −2·142	− ·957	= −3·099	0	0	0
190	2·92	140	13·188	−2·605	× ·8205	= −2·137	− ·957	= −3·094	− 9·035	38·509	29·474
200	2·52	140	13·188	−2·585	× ·8205	= −2·121	− ·957	= −3·078	−16·990	72·800	55·810
210	8·36	139	13·094	−2·541	× ·8205	= −2·085	− ·957	= −3·042	−25·431	109·464	84·033
220	11·05	138	13·000	−2·458	× ·8205	= −2·017	− ·957	= −2·974	−32·862	143·646	110·784
230	13·82	133	12·529	−2·318	× ·8205	= −1·902	− ·957	= −2·859	−39·511	173·144	133·633
240	16·01	129	12·152	−2·107	× ·8205	= −1·729	− ·957	= −2·686	−43·003	194·550	151·547
250	18·11	124	11·681	−1·784	× ·8205	= −1·464	− ·957	= −2·421	−43·844	211·539	167·695
260	19·80	117	11·021	−1·357	× ·8205	= −1·113	− ·957	= −2·070	−40·986	218·224	177·238
270	21·00	112	10·550	− ·817	× ·8205	= − ·670	− ·957	= −1·627	−34·167	221·558	187·391
280	21·52	102	9·608	− ·175	× ·8205	= − ·144	− ·957	= −1·101	−23·693	206·773	183·080
290	21·33	76	7·159	+ ·546	× ·8205	= + ·448	− ·957	= − ·509	−10·857	152·706	141·849
300	20·34	53	4·993	+1·299	× ·8205	= +1·066	− ·957	= + ·109	+ 2·217	101·549	103·766
310	18·52	32	3·014	+2·060	× ·8205	= +1·690	− ·957	= + ·733	+13·575	55·827	69·402
320	15·94	− 13	− 1·225	+2·759	× ·8205	= +2·264	− ·957	= +1·307	+20·833	− 19·520	1·313
330	12·61	− 62	− 5·840	+3·358	× ·8205	= +2·755	− ·957	= +1·798	+22·672	− 73·647	−50·975
340	8·74	−122	−11·492	+3·815	× ·8205	= +3·130	− ·957	= +2·173	+18·992	−100·444	−81·452
350	4·48	−158	−14·883	+4·102	× ·8205	= +6·366	− ·957	= +2·409	+10·792	− 66·678	−55·886
360	0	−162	−15·260	+4·200	× ·8205	= +3·466	− ·957	= +2·489	0	0	0
											1,408·702

* It will be seen for this column that, for the same position of the piston, the combined inertia and gravitation effect for the *up strokes* is the same as for the *down stroke*, but with the signs changed. The turning moments for each of the other cranks were ascertained in the same way for their down and up strokes. The mean results are given in the next table.

Effective crank leverage calculated from the following formula:—

$$\left.\begin{array}{l}\text{Effective leverage, L, of}\\ \text{crank at angle } \theta\end{array}\right\} = r \sin\theta\left(1 + \frac{r\cos\theta}{\sqrt{l^2 - r^2\sin^2\theta}}\right).$$

When, θ = angle of crank with line of stroke.
r = length of crank in feet.
And l = length of connecting-rod in feet.

Travel of piston, T, from commencement of stroke when crank is at an angle $\theta = r(1 - \cos\theta) \pm l \mp \sqrt{l^2 - r^2\sin^2\theta}$.

Down stroke :—

$$\left.\begin{array}{l}\text{Total inertia and}\\ \text{gravitation effect, F}\end{array}\right\} = \mp f\left(\text{W}r\frac{\text{N}^2}{(100)^2}\right) + \text{W}.$$

$$\text{,,} \qquad \text{,,} \qquad = \mp f\left(\cdot 9569 \times 1\cdot 75 \times \frac{70 \times 70}{100 \times 100}\right) + \cdot 9569.$$

$$\text{,,} \qquad \text{,,} \qquad = \mp f \times \cdot 8205 + \cdot 9569.$$

Up stroke :—

$$\left.\begin{array}{l}\text{Total inertia and}\\ \text{gravitation effect, F}\end{array}\right\} = \mp f\left(\text{W}r\frac{\text{N}^2}{(100)^2}\right) - \text{W}.$$

$$\text{,,} \qquad \text{,,} \qquad = \mp f\left(\cdot 9569 \times 1\cdot 75 \times \frac{70 \times 70}{100 \times 100}\right) - \cdot 9569.$$

$$\text{,,} \qquad \text{,,} \qquad = \mp f \times \cdot 8205 - \cdot 9569.$$

$$\left.\begin{array}{l}\text{Mean turning moment,}\\ \text{T M, in inch-tons for}\\ \text{No. 1 cylinder}\end{array}\right\} = \frac{\text{Total T M during down stroke} + \text{total T M during up stroke}}{36}$$

$$\text{,,} \qquad \text{,,} \qquad = \frac{2066\cdot 87 + 1408\cdot 702}{36}$$

$$= \frac{3475\cdot 572}{36} = 96\cdot 544 \text{ inch-tons.}$$

NOTE.—Piston-rod areas have been deducted in working up the mean pressures of the indicator cards.

$$\left.\begin{array}{l}\text{Mean turning moment,}\\ \text{T M, from I.H.P.}\end{array}\right\} = \frac{\text{I.H.P.} \times 33{,}000 \times 12}{2{,}240 \times 2\,\pi \times \text{revolutions per min.}}$$

TABLE SHOWING THE I.H.P., MEAN TURNING MOMENTS FOR EACH CRANK, AND THE TOTALS FOR THE S.S. "INCHDUNE."

Cylinder Number.	Indicated Horse-Power, I.H.P.	Mean Turning Moment, T M, from I.H.P.	Mean Turning Moment, T M, from above Calculation.
1	240	96·516	96·544
2	236	94·907	94·337
3	252	101·342	101·399
4F	263	105·765	106·132
4A	278	111·798	111·866
Total, .	1,269	510·328	510·278

Percentage difference between mean turning moments $= (510{\cdot}328 - 510{\cdot}278) \times \frac{100}{510{\cdot}328}$

" " = ·009 per cent. error.

EXAMPLE III.—What is the effect of inertia and gravitation on the reciprocating parts of a vertical engine having a stroke of 42 inches, connecting-rod equal 4 cranks, weight of reciprocating parts 2 tons, position of crank from commencement of down stroke 30°, revolutions per minute 72?

$$F = W\left\{\left(-fr\frac{N^2}{(100)^2}\right)+1\right\}$$

$$F = 2\left\{\left(-3{\cdot}3876 \times 1{\cdot}75 \times \frac{(72)^2}{(100)^2}\right)+1\right\} = -4{\cdot}1465 \text{ tons.}$$

LECTURE XVII.—QUESTIONS.

1. In a double-acting engine the mean pressure on the piston is 4 tons and the length of the stroke 18 inches, what is the mean pressure which can be taken from the rim of the fly-wheel, the estimated diameter of which is 8 feet? *Ans.* 1069 lbs. (about).

2. The crank of a steam engine is 2 feet long, and the mean tangential force acting upon it is 17,000 lbs., what is the mean pressure of the steam upon the piston of the engine during each stroke? *Ans.* 26703·6 lbs.

3. In a direct-acting engine the diameter of the cylinder is 17 inches, and the mean pressure of the steam 60 lbs., the crank being 12 inches long, what is the mean pressure on crank in the direction of its motion? *Ans.* 8,670 lbs.

4. Explain the manner in which the reciprocating motion of the piston in a locomotive engine is converted into the rotatory motion of the crank shaft. What are the dead points? Show by the principle of work that there is no loss of power by the intervention of the crank, friction being disregarded.

5. Explain the method of representing in a diagram, the work done during one revolution of the crank of an engine by setting off ordinates representing the tangential efforts on the crank pin.

6. In a direct-acting engine the crank and connecting-rod are as 1 to 6. Find an expression for the tangential pressure on the crank pin in any position. Construct an approximate diagram of work done upon the crank during the stroke, and give a sketch of the same, (1) when there is a single cylinder, and (2) when there are two cylinders working cranks at right angles.

7. In a horizontal direct-acting engine you are required to find an expression for the tangential force upon the crank pin in any given position of the crank. Example—The lengths of the crank and connecting-rod being 1 and 6 respectively, and the pressure on the steam piston being 2,000 lbs., estimate the tangential force on the crank when in a vertical position. Find also the vertical force acting upon the crank shaft. *Ans.* 2,000 lbs. 333 lbs.

8. In a direct-acting horizontal engine the length of the crank 1 foot and that of the connecting-rod is 5 feet. When the crank is vertical the pressure of the steam on the piston is 4,000 lbs.; find the thrust along the connecting-rod, and the pressure on the guide bars at that point of the stroke. *Ans.* 4082·5 lbs.; 816·5 lbs.

9. If the cylinder of a locomotive be 20 inches in diameter with a stroke of 2 feet, and the diameter of the driving wheel be 6 feet, find the tractive force exerted by the engine for each pound of pressure per square inch on the piston. *Ans.* 66·6 lbs.

10. Explain the effects of the inertia of the reciprocating parts in a reciprocating engine, and taking a particular case, work out a crank-pin stress diagram.

11. Draw an indicator diagram of a corliss (or some engine with instantaneous cut-off), in which the cut-off took place at $\frac{1}{4}$ stroke. From this construct a diagram of crank effort (1) for a single cylinder engine, (2) for double cylinder engine with cranks at right angles.

12. In a direct-acting engine, find the ratio of the velocity of the crank-pin to that of the piston in any given position of the crank.

13. The crank of an engine has a radius of 18 inches, the connecting-rod is 6 feet long, and the number of revolutions made by the engine is 80 per minute. Find graphically or otherwise the velocity of the piston in feet per second when the crank has passed through an angle of 30° from the dead centre during the forward stroke. *Ans.* 7·65 feet per second.

14. In a direct-acting horizontal engine, where the connecting-rod works between guides, the connecting-rod is five times as long as the crank, the pressure on the piston when the crank is vertical being 1,250 lbs.; find the thrust on the slide bar, neglecting friction, and indicate the direction in which it acts. Does the direction of the thrust change during any part of the revolution? *Ans.* 255 lbs.

15. In a direct-acting engine the crank is 2 feet in length, and the connecting-rod is 8 feet; find the distance in inches of the piston from the middle point of its stroke, when the crank is at 90° from a dead centre. Answer this by calculation as well as graphically. *Ans.* 3·05 inches.

16. A steam engine with a cylinder of D inches in diameter, receives steam at 80 lbs. absolute pressure per square inch, and the cut-off is at $\frac{7}{8}$ of the stroke. Find an expression for the diameter of the cylinder of another engine with the same stroke and piston speed which develops the same horse-power as the first engine, but which cuts off the steam at $\frac{1}{2}$ stroke. What would be the relative maximum stresses on the crank-pin and crank-shaft of the two engines when both transmit the same power, the inertia of the reciprocating parts and the obliquity of the connecting-rod being neglected?

17. What is the effect of inertia and gravitation on the reciprocating parts of a vertical engine having a stroke of 60 inches; connecting-rod equal four cranks; weight of reciprocating parts, 3 tons; position of crank from commencement of up stroke, 30°; revolutions per minute, 150? Plot the inertia and leverage diagram for this engine, making the radius of the diagram equal by scale to the calculated centrifugal force or inertia effect for an infinite connecting-rod acting at the commencement of stroke upon the crank-pin.

$$F = W\left\{\left(-f r \frac{N^2}{100^2}\right) - 1\right\} = 3\left\{\left(-2{\cdot}5099 \times 2{\cdot}5 \times \frac{150^2}{100^2}\right) - 1\right\}$$

$$F = -45{\cdot}354 \text{ tons.}$$

18. Describe and show graphically how the force transmitted to the crank-pin is affected by the inertia of the moving reciprocating parts in a stationary horizontal engine during the forward and backward strokes of the piston respectively. How is the force affected by an increase in the ratio of expansion of the steam, by the shortness of the connecting-rod, and by the momentum of the flywheel? In a horizontal non-condensing engine, whose cylinder is 16 inches diameter, stroke 28 inches, connecting-rod 5 feet 10 inches in length, making 100 revolutions per minute, and working with steam whose initial gauge pressure is 140 lbs. per square inch, when cutting-off takes place at four-tenths of the stroke, what would be the pressure on the crosshead at the beginning and at the end of the stroke if the weight of the reciprocating parts is 350 lbs.? and show, by a diagram, the variation in the pressure as the piston makes its forward stroke, the effect of the cushioning of the steam at the end of the stroke being neglected; expansion as if pv were constant. (S. & A., 1897, Hons.)

19. The piston and all that is rigidly connected with it weigh 500 lbs., the crank is 1 foot long, and the speed 200 revolutions per minute. Show, on a diagram, the forces which must be exerted at the crosshead during a

revolution if there is no steam pressure. Choose a pair of indicator diagrams, and show how we find the diagram of forces acting at the crosshead. Assume an infinitely long connecting-rod. (S. & A., 1898, H., Part i.)

20. If, on a piston of 120 square inches in area and weighing with piston-rod 290 lbs., there is at a certain instant a pressure of 130 lbs. per square inch on one side more than what there is on the other, and if the piston acceleration at that instant is 420 feet per second per second in the direction in which the steam is urging the piston, what is the total force acting at the crosshead? If this acceleration occurs when the piston is one-quarter of its stroke from one end, assuming an infinitely long connecting-rod, how many revolutions per minute is the engine making? The crank is 1 foot long. (B. of E., 1900, Adv. and H., Part i.)

21. Explain fully the influence of the reciprocating parts in modifying the effective steam pressure in an engine, and show how to make the calculations necessary in order to determine the weight of flywheel needed to keep the fluctuation of speed during one revolution of an engine within any chosen limit. (C. & G., 1900, H., Sec. B.)

22. If the connecting-rod is 5 feet long, and the crank is 1 foot; 200 revolutions per minute; what are the accelerations of the piston when it is farthest from and nearest to the crank? The piston and rod and crosshead weigh 330 lbs. Area of piston 120 square inches. At the beginning of either the in or out stroke the pressure is 80 lbs. per square inch on one side in excess of what it is on the other. Find the total forces on the crosshead in these two cases. (B. of E., 1902, H., Part i.)

23. Sketch a real probable indicator diagram for a non-condensing engine, single cylinder, 18 inches diameter, crank 15 inches, 150 revolutions per minute, cutting off at about half-stroke, with slide-valve. Neglecting inertia effects, show how we find the turning moment on the crank-shaft in every position, and how it usually varies. What effect has this change of moment upon the strength of the crank-shaft? (B. of E., 1902, H., Part i.)

24. Piston 115 square inches in area. At the beginning of either stroke there is a difference of pressure of 90 lbs. per square inch on its two sides, producing total force in the direction in which the piston is about to move. The piston and its rod weigh 410 lbs. The engine makes 130 revolutions per minute; crank 1 foot. Neglecting angularity of connecting-rod—that is, assuming that the piston has a simple harmonic motion—what is the actual force at the crosshead at the beginning of either stroke? What correction must be made when the angularity of the connecting-rod is not neglected; (B. of E., 1903, Adv. and H., Part i.)

25. Explain what is meant by the "pressure due to the inertia of the reciprocating parts" in a steam engine, and show how it modifies the effective crank effort at different points of the stroke. If the revolutions per minute are 400 and the mean piston speed 1,200 feet per minute, show that, with a 4 to 1 rod, the maximum inertia pressure is, very approximately, fifty times the weight of the reciprocating parts. (C. & G., 1903, H., Sec. B.)

Lecture XVII.—A.M.Inst.C.E. Exam. Questions.

1. What is meant by the instantaneous axis of a moving piece? How do you find it in the case of a connecting-rod, and how do you apply it to find the velocity of the piston at any point of its stroke, when the velocity of the crank-pin is known? (I.C.E., *Oct.*, 1897.)

2. Show how to find the acceleration of an engine piston at each end of its stroke when the length of the connecting-rod, the length of the stroke, and the number of revolutions per minute are given. Find the force required for acceleration per lb. mass of the piston, at each end of the stroke, in an engine with an 8-inch crank and 30-inch connecting-rod making 300 revolutions per minute. (I.C.E., *Oct.*, 1897.)

3. What is a crank-effort diagram? What data are required to allow it to be drawn, and how is it applied in finding the fluctuations of speed in an engine when the dimensions and speed of the flywheel are known? (I.C.E., *Feb.*, 1898.)

4. A piston and its rod and crosshead weigh 460 lbs. The engine makes 250 revolutions per minute, the crank is 1 foot long. Make a diagram of the horizontal force at the crosshead at every point in the stroke: (1) assuming the connecting-rod infinitely long; (2) taking the connecting-rod to be 5 feet long. State the force at some one place so that your scale may be checked. Assume no friction. (I.C.E., *Oct.*, 1898.)

5. Show how to find graphically the acceleration of the piston of a direct-acting engine in any position, the crank-pin being assumed to move uniformly. Sketch the form of the curve of acceleration (1) on a piston, and (2) on a crank angle base. Describe generally the influence of the inertia of the piston, rods and crosshead, on the stresses set up in the crank-pin. The weight of the reciprocating parts is equivalent to 3 lbs. per square inch of the area of the piston. If the length of crank be 9 inches, find how much the initial effective pressure is reduced by the inertia of the reciprocating parts when the crank makes 70 revolutions per minute, the obliquity of the connecting-rod being neglected. (I.C.E., *Oct.*, 1900.)

6. The weight of the reciprocating parts of a single-cylinder engine is 4,040 lbs., the stroke is 39 inches, and the revolutions 143 per minute; the diameter of the cylinder is 33·5 inches, and the length of the connecting-rod is twice the stroke. Find "the pressure equivalent to the inertia" at each end of the stroke. (I.C.E., *Oct.*, 1902.

7. Estimate the greatest and least forward velocity of the piston of a locomotive engine relative to the rails when the train is running at 50 miles per hour, the diameter of the driving-wheels being 66 inches, the length of stroke 27 inches, and the length of engine connecting-rod 54 inches. (I.C.E., *Feb.*, 1903.)

8. Having given the indicator diagrams of a steam engine, explain what corrections have to be made and how the corrections modify the result, before the effective turning moment on the crank-shaft can be determined. If the revolutions per minute are 400 and the piston speed 1,200 feet per minute, show that, if the connecting-rod is four times the length of the crank radius, the maximum inertia pressure is, very approximately, fifty times the weight of the reciprocating parts. (I.C.E., *Oct.*, 1903.)

9. Show how to find the instantaneous centre of the connecting-rod of a direct-acting engine. Hence deduce a graphic construction for the speed of the piston and its acceleration. (I.C.E., *Oct.*, 1904.)

10. Describe the construction of a curve of velocity of the piston of a crank and connecting-rod engine, and show that the same curve also represents the varying crank effort of the engine, supposing the pressure on the piston is constant. (I.C.E., *Feb.*, 1905.)

11. Explain how a force available for propelling a train is derivable from the pressure of the steam in the cylinders of a locomotive engine, observing that the steam pressure acts with equal force on the piston one way and on the cylinder end in the opposite direction. The weight of a portion of a locomotive engine, which is borne by the two driving wheels of 5 feet diameter, being 15 tons and the coefficient of friction being assumed to be one-fifth; determine the least intensity of pressure of steam on the two pistons which will cause the driving wheels to slip in every position of the crank-shaft, the length of the stroke being 2 feet 3 inches, and the diameter of the cylinder 1 foot 9 inches. Determine also the pressure to which the steam must be restricted if slipping is to be avoided in all positions, the length of the connecting-rod being 5 feet. (I.C.E., *Feb.*, 1905.)

12. A steam engine, whose crank rotates at a uniform speed of 200 revolutions per minute, has a stroke of 2 feet, the length of the connecting-rod being 6 feet. Show how you would proceed to draw the curve of velocity of the piston. (I.C.E., *Oct.*, 1905.)

13. It is desired to draw the turning-effort diagram for a cross compound horizontal engine in order to calculate the dimensions of the flywheel. Enumerate the data required, and explain fully the process that has to be followed, illustrating your remarks by means of hand sketches. (I.C.E., *Oct.*, 1905.)

14. What are the relative advantages and disadvantages of (i.) a piston connected to the crank by the usual piston-rod, slide, and connecting-rod; (ii.) a trunk piston with connecting-rod? Illustrate your remarks by means of hand sketches, and give instances of the use of each arrangement. (I.C.E., *Oct.*, 1905.)

15. Show how to find the inertia forces due to the piston of a steam engine:—(i.) On the supposition that the connecting-rod is of infinite length; (ii.) on the supposition that the ratio of the connecting-rod to the crank is r. Draw a diagram on a stroke base, giving the inertia at every point of the stroke, when $r = 5$, the weight of the piston is 200 lbs., and the engine makes 400 revolutions per minute. (I.C.E., *Oct.*, 1905.)

16. Enumerate the forces to which the connecting-rod of a horizontal steam engine is exposed during a complete revolution, and explain to what stresses these forces give rise. (I.C.E., *Feb.*, 1906.)

17. The cylinder of a steam engine is 24 inches in diameter, the stroke being 24 inches and the length of the connecting-rod 48 inches. Determine by calculation, or graphically, the force on the connecting-rod, the pressure on the slide-block, and the twisting moment on the crank-shaft due to an effective pressure of 90 lbs. per square inch on the piston, when the crank has turned through an angle of 30 degrees during the forward stroke. (I.C.E., *Feb.*, 1906.)

18. Give any method for finding the instantaneous value of the velocity in magnitude and direction of the middle point on the centre line of the connecting-rod of a steam engine, having given the instantaneous value of the velocity of the crank pin. (I.C.E., *Oct.*, 1906.)

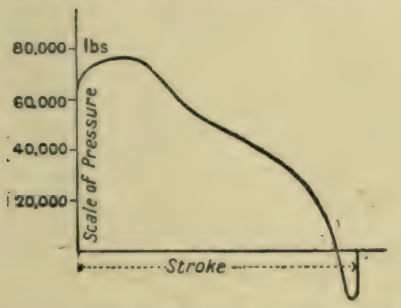
19. Show, that if θ be the crank angle subtended by the connecting-rod, the acceleration of the piston is given approximately by

$$a = \omega^2 r (\cos\theta + n \,.\, \cos 2\theta),$$

where r is the crank radius, ω is the uniform angular velocity of the crank,

and n is the ratio of the crank radius to the length of the connecting-rod. Calculate the value of the acceleration in the case where $\theta = 60°$ and the crank revolves 300 times per minute. (I.C.E., *Oct.*, 1906.)

20. The accompanying figure gives the total net pressures acting on the piston of a steam engine during the forward stroke. The ratio of the



crank to the connecting-rod is 1 to 5·0. Draw a diagram giving the tangential effort at all positions of the crank during the forward stroke. (I.C.E., *Oct.*, 1907.)

21. A vertical high-speed engine, having a 4½-inch crank, is revolving at 400 revolutions per minute. The weight of the moving parts (piston, piston-rod and slipper, and the upper half of the connecting-rod) is 150 lbs. Establish an approximate formula by means of which the vertical inertia force can be obtained at any point of the stroke when the connecting-rod is finite, and by its means calculate the inertia force at the top and bottom of the stroke—(*a*) when the connecting-rod is supposed infinite; (*b*) when the ratio of the crank to the connecting-rod is 1 to 4·2. (I.C.E., *Oct.*, 1907.)

22. To what forces is the connecting-rod of a steam engine subjected? Show that the proportionate effect of these forces is not the same in a slow-speed engine as it is in a high-speed engine. In the latter case indicate how you would proceed to calculate the necessary cross-section (supposed rectangular) at its weakest point. (I.C.E., *Oct.*, 1907.)

23. Take the case of the piston of a reciprocating steam engine and show, either analytically or graphically by means of a polar diagram, how the velocity of the piston at any point of the stroke can be obtained. Determine also the shape of the velocity curve when plotted on a stroke base, and make a sketch of this curve. (I.C.E., *Feb.*, 1908.)

24. Explain how it is that, in the case of a petrol engine running at a high speed, the effect of the inertia forces due to the motion of the piston, of the connecting-rod, and of the crank pin is to substantially reduce the irregularity of the effort diagram on the piston. Would it be practically safe to use this diagram in determining the dimensions of the connecting-rod and of the crank shaft? Give your reasons. (I.C.E., *Feb.*, 1908.)

25. Show how to construct a diagram representing the velocity of the crosshead of a reciprocating engine if the crank has uniform velocity. Show also the direction of motion of a point one-third the length of the connecting-rod from the crosshead at any three points between dead centres, if the length of the connecting-rod is five times that of the crank. (I.C.E., *Oct.*, 1908.)

LECTURE XVIII.

CONTENTS.—Governing of Engines—Watt's Governor—Action of Watt's Governor—Theory of Watt's Governor—Conical Pendulum—Example I.—Common Pendulum Governor—Crossed Arm Governor—Parabolic Governors—Galloway's Parabolic Governor—Porter's loaded Governor—Theory of Porter's Governor—Example II.—Spring loaded Governors—Proell's and Hartnell's Spring Governors—Pickering Governor—Willans' Spring Governor—Governing by Throttling and Variable Expansion—Shaft Governors—Relays—Knowles' Supplemental Governor—Inertia Governors—Thunderbolt's Marine Engine Governor—Thunderbolt's Electric Governor Regulator—Experiments upon the Action of Engine Governors—Proell Governor—Belliss Shaft Governor—Comparison of Different Governors—Results of Different Governors—To obtain the Controlling Force Graphically—Flywheels—Balancing Machinery—Weston Self-balancing Centrifugal Machine—Water-Driven Centrifugals with Automatic Cut-off—Calculations of Work Done in Overcoming Inertia, &c.—Electrically-Driven Centrifugal Machines—Considerations of the Different Ways for Driving Centrifugals—Relative Efficiencies of Three Different Ways of Driving Centrifugals—Examples III., IV., and V.—Questions.

Governing of Engines.*—For many purposes to which engines are applied, it is necessary that they should maintain a uniform speed. Owing to variations of load and of pressure on the piston, they must have some regulating device, in order to accomplish this object. Fluctuations of the speed of a steam engine are of two kinds. (1) Those which occur during the time of a revolution, and are *periodic*, being caused by the varying pressure on the piston, and obliquity of the connecting rod. (2) Those which are due to change of load, or boiler pressure, and are *not periodic*. To control the first of these as far as possible, an engine is fitted with a Flywheel, and for the second a Governor is also required.

* The following is a list of books and papers treating of governors and governing:—

Paper on "The Electrical Regulation of the Speed of Steam Engines," by P. W. Willans. *Proc. Inst. C.E.*, 1885, vol. lxxxi., p. 166.

Paper on "A New Method of Investigation applied to the Action of Steam Engine Governors," by Prof. Dwelshauvers-Dery of Liége, translated by Michael Longridge. *Proc. Inst. C.E.*, 1888, vol. xciv., p. 210.

Paper on "The Cyclical Velocity-Variations of Steam and other Engines," by H. B. Ransom. *Proc. Inst. C.E.*, 1889, vol. xcviii., p. 357.

Paper on "The Application of Governors and Flywheels to Steam Engines," by Prof. Dwelshauvers-Dery, translated by Bryan Donkin. *Proc. Inst. C.E.*, 1891, vol. civ., p. 196.

Paper on "Flywheels and Governors," by H. B. Ransom. *Proc. Inst. C.E.*, 1892, vol. cix., p. 330.

Paper on "Steam Engine Governors and their Insufficient Regulating Action with Extreme Variations of Load," by Prof. Dwelshauvers-Dery, translated by Bryan Donkin. *Proc. Inst. C.E.*, 1892, vol. cx., p. 276.

Paper on "A Method of Testing Engine Governors," by H. B. Ransom. *Proc. Inst. C.E.*, 1893, vol. cxiii., p. 194. Also, 1899, vol. cxxxvii., p. 376.

A governor is a piece of mechanism which regulates the amount of steam supplied to the engine, to suit the work it is doing, whereas, as explained in the previous Lecture, a flywheel acts in virtue of its inertia, so as to distribute throughout a whole revolution the energy developed in the cylinder. The governor can have no effect whatever on the periodic variations of speed, since it can only act during the time that steam is being admitted to the cylinder. With regard to the irregular fluctuations of speed, due to a change of load, the flywheel makes them more gradual and thus gives the governor time to act. A great many varieties of governors have been invented since the introduction of the steam engine, such as hydraulic, centrifugal, inertia, and electrical governors. By far the greatest number, however, depend for their action on centrifugal force and inertia, and since these form useful examples of the practical application of the principles enunciated in the previous Lectures, we shall now confine our remarks to such governors.

Watt's Governor.—One of Watt's important inventions was his conical pendulum governor, as applied to his double-acting engine.* This governor consists of two arms, A A, carrying heavy balls, B B, and pivoted on a pin, P, passing through the centre of the vertical spindle, V S. The upper ends of these arms are bent, as shown on the figure, and are connected by short links, L L, to the sleeve, S. This sleeve is free to move vertically on the spindle, V S, but is made to rotate with it by a feather, F, and corresponding keyway. This sleeve acts on one end of the bell crank, B C, and thus moves the rod con-

Paper on "The Mechanical and Electrical Regulation of Steam Engines," by John Richardson. *Proc. Inst. C.E.*, 1895, vol. cxx., p. 211.

Paper on "Governing of Steam Engines by Throttling and by Variable Expansion," by Capt. H. R. Sankey. *Proc. Inst. M.E.*, 1895, p. 154.

Paper on "Steam-Engine Governors," read before the Manchester Association of Engineers, by C. F. Budenberg, M.Sc. See *The Practical Engineer*, 17th April, 1891, vol. v., p. 258.

A series of articles on "Engine Governors," by R. G. Blaine, M.E., in *The Practical Engineer*, beginning 13th June, 1890, vol. iv., p. 386, and ending 24th April, 1891, vol. v., p. 277.

Article on "A New Shaft Governor," by E. J. Armstrong, in *The Practical Engineer*, 26th July, 1895, vol. xii., p. 71.

Article on "Shaft Governors," by E. T. Adams, in the *Electrical World* of New York. July, 1896.

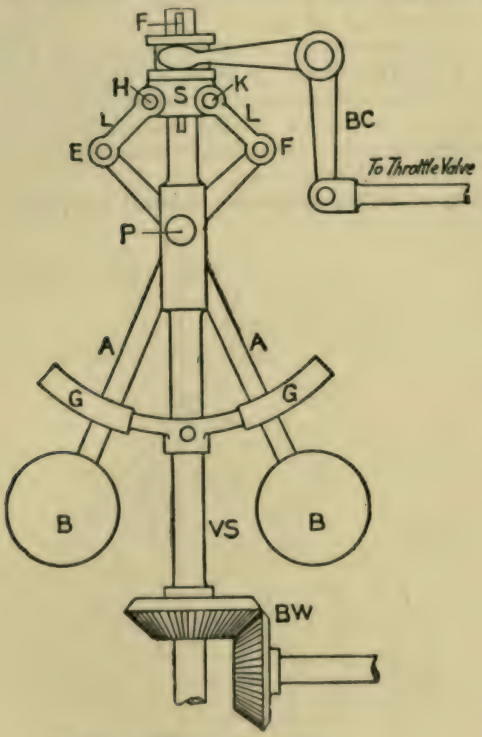
See Index for Governors in *Gas, Oil, and Air Engines*, by Bryan Donkin, published by Charles Griffin & Co.

The Steam Engine, by D. K. Clark (Blackie & Son), chap. v., on Governors, p. 65, half-vol. iii.

* See the Author's *Text-Book on Steam and Steam Engines* for a description of Watt's engines. Also Lecture XIX., Volume I., of this book for an illustration of same.

nected to the throttle valve of the engine. The vertical spindle may be driven by the engine by means of a belt or rope passing round a pulley keyed on it, or by bevel wheels, as shown at B W. In order to relieve the pin, P, the arms are driven by the guides, G G, which are fixed to the vertical spindle.

Action of Watt's Governor.—The governor is so adjusted, that when the engine is working at its normal speed, the balls rotate

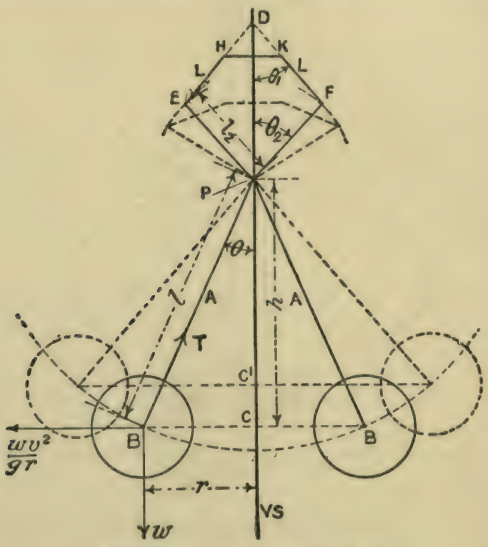


WATT'S PENDULUM GOVERNOR.

at a certain distance from the vertical spindle, and thus the throttle valve is kept sufficiently open to maintain that speed. Should the load be *decreased*, the speed of the engine, and therefore that of the governor balls, naturally becomes greater. This causes an increase of the centrifugal force of the balls, and therefore they diverge further, thereby pulling down the sleeve,

and partially closing the throttle valve, which diminishes the supply of steam and the power developed by the engine. On the other hand, should the load be *increased* the reverse action takes place, the balls come closer together, the sleeve is raised, the throttle valve opened wider, and more steam admitted to the engine. It will thus be seen that a change of speed must take place before the governor begins to act; further, that for any permanent change in the work to be done, there is a permanent alteration of speed. For each particular load on the engine, the throttle valve will be opened by a definite amount, which will be different for different loads, and each position of the valve has a corresponding position of the governor balls. But, as will be shown further on, each position of the balls corresponds to a definite speed, so that there will be a particular speed for each different load.

Theory of Watt's Governor—Conical Pendulum.—Let the balls



THEORY OF WATT'S GOVERNOR.

be rotating about the vertical spindle with a uniform velocity. Then the several forces acting on the different parts of the instrument are in equilibrium with each other. The arms, A, will describe the surface of a cone, B P B, whose height is P C, and for a given velocity of the balls there will be a definite

height of this cone. It will be sufficient to consider one ball and arm, since what is true for one will be true for the other.

Let w = Weight of one ball in lbs.
„ v = Velocity of balls in *feet per second.*
„ h = Height, P C, of cone in *feet.*
„ l = Slant height, P B, of cone in *feet.*
„ r = Radius, B C, of base of cone in *feet.*
„ T = Tension in one arm, A.

There are three forces acting on the ball, B, viz.:—

(1) The weight, w, of the ball acting vertically downward.

(2) The centrifugal force, $w v^2 \div g r$, acting in its plane of rotation, and in the direction C B.

(3) The tension in the arm, A, acting in the direction B P.

These three forces keep the ball in equilibrium, and can, therefore, be represented, in magnitude and direction, by the three sides of a triangle taken in order. If we draw a triangle, having its sides parallel or perpendicular to the directions of these forces, the lengths of the sides of this triangle will be proportional to the forces respectively. Now, such a triangle exists in the figure itself—viz., the triangle P C B—the sides of which are parallel to the three forces:—

Hence, $$h : r = w : \frac{w v^2}{g r} = 1 : \frac{v^2}{g r}.$$

$\therefore$ $$h = \frac{g r^2}{v^2}, \text{ and } \frac{r}{v} = \sqrt{\frac{h}{g}}.$$

If t = time in *seconds* of one complete revolution of balls,
n = number of revolutions per *second*,

Then, $$t v = 2 \pi r, \text{ and } n = \frac{1}{t}.$$

Substituting these in the previous equation we get the following important formulæ:—

$$t = 2 \pi \frac{r}{v} = 2 \pi \sqrt{\frac{h}{g}}. \quad . \quad . \quad . \quad . \quad . \quad \text{(I)}$$

That is, *the period of rotation is proportional to the square root of the height of the cone.*

Also, $$n = \frac{1}{t} = \frac{1}{2 \pi} \sqrt{\frac{g}{h}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(II)}$$

Or, if N be the number of revolutions per *minute* = 60 n,

Then, $$N = \frac{30}{\pi}\sqrt{\frac{g}{h}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (II_a)$$

That is, *the number of revolutions, or the speed of the engine or governor, varies inversely as the square root of the height of the cone.*

Equation (II_a) may be written in this useful form :—

$$h = \frac{30^2 g}{\pi^2} \times \frac{1}{N^2} = \frac{2936}{N^2} \text{ feet.} \quad . \quad . \quad (II_b)$$

Or, *the height of the cone depends only on the speed of rotation, and varies inversely as the square of the number of revolutions.*

Let the speed of the governor be altered from N_1 to N_2 revolutions per minute, then the heights of the cone corresponding to these speeds are :—

$$h_1 = \frac{2936}{N_1^2}, \text{ and } h_2 = \frac{2936}{N_2^2}.$$

Therefore, for a change of speed from N_1 to N_2 revolutions per minute the height of the cone will be altered by the amount :—

$$h_1 \sim h_2 = 2936\left(\frac{1}{N_1^2} \sim \frac{1}{N_2^2}\right) = \frac{2936\,(N_2^2 \sim N_1^2)}{N_1^2\,N_2^2}. \quad (III)$$

If, however, the height of the governor be kept constant, and equal to $h = \frac{900\,g}{\pi^2 N_1^2}$, the centrifugal force will change from $\frac{w v_1^2}{g r}$ to $\frac{w v_2^2}{g r}$, or from $\frac{w r \pi^2 N_1^2}{900 g}$ to $\frac{w r \pi^2 N_2^2}{900 g}$, and the difference will produce a tension, or a thrust, in the links L L. If T_2 be the tension, or thrust, in one link L; l, l_1, l_2 the lengths of B P, E D, P E; and θ, θ_1, θ_2 their inclinations to the vertical, then by taking moments about P, we have :—

$$T_2 \times l_2 \cos(\theta_1 + \theta_2 - 90) = \frac{w r \pi^2}{900 g}(N_2^2 \sim N_1^2) \times h,$$

Or, $$T_2 = \frac{w r \pi^2 (N_2^2 \sim N_1^2) h}{900 l_2 g \sin(\theta_1 + \theta_2)}.$$

Now, the vertical force acting on the sleeve, which is available for overcoming friction, and may be called the *working effort* for that change of speed, is the vertical components of

the stresses in the two links L L. These two stresses are equal.

$$\therefore \text{ The working effort } = 2\,T_2 \cos \theta_1$$

$$\text{,, ,, } = \frac{w\,r\,\pi^2\,h\,(N_2^2 \sim N_1^2)}{450\,l_2\,g} \times \frac{\cos\theta_1}{\sin(\theta_1 + \theta_2)}$$

$$\text{,, ,, } = \frac{w\,r\,\pi^2\,\frac{900\,g}{\pi^2 N_1^2}\,(N_2^2 \sim N_1^2)\cos\theta_1}{450\,l_2\,g\sin(\theta_1 + \theta_2)}$$

$$\text{,, ,, } = \frac{2\,w\,r\cos\theta_1\,(N_2^2 \sim N_1^2)}{l_2 \sin(\theta_1 + \theta_2)\,N_1^2} \quad \ldots\ldots \text{(IV)}$$

It is usual for P E and E H to be made equal in length, and then $\theta_1 = \theta_2$ nearly, unless H K be great. In that case :—

$$\text{The working effort } = \frac{2\,w\,r\cos\theta_2\,(N_2^2 \sim N_1^2)}{l_2 \sin 2\theta_2\,N_1^2}$$

$$\left.\begin{aligned} \therefore \text{ ,, ,, } &= \frac{w\,r\,(N_2^2 \sim N_1^2)}{l_2 \sin\theta_2\,N_1^2} \\ \text{Or, ,, ,, } &= \frac{w\,l\sin\theta\,(N_2^2 \sim N_1^2)}{l_2 \sin\theta_2\,N_1^2} \end{aligned}\right\} \quad \ldots\ldots \text{(IV}_a\text{)}$$

If, further, $\theta = \theta_2$, which will always be the case when the sleeve is attached to the arm below the point of suspension, as in the next form of governor, then:—

$$\text{The working effort } = \frac{w\,l\,(N_2^2 \sim N_1^2)}{l_2\,N_1^2} \quad \ldots\ldots\ldots \text{(IV}_b\text{)}$$

It should be noted, however, that this is the effort exerted by the governor when it is just starting to move. The working effort becomes smaller and smaller as the balls rise, until, when the balls have attained the position corresponding to the new speed, it is *nil*.

The movement of the sleeve, corresponding to an alteration in the height of the cone, is best determined graphically by drawing the centre lines of the arms and links to scale for different positions of the balls.

EXAMPLE I.—Find the rise of the balls of a pendulum governor, when its speed is increased from 60 to 62 revolutions per minute. Find also the height of the cone of revolution at the lower speed, and the working effort, if the balls weigh

22 lbs. each, and each arm is jointed to the link at two-thirds of its length below its point of suspension.

Here, $N_1 = 60$, $N_2 = 62$, rise of balls $= h_1 - h_2$, $w = 22$, and $\frac{l}{l_2} = \frac{3}{2}$.

Therefore, from equation (III) we get:—

$$h_1 - h_2 = 2936\left(\frac{N_2^2 - N_1^2}{N_1^2 \times N_2^2}\right) = 2936\left(\frac{62^2 - 60^2}{60^2 \times 62^2}\right).$$

$$\therefore \quad h_1 - h_2 = \cdot 0518 \text{ foot or } \cdot 62 \text{ inch.}$$

Also from equation (II_b):—

$$h_1 = \frac{2936}{N_1^2} = \frac{2936}{3600}.$$

$$\therefore \quad h_1 = \cdot 816 \text{ foot} = 9 \cdot 79 \text{ inches.}$$

And from equation (IV_b):—

$$\textit{The working effort} = \frac{w\, l\, (N_2^2 \sim N_1^2)}{l_2\, N_1^2}$$

$$\text{,, ,,} = \frac{22 \times 3\,(62^2 - 60^2)}{2 \times 60^2}$$

$$\text{,, ,,} = 2 \cdot 237 \text{ lbs.}$$

In this case, as we assume $l_1 = l_2$ and $\theta_1 = \theta_2$, the travel of the sleeve will be twice the rise of the point where the link joins the arm, and this will be two-thirds of the alteration in height.

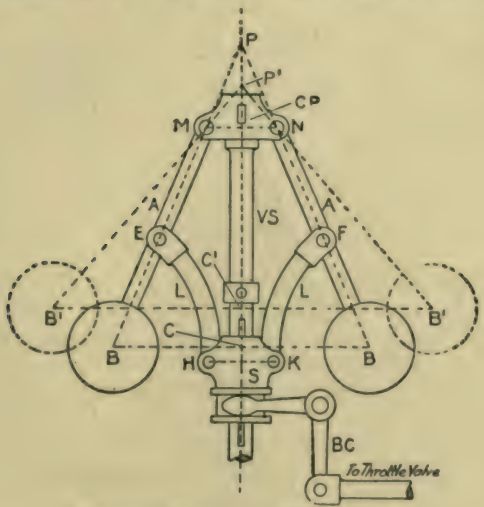
$$\therefore \textit{ Travel of sleeve} = 2 \times \frac{2}{3}(h_1 - h_2) = \frac{4}{3} \times \cdot 62 = \cdot 827 \text{ inch.}$$

Common Pendulum Governor.—A common modification of Watt's governor is shown by the following figure. Here, the arms A A, carrying the balls B B, instead of being jointed together by a pin passing through the vertical spindle V S, are pivoted at M and N to a cross-piece, C P, which is rigidly connected to the spindle. The links L L, carrying the sleeve S, are attached to the arms at the points E and F.

The formulæ deduced for Watt's pendulum governor are equally applicable to this case. The only thing requiring special attention here, is the height of the cone of revolution. The vertex of the cone is always at the point where the centre lines of the arms meet. In this case, the arms terminate at M and N, which are at a short distance from V S, and thus the vertex of the cone will be a variable point on the centre line of

the vertical spindle. When the balls are in the position shown by the full lines, the vertex is at P, and the height of the cone is P C; but, when the balls move into the new position, shown by dotted lines, the vertex of the cone is at P′, and the height of the cone is P′ C′.

The effect of suspending the arms at a short distance from the vertical spindle, is to cause the movement of the sleeve to be less for a given variation in the height of the governor, than would be the case were the centres of suspension in the vertical spindle. It will be apparent from the figure, that the effective variation of height is $C C' = (P C - P' C') - P P' = (h_1 - h_2) - P P'$, instead of $h_1 - h_2$, as in the previous case. Hence,



COMMON PENDULUM GOVERNOR.

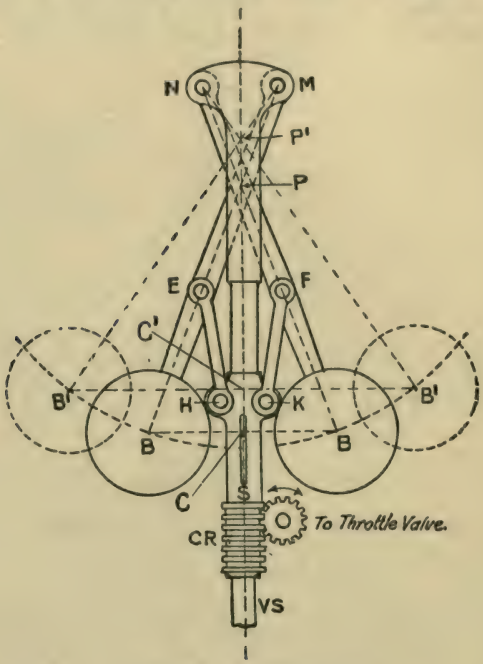
this governor is *less sensitive* than the former, since the speed must vary between greater limits for a given movement of the sleeve. The *sensitiveness* of a governor depends on the movement given to the sleeve for a given variation in speed, and also on the smallness of the time taken by the governor in adapting itself to its new position. In order, therefore, to increase the sensitiveness of this form of governor it is necessary that the points M and N should be as near the vertical spindle as possible.

Crossed-Arm Governor. — The special feature in which this governor differs from the former ones is, that the arms are

suspended from pins placed on the opposite sides of the spindle to that of their balls. From an inspection of the figure, it will be apparent that crossing the arms in this manner causes a greater movement of the sleeve for a given variation in the height than in the pendulum governor. The rise of the balls in this case is :—

$$C\,C' = P\,C - P'\,C' + P\,P' = (h_1 - h_2) + P\,P'.$$

The sensitiveness of this governor is therefore much greater than either of the two previous forms. By properly propor-



CROSSED-ARM GOVERNOR.

tioning the lengths of the arms and N M, so that the balls move out and in, along a curve which is approximately a parabola, this governor may be made almost isochronous, and, therefore, extremely sensitive.* It will be noticed, that the sleeve of the

* A governor is said to be *isochronous* when its speed of rotation (and, therefore, the height of the cone) is the same for all positions of the balls within its range.

governor shown, has a circular rack C R, which gears with a pinion on the throttle valve spindle.

Parabolic Governors.*—Governors have been so constructed that their balls were guided to move in a truly parabolic path, and thus be absolutely isochronous, but owing to their complication they have not come into general use. With any centrifugal governor, the speed must increase somewhat before the extra centrifugal force is able to overcome the friction resisting the motion of the links, sleeve, valve, &c., and if it be absolutely isochronous, whenever the friction is overcome the balls would rise right up to the top of their range, and remain there until the speed has fallen sufficiently for gravity to reassert itself and overcome the friction, which would now tend to keep the balls up. They would then come down to the bottom of their range, and there would thus be continual hunting. Such a governor would therefore be wanting in stability or steadiness.

LOADED PARABOLIC GOVERNOR, BY GALLOWAYS, LD.

Galloway's Parabolic Governor.—From the illustration it will be seen, that in this type two cylindrical rollers take the place of the ordinary balls in the previously mentioned governors. These rollers are suspended at each end by links from a crosshead fixed to the top of the governor spindle, and naturally rise and fall in circular arcs with these links as radii. They move along parabolic slots cut in a weight W, which rotates with the spindle, but is free to rise and fall along the same. By this arrangement, the moment of the centrifugal force of the rollers is balanced by that of the weight at nearly the same speed for all positions. Hence, this governor may be considered practically insochronous. To the bottom of the slotted weight there is sometimes attached a sleeve termi-

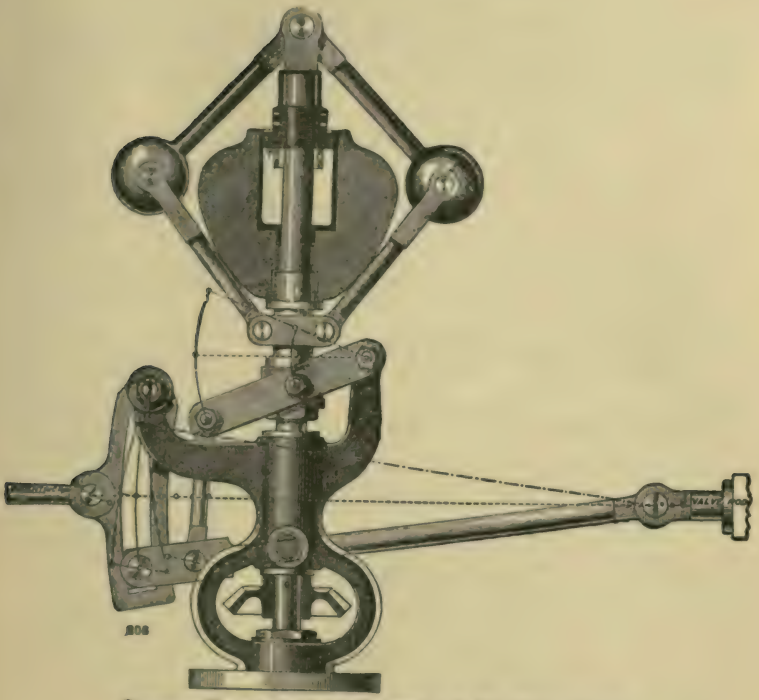
* See the Appendix to *The Steam Engine*, by Prof. Rankine (Chas. Griffin & Co.), and Chapter XV. of *Practical Treatise on the Steam Engine*, by Arthur Rigg (E. & F. Spon), for descriptions of guided parabolic governors.

nating in a collar, which engages the forked lever connected to the throttle valve, or expansion gear; but in this case, a central spindle C S, which is carried up inside the main governor spindle, rises and falls with the weight W, and acts directly on an equilibrium valve. The governor is driven through gearing contained in the cast-iron box seen at the foot of the vertical column.

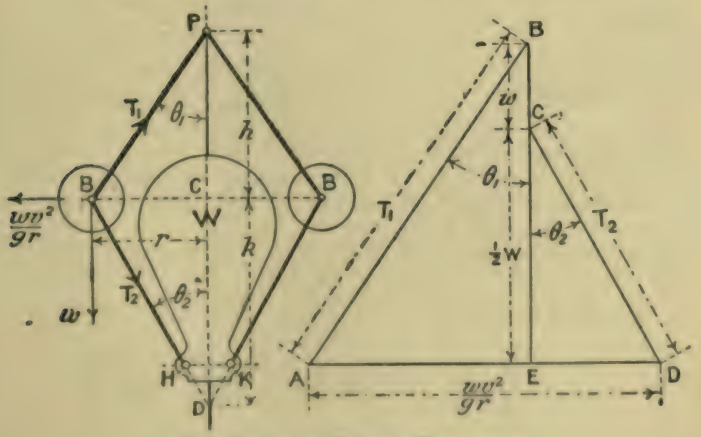
Porter's Loaded Governor.—From equation (IV) and Example I. we see that the simple pendulum governors possess a comparatively small working effort, unless the balls are very heavy. To overcome this objection Porter made the balls smaller, and loaded the sleeve with a heavy weight. This increases the height of the cone, corresponding to any particular speed, and all the forces concerned, and thus gives a greater working effort. It can be used both in connection with throttle valves and some forms of expansion gear. To minimise the oscillations of the ordinary Porter governor, Messrs. Clayton & Shuttleworth have made a cylindrical hole in the top of the central weight, and fixed a piston on the vertical spindle, thus forming a simple air cushion.

PORTER LOADED GOVERNOR, BY TANGYES, LIMITED.

Theory of the Porter Governor.—Each of the balls is in equilibrium under the action of four forces acting in a plane passing through the axis of rotation. These forces are:—(1) the weight of the ball w, (2) the centrifugal force $w v^2 \div g r$, and (3) the tensions in the two links, T_1 and T_2. Let A B C D A be a polygon representing these forces, A B being parallel and equal to T_1, B C to w, C D to T_2, and D A to the centrifugal force. If B C be produced to meet A D in E, then C E is equal to the vertical component of T_2, and must therefore be equal to half the load W, since this weight is supported by the vertical components of the tensions in the two bottom links.



PORTER GOVERNOR, BY CLAYTON & SHUTTLEWORTH.



ACTION OF THE PORTER GOVERNOR.

If the inclination of the links B P, B H to the vertical, be θ_1 and θ_2 respectively, and the heights C P, C D be denoted by h and k, then, the other letters being the same as before, we have :—

$$T_1 \sin \theta_1 + T_2 \sin \theta_2 = \frac{w v^2}{g r}.$$

$$T_1 \cos \theta_1 = w + \tfrac{1}{2} W, \text{ or } T_1 = \frac{w + \frac{1}{2} W}{\cos \theta_1}.$$

And, $$T_2 \cos \theta_2 = \tfrac{1}{2} W, \quad \text{or } T_2 = \frac{\frac{1}{2} W}{\cos \theta_2}.$$

$$\therefore \frac{w + \frac{1}{2} W}{\cos \theta_1} \sin \theta_1 + \frac{\frac{1}{2} W}{\cos \theta_2} \sin \theta_2 = \frac{w v^2}{g r}.$$

But, $\tan \theta_1 = \dfrac{r}{h}$; $\tan \theta_2 = \dfrac{r}{k}$; and $v = 2 \pi r n$.

$$\therefore \quad (w + \tfrac{1}{2} W) \frac{r}{h} + \tfrac{1}{2} W \frac{r}{k} = \frac{w \times 4 \pi^2 r^2 n^2}{g r}.$$

Or, $$(w + \tfrac{1}{2} W) \frac{1}{h} = \frac{w \times 4 \pi^2 n^2}{g} - \frac{\frac{1}{2} W}{k}.$$

$$\therefore \quad h = \frac{w + \frac{1}{2} W}{\frac{4 \pi^2 n^2 w}{g} - \frac{\frac{1}{2} W}{k}}.$$

Hence, $$h = \frac{(2 w + W) g k}{8 \pi^2 n^2 w k - W g}$$

Or, since $n = \dfrac{N}{60}$, $$h = \frac{450 (2 w + W) g k}{\pi^2 N^2 w k - 450 W g} \qquad \text{(V)}$$

This governor is usually constructed with all four links of equal length; then $k = h$, and $\theta_1 = \theta_2 = \theta$, very nearly, unless the distance H K is great, and in our further investigations we shall assume that this is so.

In this case we have :—

$$T_1 \sin \theta + T_2 \sin \theta = \frac{w v^2}{g r}.$$

$$T_1 \cos \theta = w + \tfrac{1}{2} W, \text{ or } T_1 = \frac{(w + \frac{1}{2} W)}{\cos \theta},$$

And, $T_2 \cos\theta = \frac{1}{2} W$, or $T_2 = \frac{\frac{1}{2} W}{\cos\theta}$

$\therefore \quad (w + \frac{1}{2} W) \tan\theta + \frac{1}{2} W \tan\theta = \frac{w v^2}{g r}$.

But, $\tan\theta = \frac{r}{h}$ and $v = 2\pi r n$.

Hence, $(w + W)\frac{r}{h} = \frac{w \times 4\pi^2 r^2 n^2}{g r}$.

$$\therefore \quad h = \frac{w + W}{w} \times \frac{g}{4\pi^2 n^2}$$

$$\text{Or,} \quad h = \frac{w + W}{w} \times \frac{900 g}{\pi^2 N^2} = \frac{w + W}{w} \times \frac{2936}{N^2} \qquad \text{(VI)}$$

We might also have arrived at this result by putting h for k in equation (V).

If the speed of rotation change from N_1 to N_2 revolutions per minute, the corresponding heights of the governor will be:—

$$h_1 = \frac{w + W}{w} \times \frac{900 g}{\pi^2 N_1^2}; \qquad \text{and } h_2 = \frac{w + W}{w} \times \frac{900 g}{\pi^2 N_2^2}.$$

The alteration in the height of the cone of revolution would therefore be:—

$$h_1 \sim h_2 = \frac{w + W}{w} \times \frac{900 g}{\pi^2} \times \left(\frac{N_2^2 \sim N_1^2}{N_1^2 N_2^2}\right)$$

$$\text{Or,} \quad h_1 \sim h_2 = 2936 \frac{w + W}{w} \left(\frac{N_2^2 \sim N_1^2}{N_1^2 N_2^2}\right) \qquad \text{(VII)}$$

With the arrangement of links usually adopted in this governor, the travel of the sleeve is twice the change in the height of the balls and equal to $2 (h_1 \sim h_2)$.

Using the simpler equation (VI) we see that:—

$$W = \frac{\pi^2 N^2 w h}{900 g} - w.$$

And, therefore, if the speed alter from N_1 to N_2 revolutions per minute, the load necessary to keep the height of the cone constant, and equal to h, $\left(= \frac{w + W}{w} \times \frac{900 g}{\pi^2 N_1^2}\right)$, will change from—

$$W = \frac{\pi^2 N_1^2 w h}{900 g} - w \text{ to } W' = \frac{\pi^2 N_2^2 w h}{900 g} - w.$$

But if the actual load be W, the difference $W' \sim W$ is available for overcoming friction and moving the valve. This may be called the *working effort* for that change in speed, and is equal to:—

$$W' \sim W = \frac{\pi^2 w h}{900 g} (N_2^2 \sim N_1^2)$$

$$\text{Or,} \quad W' \sim W = \frac{\pi^2 w \dfrac{w + W}{w} \times \dfrac{900 g}{\pi^2 N_1^2} (N_2^2 \sim N_1^2)}{900 g}$$

$$\therefore \quad W' \sim W = (w + W) \frac{N_2^2 \sim N_1^2}{N_1^2}. \quad \ldots\ldots \text{(VIII)}$$

EXAMPLE II.—The balls of a Porter governor weigh 4 lbs. each, and the central weight 36 lbs. If all the links are of equal length, find the height of the governor when revolving 240 times per minute. If the speed increase to 248 revolutions per minute what will be the working effort and the rise of the balls and the sleeve?

Here, $N_1 = 240$; $N_2 = 248$; $w = 4$; and $W = 36$.

From equation (VI), $\quad h_1 = \dfrac{w + W}{w} \times \dfrac{2936}{N_1^2}$,

Or, $\quad h_1 = \dfrac{4 + 36}{4} \times \dfrac{2936}{240 \times 240}$.

$\therefore \quad h_1 = \cdot 51$ foot or 6·12 inches.

From equation (VIII), *The working effort*, $= (w + W) \dfrac{N_2^2 - N_1^2}{N_1^2}$,

" " $= (4 + 36) \dfrac{248^2 - 240^2}{240^2}$,

" " $= 2 \cdot 71$ lbs.

And from equation (VII), $h_1 - h_2 = 2936 \dfrac{w + W}{w} \left(\dfrac{N_2^2 - N_1^2}{N_1^2 N_2^2} \right)$,

Or, $\quad h_1 - h_2 = 2936 \times \dfrac{4 + 36}{4} \times \dfrac{248^2 - 240^2}{240^2 \times 248^2}$

$\therefore \quad h_1 - h_2 = \cdot 0324$ foot or $\cdot 389$ inch.

$\therefore$ *Travel of sleeve* $= 2 (h_1 - h_2) = 2 \times \cdot 389$,

" $= \cdot 778$ inch.

On comparing these results with those of Example I., it will be noticed that while both governors weigh about the same, the loaded governor has a working effort of about 20 per cent. greater than that of the Watt governor, but its travel and height are less. It will also be seen from equations (III) and (VII), by putting $N_2 = c N_1$, where c is a constant, that the change in height corresponding to a given percentage variation in speed gets smaller as the speed increases.

Spring Loaded Governors.—Soon after the introduction of the Porter governor, Mr. John Richardson, of Messrs. Robey & Co., Lincoln, designed one in 1869, in which a spring was substituted for the weight. This improvement produces a greater working effort with less weight, bulk, and cost. A spring has less inertia, and acts much more quickly than a weight, and it has also a certain amount of cushioning action. A governor loaded with a spring can act in any position, whereas one with a weight must work vertically. The equations for a spring governor may be obtained in the same way as for the weighted governor, but the load W will be different for different positions of the balls, owing to the varying compression of the spring.

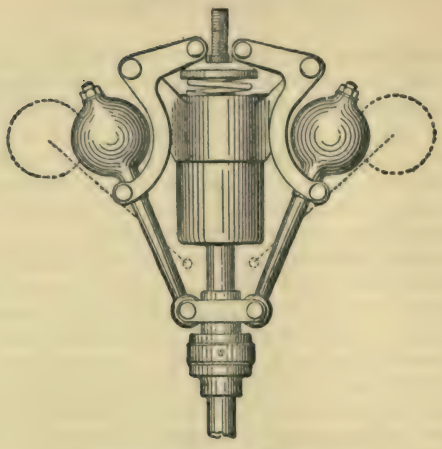
Proell and Hartnell's Spring Governors.*—The first of the two following figures illustrates the well-known Proell spring governor. It will be observed that a helical spring, contained in a cylindrical case, surrounds the governor spindle, and bears upon the inner ends of the two bell crank levers, which are connected to the arms carrying the governor balls. The dotted lines show the positions of the balls for a speed above the normal. As they move out to this position the spring is compressed and the sleeve is raised. It will further be noticed that the links are so proportioned that the balls diverge in nearly a straight line. Consequently, when working vertically, the balls do not move either with or against gravity.

It will be observed that there are no less than three pin joints on each side of the Proell governor above the sleeve, at each of which there must be friction. In Hartnell's governor, illustrated by the next figure, there is but one.†

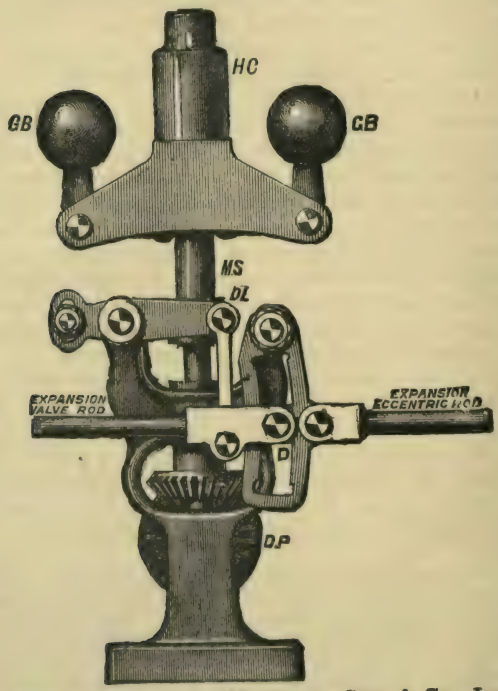
Here, the governor balls are fixed directly to the outer ends of the bell crank levers, the inner ends of which bear upon a collar on the upper end of the movable tube or sleeve, M S.

* The figure of Proell's governor is from *The Proc. Inst. C.E.*, vol. cxx., Session 1895-96, by kind permission of the Council, from a paper read by John Richardson, M.Inst.C.E., on "The Mechanical and Electrical Regulation of Steam Engines," which the student should consult.

† See Lecture XVIII. of the Author's *Text-Book on Steam and Steam Engines* for a description of an engine to which this governor is applied.



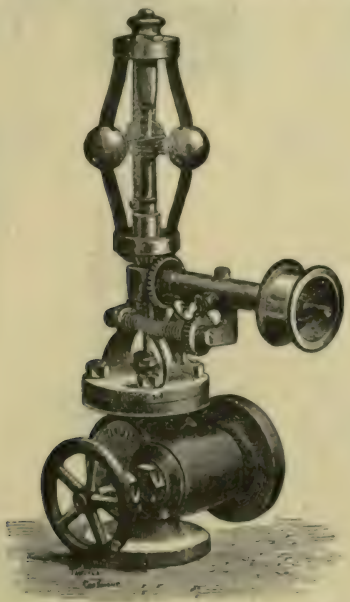
PROELL'S SPRING GOVERNOR.



HARTNELL'S SPRING GOVERNOR, BY MARSHALL, SONS & CO., LTD.

Between the top of this collar and the upper end of the hollow casting H C, which is keyed to the top of the governor spindle, there is placed a strong helical spring. The lower end of M S has a double collar engaged by a forked lever, connected to the drag link D L, and expansion valve rod.

Pickering Governor. — A very simple and direct acting governor which has been introduced for small electric light engines is shown by the next figure. Here the balls are supported by flat springs, which act directly on the throttle-valve spindle. There is also an auxiliary spring, as seen just



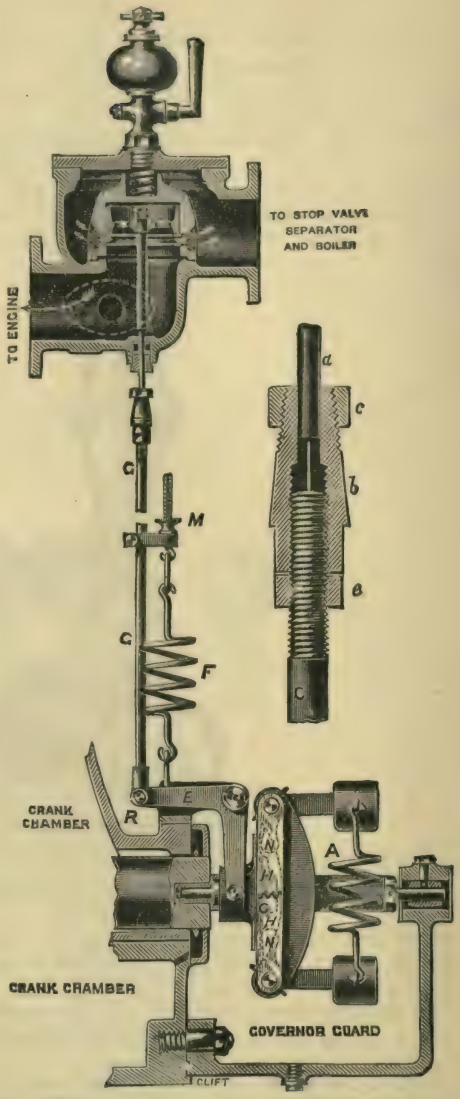
PICKERING GOVERNOR, BY TANGYES, LTD.

below the driving spindle, actuated by a thumb screw and worm wheel, which enables the attendant to adjust the speed of the engine whilst running.

Willans' Spring Governor.*—In the previous cases, the pressure of the spring has to be transmitted through the pin joints of the

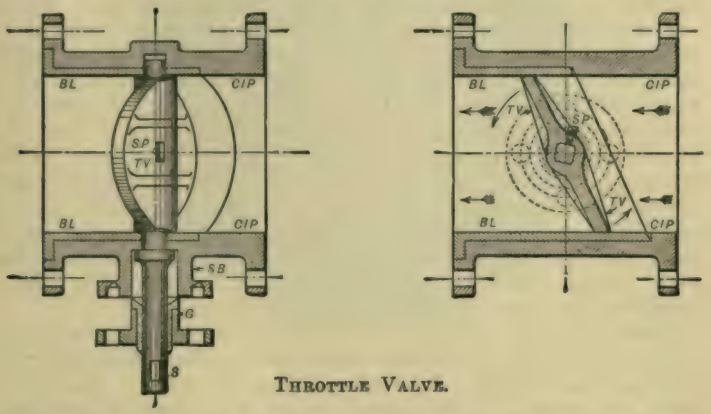
* For a description of Willans' central-valve triple expansion engine, to which this governor is fitted, see the Author's *Text-Book on Steam and the Steam Engine.*

governor arms, and thereby causes more friction and wear and tear than would be the case if the springs were directly connected to the balls. In Willans' governor, as will be seen from the figure, the balls are connected directly by a helical spring A, on each side of the governor spindle. Another spring F, is clamped at its upper end to the throttle-valve spindle G, and hooked at its lower end to the bracket carrying the bell crank lever E. This spring pulls the valve rod downwards, in opposition to the springs A, and thus pushes the sleeve against the toes N N (shown dotted), of the governor arms. By adjusting the tension in F, by the nut M, the governor can be set to the required speed while the engine is running. It will be noticed that this governor works horizontally, and is driven directly by one end of the shaft.



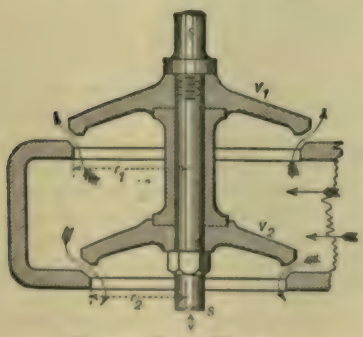
WILLANS' SPRING LOADED GOVERNOR.

Governing by Throttling and Variable Expansion.—Prior to 1876, governors generally controlled steam engines by actuating a butterfly throttle valve of the form shown in the figure. This valve, although simple in construction, is difficult to fit so as to remain steam tight, and hence the double-beat valve shown in



THROTTLE VALVE.

next illustration, or still better, a grating piston valve like that shown attached to the Willans' governor, has been adopted in preference. The ordinary butterfly throttle valve is not, as at one time supposed, a balanced valve, since the action of a fluid rushing past an oblique plane is such, as to cause a greater pressure on the forward edge and thus tend to close the valve. A good throttle valve should be able to entirely stop the admission of steam to the cylinder.



DOUBLE-BEAT VALVE.

Recently, many patents have been taken out for controlling the speed of an engine by altering the point of cut-off. In most cases, this enables the engine to work more economically; but as shown by Captain Sankey in his paper on "Governing of Steam Engines by Throttling and by Variable Expansion" (read before the Institute of Mechanical Engineers, in April, 1895), the indicator diagrams obtained from engines governed by this method are often "cloaks for exaggerated initial con-

densation," and it may be found that the actual feed water used, is less with ordinary throttling than with variable expansion. Throttling the steam varies the amount supplied by varying the pressure, while the volume used remains constant. On the other hand, automatic expansion supplies the steam at a constant pressure but alters the volume used per stroke.

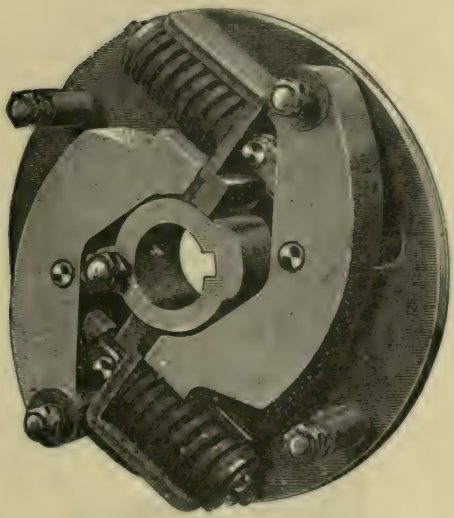
The point of cut-off may be controlled either by means of a separate expansion valve, or by acting directly on the main valve or valves. In the first case, there are two eccentrics which work the main and the expansion valves. As will be seen from the illustrations of the Hartnell, and Clayton & Shuttleworth's governors, the stroke of the expansion valve is altered by a drag link and a block connected to the governor sleeve. In the second case, when a slide valve is used, either the throw or the position of the main eccentric is varied by a shaft governor, and no second eccentric is required. With "trip gear" the governor automatically releases the admission valves sooner or later, according to the load on the engine.*

Shaft Governors.—A large number of these have been designed, but the following illustrations will serve to show their general principle and action. A circular casting is keyed to the crank shaft, and carries on one side a pair of symmetrically arranged weights jointed thereto at one end, but whose other ends are free to move in a plane perpendicular to the shaft against the resistance of the interposed helical springs. On the other side of this casting there is fixed a pair of straps embracing a circular disc carrying the eccentric which works the valve. The centre of this disc is some distance from the centre of the shaft and that of the eccentric. The governor weights have bosses which pass through slots in the circular casting, and are connected by links to studs on the disc. In moving outwards by centrifugal force, these weights compress the springs and rotate the disc, thus changing the position of the eccentric, and varying the cut-off of the slide valve.

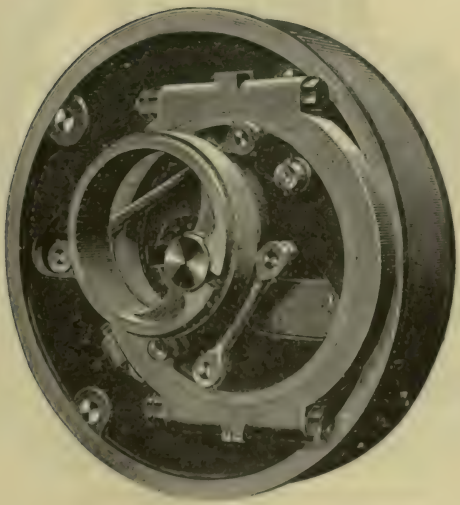
Relays.†—Except in the case of "trip gear," the effort required to work the throttle valve, or expansion gear, may be considerable, and can only be satisfactorily supplied by a relay—that is, by making the engine itself, or steam from the boiler, or water pressure, or electro-magnetic mechanism, move the valve, while

* See Index for page in the Author's *Text-Book on Steam and Steam Engines* for illustrations.

† See *Engineering*, 1st January, 1886, p. 4, for a description of Lüdes steam relay governor. Also "Regulation of Steam Engines," by John Richardson, *Proc. Inst. C.E.*, vol. cxx., 1895, Part II., for description and discussion on electrical and other relays for governors.

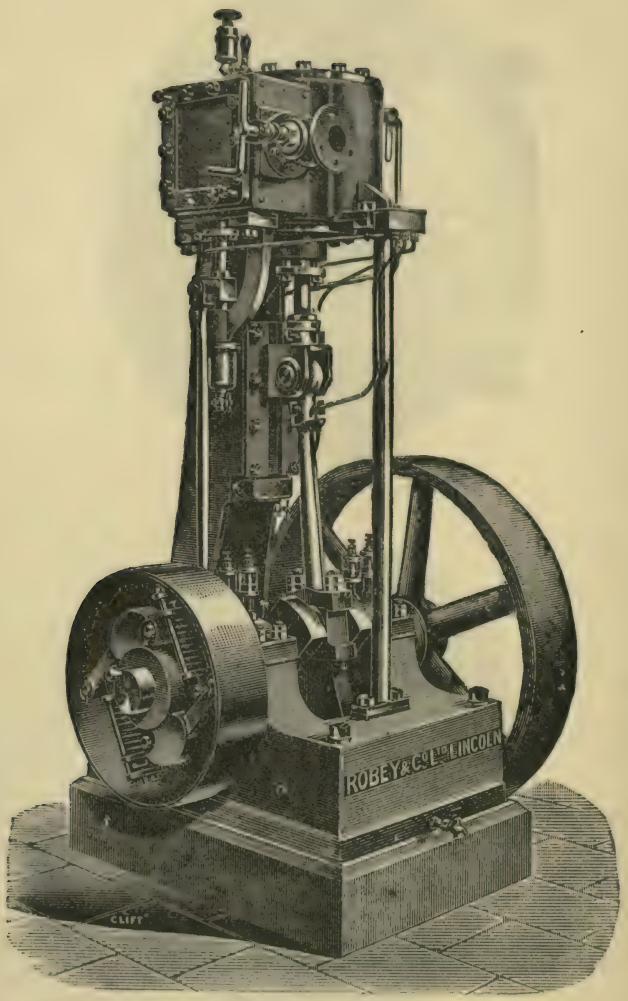


FRONT VIEW.



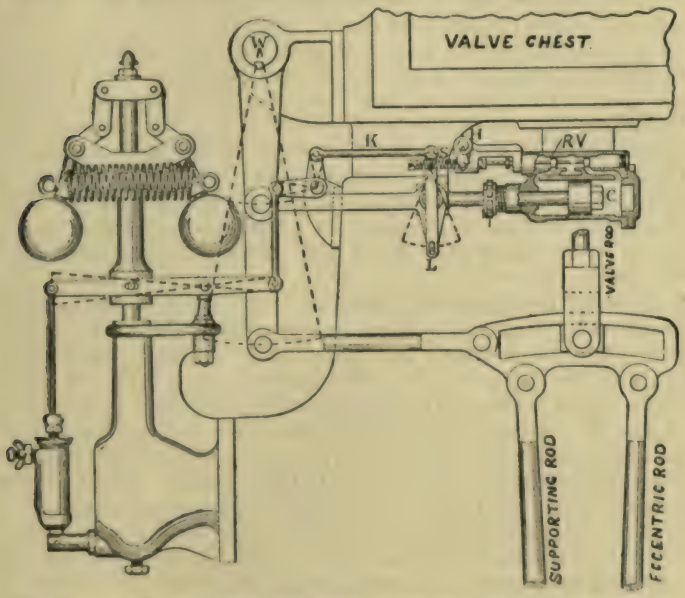
BACK VIEW.

AUTOMATIC EXPANSION SHAFT GOVERNOR,
BY MESSRS. RANSOMS, SIMS & JEFFERIES, LD.



SHAFT GOVERNOR APPLIED TO A HIGH-SPEED ELECTRIC LIGHT ENGINE.

the governor has simply to control the relay. In most relays that have been used for governing, the governor starts the relay, and then the latter goes on without any control, until the position of the governor is altered, and it is set into motion in the opposite direction. All such relays necessarily have the fault of hunting, but this is not so for one of the steering-gear type. The governor, as it were, informs such a relay when to move and how far, and the extent of the change in the height of the governor cone determines the travel of the relay.



GOVERNOR WITH RELAY FOR COMPOUND AND TRIPLE EXPANSION ENGINES, BY DAVEY, PAXMAN & Co.

The steam relay shown is applied by Messrs. Davey, Paxman & Co. to compound and triple expansion engines. The weigh-shaft W, which works the expansion gears of all the cylinders, is connected with the piston of the small relay cylinder C. The relay valve R V, which admits steam to this cylinder, is worked by the floating lever L, and allows steam to enter at its middle and exhaust at its ends. The lower end of the floating lever is attached at L, to the crosshead of the relay piston rod, and its upper end through the links K, &c., to the governor sleeve,

while the small piston valve of the relay is connected to an intermediate point. It will thus be seen that when the governor balls rise, R V will move to the left, and so admit steam to the left of the relay piston. This forces the piston inwards, and pushes the eccentric-rod end of the drag link further from the block attached to the engine valve rod, and therefore reduces the travel of the valve and the power of the engine. In addition, as the relay piston moves one way or the other, it rotates the floating lever L, about its upper end, and brings R V back to its mid position, and so automatically comes to rest. By this means, the relay piston and main slide valves are made to follow all the motions of the governor, and the amount of the motion of the relay piston will depend on the change in the height of the balls. The governor itself has very little work to do, since it has only to move the small valve R V. By means of the weigh shaft W, and levers attached to it the valve rods of all the cylinders are moved simultaneously, in the same way as the one shown. Minor adjustments of the speed may be made while the engine is running by altering the tension in the spring S, by means of a worm and worm wheel on the end of the spindle H.

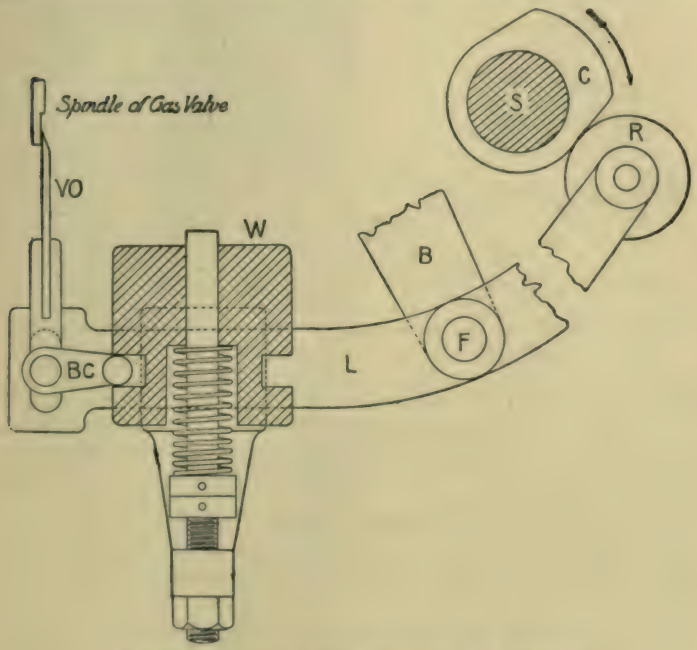
Knowles' Supplemental Governor.*—Another method is that invented by Knowles. Here two governors are used, a large one to control the valve in the ordinary way, and a smaller one to alter the length of the rod connecting the first one to the valve. This is effected by fitting two friction cones to the sleeve of the smaller, or supplemental, governor, and having a third between them, which will gear with one or other if the governor rises or falls by more than a prescribed amount. The valve rod is in two parts, having a right- and left-handed screw respectively at their adjacent ends, and the nut which joins these screws is rotated by the third friction cone. This governor has been extensively employed in spinning mills, where the fluctuations in load are neither great nor sudden, but where the speed must remain very constant.

Inertia Governors.†—For small gas engines, which always receive a full charge of gas during each cycle or none at all, a form of governor known as the inertia governor, has been found suitable. In the one first illustrated, the gas valve is opened by a valve opener V O, which is actuated through the lever L, by the cam C, fixed on the side shaft S. On the lower end of the valve opener there is a bell crank B C, engaging a slot on the

* See the *Practical Engineer*, vol. v., p. 205, March 27, 1891.

† See *Gas, Oil, and Air Engines*, by Bryan Donkin, for other forms of gas engine governors.

governor weight W. The weight is supported by a spring fixed to a bracket on the lever L. As the lever is moved upwards, the inertia of the weight W, causes it to lag behind, and thus compress the spring, but the latter is so adjusted that as long as the speed does not exceed the normal, B C is not moved down sufficiently to cause V O to miss the gas valve spindle.



INERTIA GOVERNOR FOR STOCKPORT GAS ENGINE.

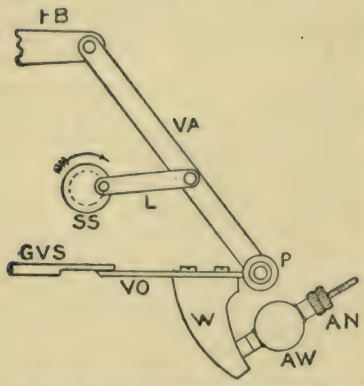
INDEX TO PARTS.

C for Cam.	B for Bracket.
S ,, Side shaft.	W ,, Inertia weight.
R ,, Roller.	B C ,, Bell crank.
L ,, Lever.	V O ,, Valve opener.
F ,, Fulcrum.	

If, however, the speed should rise above the normal, the inertia of the weight is sufficient to press B C down far enough to cause V O to pass to the right of the spindle, and then no gas is admitted for that cycle. As the direction of the thrust necessary to open the valve passes through the centre of the pin supporting

the bell crank B C, it is received direct from the lever L, and does not affect the governor weight W.

Another, and simpler form of inertia governor, as used in small Otto gas engines, depends for its action on the inertia of a small weight W, and adjusting weight A W. The vibrating arm V A, is driven by the link L, from a pin on the end of the side shaft S S, and causes the valve opener V O, and the weight W, to move backwards and forwards. The centre of gravity of the weights and valve opener being to the right of the pin P, the point of V O is pressed against the flat of the gas valve spindle. The effect of the inertia of these weights acts below P, and therefore tends to turn V O downwards when P is moved to the left. The position of A W is so regulated by the adjusting nuts A N, that when the speed exceeds what is desired, the latter tendency will predominate and cause the end of V O to pass below the valve spindle and leave the valve unopened.



INERTIA GOVERNOR FOR OTTO GAS ENGINE.

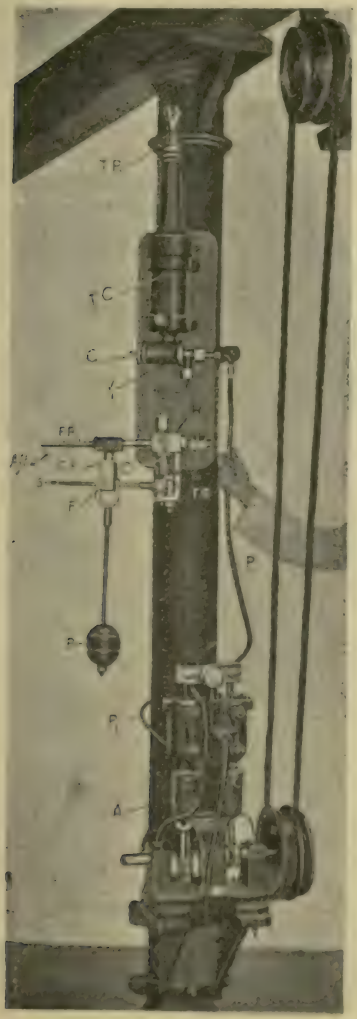
Thunderbolt's Marine Engine Governor. *—When a short or a lightly loaded screw-driven steamship encounters heavy weather with "head seas," the screw propeller frequently rises out of the water. This action naturally diminishes its resistance to rotation, since the screw is then revolving more or less in the air, instead of in the denser salt water. Consequently, the engines commence to "race," which is not only very uncomfortable to passengers and crew, but may also be dangerous to the machinery; for, immediately after such a rise, the screw will undoubtedly be as suddenly immersed again in the water, thus alternately subjecting the screw shaft and other moving parts to very severe stresses. To such rough usage may be attributed some of the "delays," "breakdowns," and even total losses of steamships.

Many marine engine governors have been devised and applied, with more or less success, for the object of so automatically adjusting the supply of steam to the engines, that "racing" should not take place. As far as we are aware, one of the most successful of these governors, which has lately been put into

* See Vol. IV. for Application of Governors to Water Motors and Turbines

extensive daily practice, is that made and supplied by "The Thunderbolt Governor Coy., Ltd.," of Middlesbro'-on-Tees.

General Arrangement.—As will be seen from the accompanying outside view (Fig. 1) this governor consists of—



INDEX TO PARTS.

Figs. 1, 2, and 3.

A for Air Compressors.
P_1, P_2 ,, Copper Pipes.
R ,, Regulator.
O ,, Outlet Port.
V_1, V_2 ,, Valves.
TC ,, Throttling Cylinder.
TR ,, Throttle Valve Rod.
G ,, Gland for V_1 box.
FR ,, Fixed Rod.
CF ,, Clamped Fork.
S ,, Spindle.
F ,, Fulcrum.
B ,, Ball of Pendulum.
TS ,, Thumb Screw.
V_3 ,, Safety Valve.
EG ,, Emergency Gear.
→ ,, Direction Aft.

FIG. 1.—THUNDERBOLT'S MARINE ENGINE GOVERNOR.

Connected-up for Testing Prior to being Fitted on Board Ship.

1. A set of duplex double-acting air compressors, A, driven by rope, belt, or wheel gearing from the main screw shaft.

2. An automatic regulator, R, actuated by a heavy pendulum ball, B.

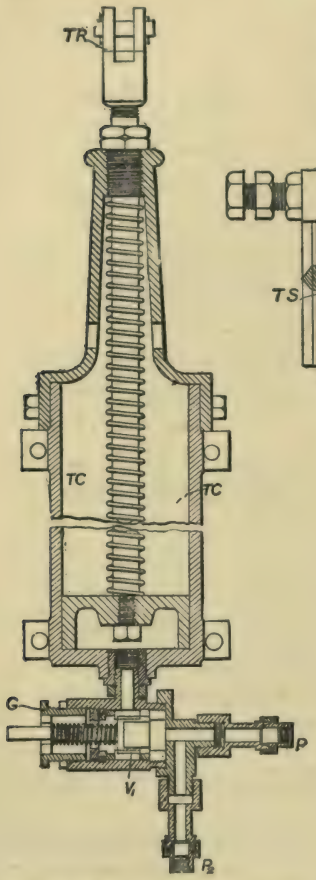


FIG. 2.—THROTTLING CYLINDER.

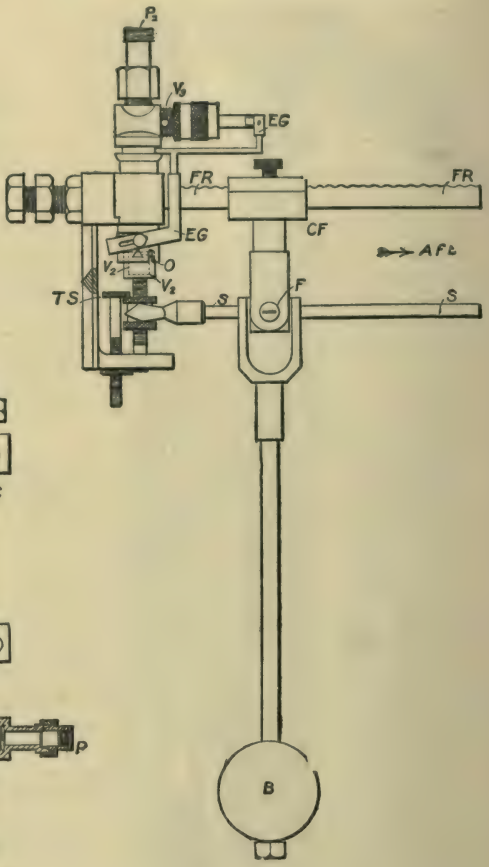


FIG. 3.—REGULATOR.

3. A throttling cylinder, T C, whose piston is connected through, T R. to the main steam-pipe throttle valve.

In this general view, these parts are all shown connected together and fixed to one of the vertical columns of the workshop, where they are made. The whole is just ready for being tested and adjusted, before being fitted on board a vessel. A ½-inch copper pipe, P_1, connects the air compressors, A, with the inlet spring piston valve, V_1, for its cylinder, T C; and another similar pipe, P_2, connects the regulator, R, also with the said valve, V_1.

Action of the Governor.—When the governor is in action and the speed of the main engine is uniform, the compressed air from A passes freely through the pipes, P_1, P_2, and outlet port, O (see Figs. 2 and 3 as well as Fig. 1). Now, should the stern of the vessel rise out of the water, every fore and aft fixture in the vessel, which may have been previously level or vertical, is thereby inclined to its normal position. But the heavy pendulum with its ball, B, being free to swing from its fulcrum, F, as an axis of motion, tends to remain in a *vertical position* due to its inertia. This naturally raises the *forward* conical end of the spindle, S (to which the upper forked end of the pendulum rod is attached), and lifts the valve, V_2, thus partially or wholly closing the outlet port, O. The air from the compressor pump is consequently prevented from escaping as freely as before by the outlet port, O. The pressure of the air in P_2 is thereby very quickly increased, and acting at once upon the piston valve, V_1, against the reaction of its spring, the air passes up to and raises the piston of the cylinder, T C, against the downward pressure of its spiral spring. As this piston rises, it elevates the throttle valve rod, T R, and cuts off the steam supply from the main engines. The speed is automatically prevented from increasing to a dangerous or even inconvenient degree.* On the other hand, when the bow of the ship rises and the screw is thereby suddenly immersed, precisely the opposite action takes place. The heavy pendulum with its ball, B, remains vertical, the conical end of S is depressed and the valve, V_2, uncovers more than the normal amount of the outlet port, O. The air pressure in P_2 and on V_1 is thus reduced below the normal, and the spring in the cylinder, C, presses down its piston, which lowers T R and opens the main steam-pipe throttle valve to the full extent;

* In the case of compound, triple, or quadruple expansion engines, where the steam once admitted naturally applies its force throughout the several cylinders placed in series, then this governor may be also connected to an exhaust throttle valve, as well as to the steam one.

thereby tending to supply the desired quantity and pressure of steam, to keep the engines moving at as uniform a rate as possible, under these erratic circumstances.

When the piston of the throttling cylinder, T C, descends, the air which it contains, escapes freely through holes in the gland, G.

It will be seen, that the action of the pendulum causes the throttle valve of the main engines to synchronise, with the up and down pitching movements of the vessel in such a manner, as to supply steam in proportion to the resistance offered to her screw. Also, the quicker the change in speed of the engines, the quicker will be the alteration in the air pressure, as the compressors are driven by the engines.

With "following seas" the valve, V_2, should be so adjusted by the thumb screw, T S, as to partly close the outlet port, O, and make the governor act more like a stationary or standard speed governor. This governor can be rendered more or less sensitive, or quick in its action (to suit different lengths and kinds of vessels, when running light or loaded), by shifting the fore and aft position of the fulcrum, F, with the suspended clamped fork, C F, along the fixed rod, F R.

Emergency Cut-off.—Should the screw shaft break, or from any other cause the engines "run off," then, the sudden and great increase of air pressure, forces out a small safety valve, V_3, connected directly to the emergency gear, E G, which falls and thus entirely closes the outlet port, O. The pressure of air in, T C, consequently increases and keeps its piston with its throttle valve rod, T R, fully raised, until the engineer can close the main boiler stop valve.

General Behaviour. — From reports of well-known superintending and chief engineers of steamships it is said, that the main stop valve may be left full open, whilst encountering heavy "head seas," when this governor is properly applied and worked, for it has been well designed and made.

Thunderbolt's Electric Governor Regulator.—The same firm make an "Electric Governor Regulator," where a vertical electro-magnetic solenoid, takes the place of the pendulum in the previously described marine form. The terminals of this solenoid are connected to those of the dynamo or switchboard. If an abnormal rise in voltage takes place, then an adjustable movable iron core is attracted further than usual, into the hollow cylindrical centre of the solenoid, thus raising the conical end of the spindle, S, and closing the throttle valve in the manner previously described. Should the voltage fall below the normal then the counter-spring attached to the upper end of the iron

core asserts itself and depresses the conical end of S, which naturally opens the throttle valve and gives the engines more steam.

"Experiments upon the Action of Engine-Governors."*—Although the subject is admitted to be of great importance, comparatively few experiments have been made on engine-governing. The performances of different governors seem to vary so widely, that it is by no means easy to choose the type best suited to an engine working under given conditions. With a view, therefore, to observe in closer mechanical detail the behaviour of different kinds of governors, experiments were made at Mason College, Birmingham, under the superintendence of Professor R. H. Smith, M. Inst. C.E., on nine governors of different types.

From speed-curves and other data connected with any particular governor which you may consider, it is possible to obtain a practical measurement of those quantities which are mainly concerned in the efficient action of a governor—viz., its sensitiveness, its controlling force, and its controlling energy. These quantities must be expressed in terms of some unit of speed-variation, and may be referred to a change of speed amounting to 1 per cent. of the mean speed for which the governor is designed.

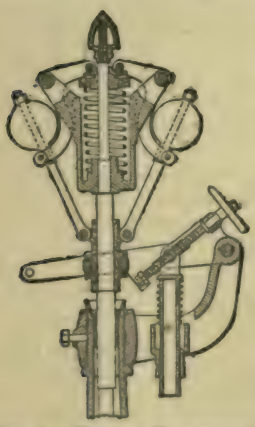


FIG. 1.—PROELL GOVERNOR.

The term "sensitiveness" will be used to indicate the distance in inches through which the sleeve would move in either direction from its mean position in response to a 1 per cent. variation of speed, if there were no frictional resistances to overcome, and when no controlling force is exerted.

The term "controlling force" will be used to indicate the force which would be exerted by the governor upon the controlling gear, if the given variation of speed were to take place without as yet producing any motion of the sleeve.

* I have to thank Mr. W. G. Hibbins, A.M.Inst.C.E., the author of this "Selected Paper," and the Council of the Institution of Civil Engineers for their kind permission to make the following extracts. See *Proc. Inst. C.E.*, vol. cxxxvii., pp. 376 to 401, of 1899, for the complete paper. These detailed experiments will form a useful guide to students, in any similar investigations which they may have to undertake.—A. J.

The term "controlling energy" will be used to indicate the inch-pounds of work that would be done by the sleeve, if, starting from its mean position, it were caused to travel through a distance equal to the sensitiveness, first upwards and back again, then downwards and back again. The speed during each ascent being always 1 per cent. greater, and during each descent 1 per cent. less, than the speed corresponding to the momentary position of a sleeve working without friction. Hence, the "controlling energy" is very nearly the same quantity as the "controlling force," multiplied by four times the distance which represents the "sensitiveness."

Proell Governor.—From tests of this governor (Fig. 1), the curves shown in Fig. 2 were obtained.

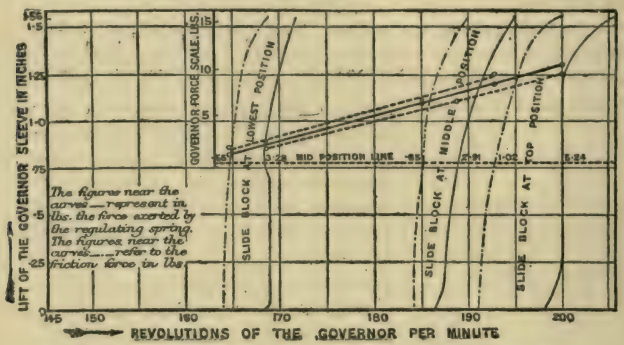

LIFT OF THE GOVERNOR SLEEVE IN INCHES
GOVERNOR FORCE SCALE LBS.
SLIDE BLOCK AT LOWEST POSITION
SLIDE BLOCK AT MIDDLE POSITION
SLIDE BLOCK AT TOP POSITION
MID POSITION LINE
The figures near the curves represent in lbs. the force exerted by the regulating spring. The figures near the curves refer to the friction force in lbs.
REVOLUTIONS OF THE GOVERNOR PER MINUTE

FIG. 2.—SPEED-CURVES FOR PROELL GOVERNOR.

The following are details of this type :—

Weight of each ball, ...	=	4·8 lbs.
Unstrained length of large spring of the governor, ...	=	$7\frac{3}{4}$ inches.
Compressive strength, ...	=	120 lbs. per inch extension.
Pitch of the coils, ...	=	$\frac{5}{8}$ inch.
Outside diameter of coils, ...	=	$2\frac{9}{8}$ inches.
Diameter of steel of which the spring was made as given by the micrometer gauge, .	=	0·313 inch.

When placed in the governor, the length of the spring was $5\frac{13}{16}$ inches, which was equivalent to a pressure of 232·5 lbs. on the cap with the governor at rest.

Natural length of small regulating spring,	=	$7\frac{3}{4}$ inches.
Pitch of coils, ...	=	$\frac{1}{2}$ inch.
Outside diameter, ...	=	$1\frac{3}{4}$,,
Diameter of its steel, ...	=	0·177 inch.
Mean strength, ...	=	16·7 lbs. per inch.

With the slide-block at the bottom of the slide, the regulating spring was compressed $\frac{3}{4}$ inch, when the governor was at rest; the pressure then exerted is 12·52 lbs., which is equivalent to 8·24 lbs. acting vertically upwards at the centre of the governor spindle. With the slide-block at the centre of the slide, the small spring was stretched $\frac{5}{32}$ inch with the governor at rest. The downward pull then exerted is 2·6 lbs., equivalent to 1·33 lbs. at the centre of the governor spindle. With the slide-block at the top of the slide, the small spring is extended $\frac{5}{8}$ inch, the governor being at rest; the downward pull then exerted is 10·43 lbs., equivalent to 3·97 lbs. at centre of governor spindle.

The downward force exerted at the centre line of the governor spindle by the small spring has the following values when the balls are fully extended—

Block in lowest position,	. . . . .	1·87 lbs.
,, central ,,	. . . . .	5·2 ,,
,, highest ,,	. . . . .	6·56 ,,

The pressure exerted by the large spring with the balls fully extended was 326·25 lbs.

Fig. 3 shows the distances from the centre line of the governor spindle to the slide-block in its different positions.

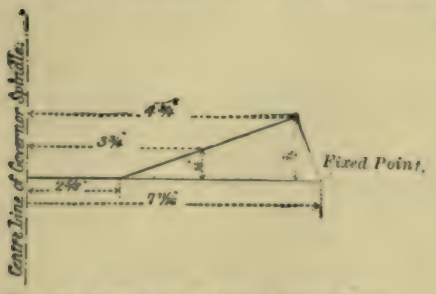


FIG. 3—DISTANCES FROM CENTRE LINE OF GOVERNOR SPINDLE TO SLIDE-BLOCK.

FIG. 4.—BELL-CRANK LEVER.

Diameter of each ball, ...	=	3¼ inches.
Weight of ball and arm, ...	=	6 lbs. 4½ ozs.
Length of governor arm from ball centre to pin centre, ...	=	8 inches.
Full movement of governor-sleeve, . .	=	1·56 inches.

Fig. 4 shows the leverage exerted by the bell-crank levers which are attached to the governor arms. As shown by the curves in Fig. 2, the governor was exceedingly sensitive, a difference of only 9 revolutions per minute being sufficient to

send the governor sleeve from its lowest to its highest position, the slide-block being at the centre of the slide.

The governor sleeve is at mid-position when 0·78 inch high. The loads placed on the sleeve in this case are those due to the weight of the lever, &c., and the tension or compression of the regulating spring. As it is difficult to estimate the pressure on the governor sleeve due to the weight of the lever, this was measured by a spring-balance attached to the lever at the centre line of the governor spindle, and found to be 3 lbs. There was also a change in the pressure on the sleeve due to the shifting of the slide-block and its attachments.

The pressures exerted at centre line of governor spindle with the block in the three positions were—

2·01 lbs. when the slide-block was in its lowest position,
1·56 ,, ,, ,, ,, middle ,,
and 1·16 ,, ,, ,, ,, highest ,,

The loads due to the regulating spring, with the sleeve at mid-position, were—

3·28 lbs. acting vertically upwards with slide-block in lowest position,
2·91 ,, ,, downwards ,, middle ,,
and 5·24 ,, ,, ,, ,, highest ,,

The total loads for the three positions are therefore—

$(3 + 2{\cdot}01 - 3{\cdot}28)$ lbs. = 1·73 lbs. acting downwards for 1st position,
$(3 + 1{\cdot}56 + 2{\cdot}91)$,, = 6·47 ,, ,, ,, 2nd ,,
and $(3 + 1{\cdot}16 + 5{\cdot}24)$,, = 9·4 ,, ,, ,, 3rd ,,

These loads have been plotted upwards from the mid-position line, as shown in Fig. 2, and to the given scale. Two straight lines are obtained, as shown; and, measuring their inclination by the divisions of the squared paper as before, the following results are obtained:—

With slide-block in lowest position, friction = 0·43 lb.
,, ,, middle ,, ,, = 0·708 ,,
,, ,, highest ,, ,, = 1·04 ,,

The normal speed of the governor was supposed to be 190 revolutions per minute, and 1 per cent. of this is 1·9 revolutions, equivalent to 1·9 horizontal divisions. Hence, controlling force = 0·50 lb.

Considering the middle pair of curves—*i.e.*, with the slide-block at mid-position—it requires an increase in speed of from 187·5 revolutions to 194·3 revolutions per minute to lift the sleeve from a height of 0·25 inch to 1·5 inch. Therefore—

$$\text{Sensitiveness} = \frac{1{\cdot}9}{6{\cdot}8} \times 1{\cdot}25 = 0{\cdot}349.$$

$$\text{Controlling energy} = (4 \times 0{\cdot}5 \times 0{\cdot}35) = 0{\cdot}70 \text{ inch-lb}$$

The Tangye, Acme, Lüde, and Proell governors were driven by belting from a counter-shaft, which was in its turn driven by means of a vertical steam engine. The Watt, Porter, Pickering, Turner-Hartnell, and Belliss governors as tested, were driven by means of an electric motor, the speed of which could be regulated exactly by means of suitable resistance frames.

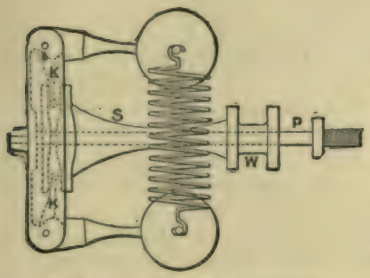


FIG. 5.—BELLISS GOVERNOR.

Belliss Shaft Governor.—The governor of this type (Fig. 5) was supposed to run at about 300 revolutions per minute, and to control an engine of 60 H.P. The brass sleeve, S, is moved by the levers, K K, along the shaft, P. A lever is fixed on to the sleeve at W, which moves the valve-gear. In order to test the governor, it was placed between the centres of a lathe, and driven by the electric motor. The regulating arrangement was attached as shown in Fig. 6.

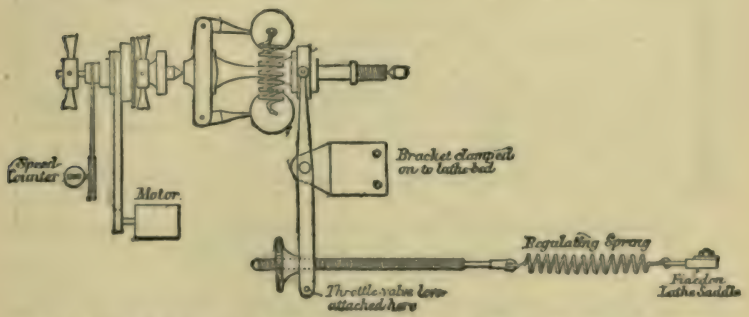


FIG. 6.—REGULATING ARRANGEMENTS WHEN TESTING THE BELLISS GOVERNOR.

Three pairs of curves (Fig. 7) were obtained as follows:—Curves 1, 1, with no regulating spring attached to the lever. Curves 2, 2, with regulating spring attached, but with no tension at starting. Curves 3, 3, with the regulating spring stretched 1·28 inches at the start = 34·12 lbs. pull.

The following are details relating to this governor :—

Mean strength of governor springs	=	5·6 lbs. per inch extension.
,, regulating spring	=	26·66 ,, ,,
Weight of throttle valve and vertical gear, ...	=	4·75 lbs.
Weight of one ball, with arm, &c.,	=	7·42 ,,

This governor appeared to be very powerful, and at the same time, fairly sensitive. Curves 3, 3 give the nearest approach to the actual working curves.* The movements of the sleeve, S, were in this case measured directly, which could easily be done as the sleeve kept very steady, and the speed of rotation could be regulated exactly. The speed was recorded by a speed counter, driven by the cone pulley of the lathe.

The sleeve is at mid-position when about 0·44 inch from its starting point. The loads calculated are zero for curves 1, 1,

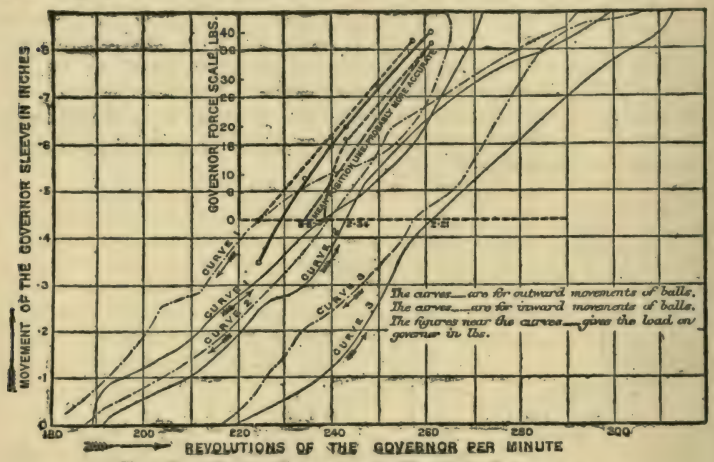

MOVEMENT OF THE GOVERNOR SLEEVE IN INCHES
GOVERNOR FORCE SCALE LBS.
CURVE 1
CURVE 2
CURVE 3
The curves—are for outward movements of balls.
The curves—are for inward movements of balls.
The figures near the curves—give the load on governor in lbs.
REVOLUTIONS OF THE GOVERNOR PER MINUTE

FIG. 7.—SPEED-CURVES FOR BELLISS GOVERNOR.

owing to the fact that no regulating spring is on the governor lever ; 17·07 lbs. for curves 2, 2 ; and 37·76 lbs. for curves 3, 3. These are plotted off above the mid-position line, according to scale ; and, tracing the two lines shown in Fig. 7, the following results are obtained by the methods already described :—

For the curves 1, 1, ...	friction	=	8·8 lbs.
,, 2, 2, ...	,,	=	2·34 ,,
,, 3, 3, ...	,,	=	2·21 ,,

* It was intended to obtain curves such that the mean speed of rotation was 300 revolutions per minute, the speed given by the makers for this particular governor, but this was impossible, owing to the shortness of time during which the governor could be spared for the experiments.

The mean speed of the governor is given as 300 revolutions per minute, and 1 per cent. of this is 3 revolutions, equivalent to 1·5 horizontal divisions. Hence, the controlling force of the governor = 3·89 lbs.

Considering curve **3**, which is the nearest approach to the working curve, if the speed is altered from 238 to 305 revolutions per minute, the sleeve is moved from 0·1 inch to 0·8 inch; hence, a change in speed of 67 revolutions per minute moves the sleeve 0·7 inch; therefore, the sensitiveness = 0·031. The controlling energy = (3·89 × 0·031 × 4) = 0·482 inch-lb.

Comparison of Different Governors.—The frictional resistance was reduced to a minimum in each governor, by carefully cleaning and oiling the working parts, but even then it varied considerably in amount, as is shown by the forms taken by several of the curves, and also by the calculations obtained from them. It will be observed that the Proell governor possessed very little frictional resistance in its working parts, as compared with some of the others, and was closely followed in this respect by the Tangye and four-pendulum governors. It should be remembered, however, that the Proell governor had no valve attached, and therefore the frictional resistance of the valve-spindle at the stuffing-box is an element not included in the experiments. The Pickering governor curves exhibit a frictional resistance much greater than might be expected, considering that it possesses no pin joints. It is, therefore, certain that the greater part of the frictional resistance in many governors is produced at the stuffing-box of the valve-spindle. In the governors tested, the stuffing-box was not screwed tighter than is usual in practice. It is also probable, that in a Pickering governor, there is a considerable amount of frictional resistance at the points where the lever of the regulating spring presses on the governor valve-spindle, and also inside the cap at the top of the governor.

The table on the following page shows the chief results obtained from the nine governors tested.

Comparing the values of the controlling force, it will be noticed that the small Tangye governor exhibits a high value; it is, however, lacking in sensitiveness. Another notable feature is the large controlling force of the Turner-Hartnell governor, and the large amount of frictional resistance in its working parts. It is evident that the controlling force depends largely on the weight of the balls, and the central weight or "muff" of the governor, for the highest values are exhibited by the Lüde, Watt, Porter, Turner-Hartnell, and Belliss governors, which all had either heavy balls, weights, or a heavy central load. As regards sensitiveness, the Proell governor appears to be far

superior to the rest. It is striking how favourably the Porter governor compares with some of the more modern types, possessing, as it does, both sensitiveness and great controlling force. The Watt governor also shows large controlling force, due no doubt to its very heavy balls.

Type of Governor.	Mean Speed Revolutions.		Sensitiveness.	Controlling Force.	Controlling Energy.	Approximate Mean Frictional Resistance in Lbs. for Working Curves.
		Per Minute.		Inch-lbs.	Lbs.	
Tangye (Soho),		400·0	0·009	1·54	0·056	2·6
Acme,		400·0	0·016	1·98	0·13	3·22
Lüde (4-pendulum governor),	Load = 0	200·0	0·056	0·95	0·213	2·62
	,, = 20 lbs.	229·25	0·102	1·96	0·799	1·90
Proell,		190·0	0·35	0·50	0·70	0·71
Watt,		90·0	0·113	4·18	0·189	4·3
Porter,		260·0	0·08	1·04	0·33	2·87
Pickering,		400·0	0·018	0·42	0·03	3·85
Turner - Hartnell (shaft-governor),		200·0	0·016	7·99	0·511	...
Belliss (shaft-governor),		300·0	0·031	3·89	0·482	2·21

Results.—The following may be regarded as a summary of the results arrived at :—

1. In testing governors by running them steadily at different speeds, great difficulties have to be overcome to obtain the natural characteristics of the governors considered, chiefly owing to frictional resistances, which appear to vary erratically.

2. If the different governors were placed on the engine and tested by suddenly altering the load, there would still be uncertainty in the results, owing to the variations of the frictional resistance in the valve-gear, which is connected with the sleeve (or the eccentric) of the governor.

3. The controlling force of a governor is increased by increasing the weight of the balls or the central load.

4. The controlling force and sensitiveness are increased by providing the governor with powerful springs.

5. Increasing the masses of the revolving balls or weights also increases the amount of frictional resistance.

6. Shaft-governors can be made much more powerful, bulk for bulk, than vertical governors.

7. The advantages of using the more modern types of governor over using the older types, such as well-designed Watt and Porter governors, are not so great as perhaps might be expected.

APPENDIX.

To Obtain the Controlling Force Graphically.

"A method of obtaining graphically the controlling force of a vertical governor is shown in Fig. 8.

Let C = centripetal force in lbs. acting at the centre of the governor ball in order to maintain equilibrium at the normal speed.
W = weight in lbs. of each of the governor balls.
T, t = tension in lbs. on the upper and lower arms respectively.
N = number of revolutions of the governor per minute at the normal speed.
F = the total load on the sleeve divided by the number of balls.
f = the controlling force exerted at the sleeve for each ball in the governor, or the whole controlling force divided by the number of balls.
θ and ϕ = the angles made by the centre lines of the upper and lower arms with the vertical centre line of the spindle when the governor is running at normal speed.
R = radius, in feet, of the circle described by the centres of the governor balls.

The force diagram is constructed as follows :—

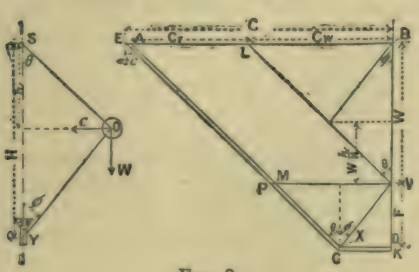


Fig. 8.

Draw a vertical line and mark off B V, V D, representing, to the same scale, W and F * respectively in magnitude and direction.

Draw V L parallel to the upper arm, O S, and V G parallel to the lower arm, O Y.

Draw V M and D X both perpendicular to B D. Through X draw X A parallel to V L, and through B draw B A parallel to V M and D X.

It is evident that the line, B A, represents the force, C, in magnitude and direction.

Now, the force, C, is partly due to the effect of W and partly to the effect of F.

Calling the former part C_w and the latter C_f,

$$C = C_w + C_f.$$

In the diagram, the line B L represents C_w, and L A represents C_f. Now,

$$C = \frac{4\pi^2 N^2 R}{g} . W,$$

* It will be noticed that only half the central weight is here taken into consideration, the other half being supposed to act on the other side of the centre line of the governor spindle.

so that C is proportional to N^2 and 1 per cent. increase of N is equivalent to an increase of 0·02 C in C.

Produce B A to E, making A E equal to 0·02 C.

Draw E G parallel to A X, produce V X to G, and draw G K parallel to X D.

Then the line, D K, represents half the force, f, to the same scale as F and W.

To obtain its magnitude from the similar figures, V D X M, V K G P :—

$$\frac{f}{F} = \frac{0{\cdot}02\,C}{C_f} = 0{\cdot}02\left\{\frac{C_w + C_f}{C_f}\right\}.$$

i.e.,
$$f = 0{\cdot}02\,F\,\frac{C_w + C_f}{C_f} = 0{\cdot}02\left\{1 + \frac{C_w}{C_f}\right\}F.$$

f is the force for 1 ball only. The controlling force for 2 balls is $2f$, and for 4 balls it is $4f$.

This expression for the magnitude of the controlling forces is applicable to any vertical governor, no matter what may be the train of mechanism between the balls and sleeve.

For the special kind of governor shown in Fig. 8, f can be obtained as follows :—

$$\frac{f}{0{\cdot}02\,C} = \frac{F}{F(\tan\theta + \tan\phi)} = \frac{1}{\tan\theta + \tan\phi}.$$

i.e.,
$$f = \frac{0{\cdot}02\,C}{\tan\theta + \tan\phi} = 0{\cdot}02\,\frac{W\tan\theta + F(\tan\theta + \tan\phi)}{\tan\theta + \tan\phi}$$

Or,
$$f = 0{\cdot}02\left\{F + \frac{W}{1 + \dfrac{\tan\phi}{\tan\theta}}\right\} = 0{\cdot}02\left\{F + W\,\frac{H - h}{H}\right\}.$$"

Flywheels.—We have already mentioned that the function of the flywheel is to take up and give out energy so as to minimise the fluctuations in speed due to the periodic changes in the crank effort, and also to reduce the suddenness of other changes in the speed of the engine.

In Lect. XII., Vol. I., we found an expression for the tension per square inch in the rim of a flywheel due to centrifugal force, and saw that it was independent of the cross area of the rim. The highest speed at which a flywheel can be run with safety, will therefore depend upon the tensile strength and the density of the material of which it is made, for it is evident that we cannot make it able to go faster by enlarging the cross area of the rim, since that increases the total stress in exactly the same proportion as it increases the total strength. Consequently, for very high speeds it is necessary to select a material, and so dispose of it, as to have the greatest possible strength for a given mass. Flywheels are usually made of cast iron, either moulded and cast in one piece, or built up in several segments; but sometimes,

they have been made of wrought iron or steel, so as to offer a much greater resistance to bursting, or by winding steel wire into an annular trough made of steel plates at a definite distance from the crank shaft.*

Balancing Machinery.†—If a flywheel be not accurately balanced, it will cause wobbling stresses, which produce vibration and wear, and which become greater as the speed is increased. Owing to the recent demand for high-speed machinery, such as sugar-drying, cream separating, hydro-extracting, and electric light machinery, the attention of engineers has been specially directed of late to the necessity for more perfect balancing, with a view to reducing vibration and its attendant noise, tear, and wear. Even in the case of express trains, it has been found advisable to balance the carriage wheels. This is done by placing their axles and their wheels on a framing with springs of exactly the same kind as those to be used on the carriage for which they are intended, and running them at their highest speed of, say, 60 to 70 miles per hour. Pieces of clay are placed upon the inside of their rims until they run perfectly smoothly. These lumps are then replaced by pieces of cast iron or lead of the same weight, and the process repeated until as perfect a balance as possible has been obtained.

In works where the importance of balancing machinery is recognised, the machine to be balanced is placed upon a testing table and run at gradually increasing speeds. At each speed the balance is made as perfect as possible, by trial, in a manner similar to that just described for railway carriage wheels, until the maximum working speed has been reached, and the whole is capable of running practically free from vibration even when not secured by bolts or clamps.

A common method of balancing pulleys in the workshop is to mount them on a shaft, or mandril, which is then placed on two parallel and perfectly level straight edges. This is a delicate method of procuring a *statical* balance, but it does not follow that there is a true *dynamic* balance, as there may be a *centrifugal couple*, which will cause vibration, and needless pressure on the bearings. To take a simple case, con-

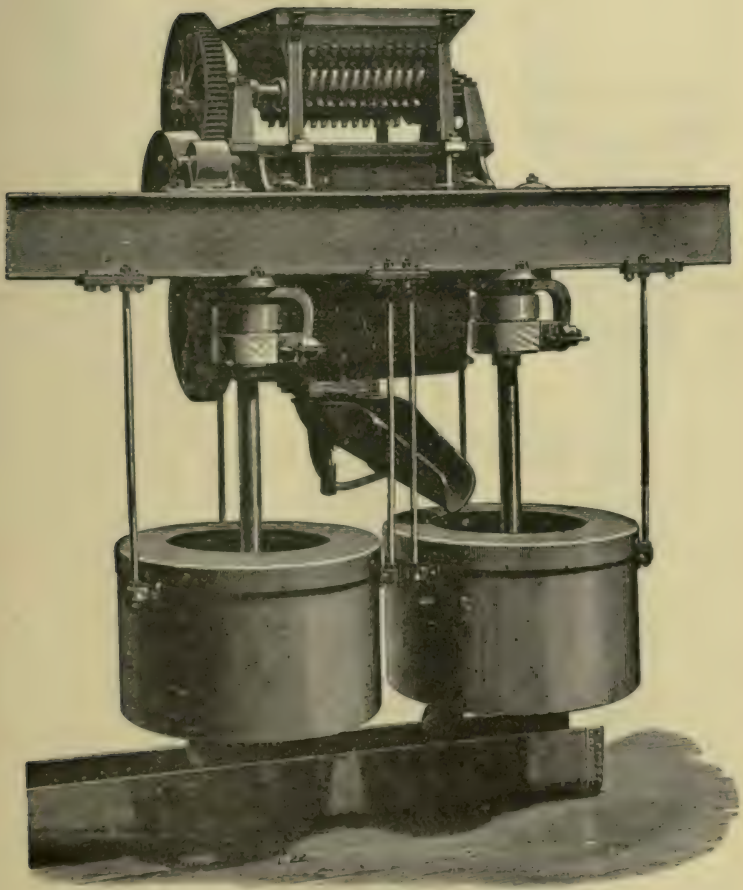
* See Prof. Sharp's pamphlet on "A New System of Wheel Construction" (Technical Publishing Co., Manchester). Also, "Flywheels for Slow-Speed Electric Traction Steam Dynamos," by A. Marshall Downie, B.Sc., and the full discussion in vol. xlv.—Parts I. and II., Nov. and Dec., 1901, *Trans. Inst. Engs. and Shipbuilders in Scotland.*

† See *Proc. Inst. Eng. and Shipbuilders in Scotland*, vol. lxiii., 1899, for a paper on "The Mechanics of the Centrifugal Machine," by C. A. Matthey. Also, *Proc. N.E. Coast Inst. of Eng. and Shipbuilders*, vol. xii., 1896, for a paper on "An Investigation into the Force tending to produce Vibration in High-Speed Engines," by J. M. Allan.

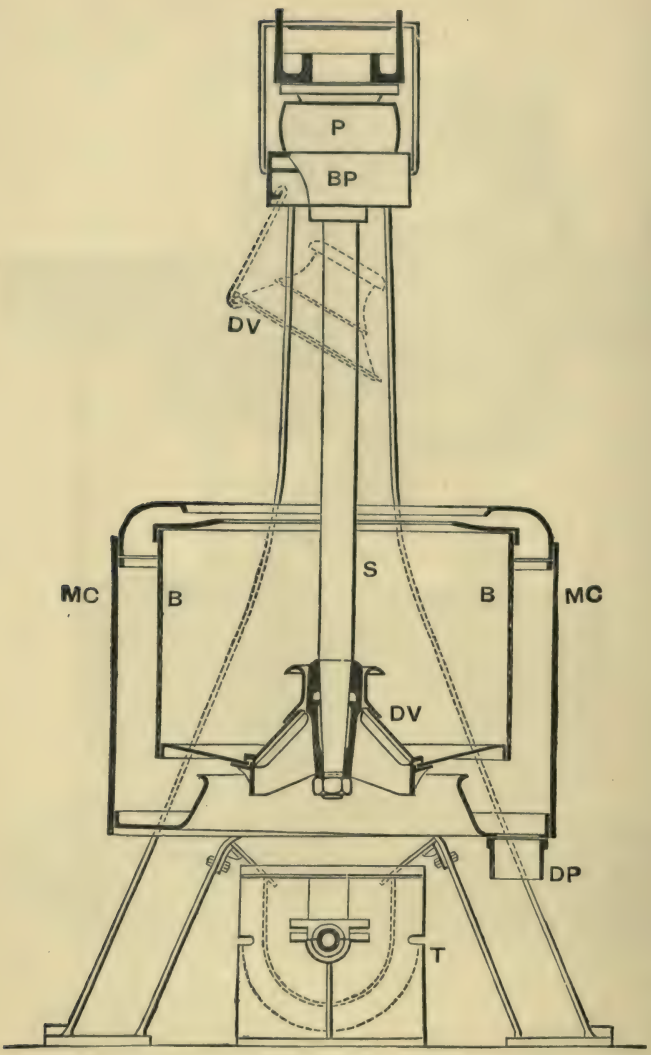
sider a crank shaft with two cranks 180° apart. The static balance may be perfect if the cranks are similar; yet, it is clear that the centrifugal forces of the two cranks, although equal, parallel, and opposite in direction, are not in the same straight line, and therefore form a couple in a plane passing through the axis of the shaft. The plane of this couple revolves with the cranks, and it consequently tends to make the axis describe a double cone in space, the common vertex of this cone being at the centre of gravity of the whole rotating mass. Similarly, with a pulley there may be an excess of material on one side at one extremity of a diameter, and at the other extremity an excess on the other side, which, while the static balance is perfect, cause a centrifugal couple, and set up objectionable vibrations at a high speed. The final adjustment of the balance of a wheel or pulley should therefore always be made at the highest speed at which it is intended to run. In order to have a *statical* balance about an axis, it is sufficient that the axis should pass through the centre of gravity of the *whole* mass, but for a perfect *dynamic* balance, it must *also* pass through the centre of gravity of *every* section taken at right angles to the axis. It is possible, however, in some cases to have the body as a whole balanced without this last condition, but in such cases there will be several centrifugal couples whose resultant is zero, but which tend to bend the shaft at several places.

Weston Centrifugal Machine. — As a useful application of centrifugal force, and an example of a self-balancing high-speed machine, we here illustrate the Weston centrifugal for drying sugar. The first figure gives a general view of a pair of 30-inch centrifugals suspended from the house framing, with sugar-breaker, pug mill, swivel shoot, and molasses gutter. The baskets of these machines are driven at 1,200 revolutions per minute, and give an output of 12 to 16 tons of dried raw sugar, or 12 to 20 tons of dried refined sugar, per day of ten hours, and require about seven horse-power to drive them.

In order to charge the machine, the valve at the bottom of the pug mill is opened, so as to allow the sugar to gravitate down the scoop into the basket B, seen in the vertical section. When a sufficient charge has been given, the pug mill valve is closed, and the basket started rotating by a friction pulley of the kind shown in Lecture VIII., p. 158 of Vol. I. The belt which drives the pulley P, connected to the spindle S, thus gradually brings the speed up to its normal. The centrifugal force causes the water and molasses to pass through the numerous holes in the periphery of the basket into the monitor case M C, from whence it escapes by the discharge pipe D P; while



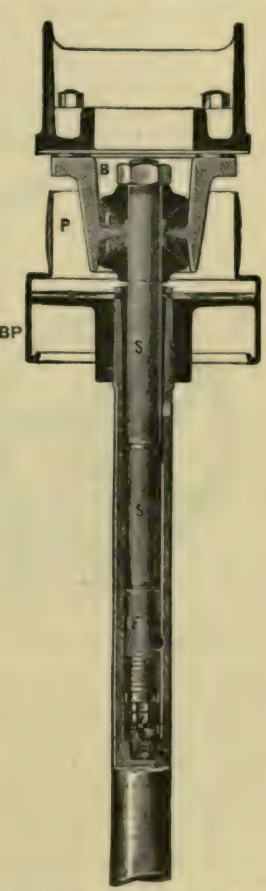
WESTON CENTRIFUGAL SUGAR-DRIERS, BY WATSON, LAIDLAW & Co.



VERTICAL SECTION OF WESTON CENTRIFUGAL.

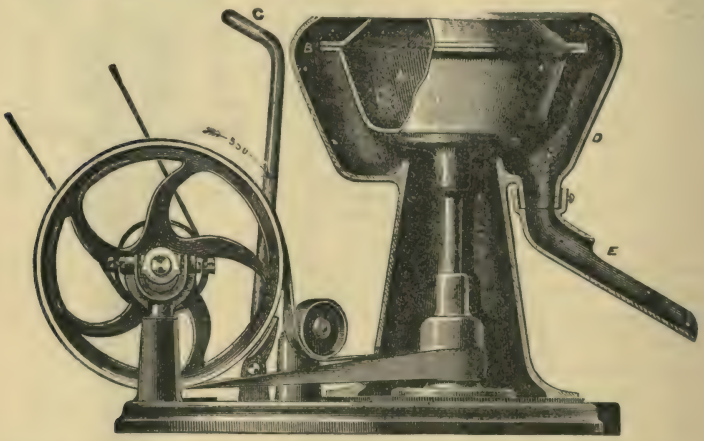
the sugar forms a wall around the inside of the basket. When the sugar is dried, the friction grip of the driving pulley is relieved, and the brake-strap applied to the brake pulley B P, so as to bring the basket and its contents quickly to rest. The conical cover, or discharge valve D V, is then raised and hung on the brake pulley, as seen in dotted lines. The wall of sugar is broken down and swept through the central opening into the conveying trough T. It is then forced along this trough by a large screw to wherever it may be wanted.

The basket is not compelled to revolve about a fixed axis, but is permitted to choose its own centre of rotation by the use of elastic bearings. By allowing the revolving basket to oscillate within certain limits, it assumes as its centre of gyration the centre of gravity of the basket and its contents, and so becomes self-balancing. This reduces to a minimum the power required to drive the machine, severe stresses, wear and tear, and the vibrations transmitted to the building. By referring to the sectional view of the spindle, it will be easily understood how this is accomplished. A strong block B, is bolted to the overhead beam, and inside this block are placed two india-rubber buffers I I, the upper of which sustains the suspended spindle S, by a nut and washer. This spindle does not rotate, but it carries, at its lower end, a series of washers which support the bearing F, fixed to the outer revolving spindle. The pulley P, and brake pulley B P, are attached to the upper end of this outer spindle, and the perforated basket to its lower end. The hollow portion of this spindle is filled with oil, so that the bearing F runs in an oil bath, and is always well lubricated.



SECTION OF SPINDLE AND BEARINGS FOR WESTON CENTRIFUGAL.

There are many other applications of this principle, such as in hydro-extractors, cream-separators, &c. The next illustration shows a modification of the above machine adapted for extracting oil from engine waste, turnings, screws, &c., or drying crystals and ores. The material to be dried is put into the hollow pan A, which is then rotated at about 2,000 revolutions per minute. The oil, or water, escapes through the narrow opening between the upper and lower parts of the pan at B, into the outer casing D, and thence to the spout E. The pan is emptied by lifting it

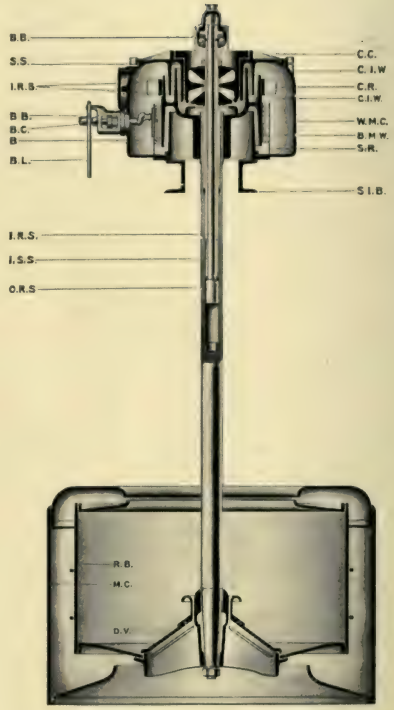


UNDER-DRIVEN CENTRIFUGAL EXTRACTOR, BY WATSON, LAIDLAW & Co.

bodily from the top of the spindle and turning it upside down. It rests on a leather-faced disc on the top of the spindle, and is kept central by a continuation of the same, which fits easily into a recess in the bottom of the pan. This arrangement permits of a little slip at starting, by which the driving belt is relieved from any sudden or severe stress. The spindle is similar in construction to the one just described, but inverted, so that this machine is also self-balancing.

INDEX TO PARTS.

B B	Ball-Bearing carrying weight of Basket.
S S	Suspension Sleeve.
I R B	India Rubber Buffers.
C I W	Cast Iron Washers.
W M C	Water Motor Case.
C C	Case Cover for W M C.
B M W	Brake Motor Wheel.
C R	Cup Ring for Wheel.
B	Brake.
B B	Brake Bracket.
B C	Brake Crank.
B L	Brake Lever.
S R	Splash Ring.
S I B	Supporting Iron Beam.
I R S	Inside Revolving Spindle.
I S S	Inside Stationary Spindle.
O R S	Outside Revolving Spindle.
D V	Discharge Valve.
R B	Revolving Basket.
M C	Monitor Case.



Vertical Section of Water-driven Centrifugal, Patented and made by Watson, Laidlaw & Co., Glasgow.

For description, see Professor Jamieson's Text Book of Applied Mechanics.

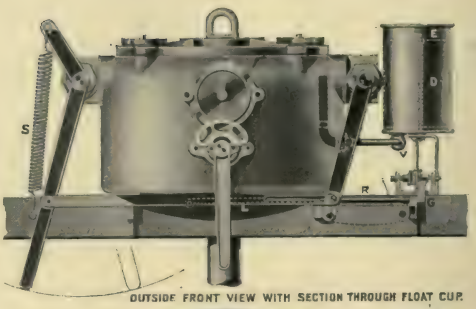
WATER-DRIVEN CENTRIFUGAL

WITH

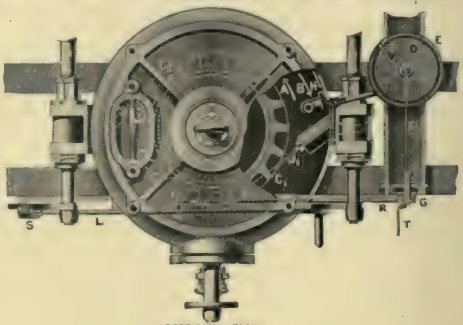
AUTOMATIC CUT-OUT.

INDEX TO PARTS.

A, B } Flow of Water.
C_1, C_2 } Pelton Wheel Cups.
D Float.
E Float Cup.
F Lever.
G Detent.
H Scoop.
J_1, J_2 } Water Jets.
L Link.
R Rod.
S Spring.
T Trigger.
V Valve.



OUTSIDE FRONT VIEW WITH SECTION THROUGH FLOAT CUP.



SECTIONAL PLAN.

Automatic Cut-out for Water-driven Centrifugal, Patented and made by Watson, Laidlaw & Co., Glasgow.

For description, see Professor Jamieson's Text Book of Applied Mechanics.

Water-Driven Centrifugals with Automatic Cut-off (*see Accompanying Plates with Index to Parts*).* — *General Features and Objects.*—In order to suit the different classes of sugars or other materials to be dried, these centrifugals may be made to attain full speed in a longer or shorter time by fitting each water motor with at least two water jets, and a separate valve to each jet. Consequently, should it be desired to attain full speed in a short time, both valves are opened at starting. When full speed is reached, one valve is closed automatically, thus leaving the other valve open until the machine has to be stopped. This valve is then closed by hand and the brake applied in the usual way. The jet which is first closed has been termed the "*accelerating jet*," and the one which maintains a uniform speed until shut off by hand is called the "*maintaining jet.*"

When drying very low grade sugars, or other materials which have a very small grain (and are liable to become impervious to the passage of the liquid to be extracted), the "*maintaining jet*" should be used *alone;* for, while it will ultimately bring the centrifugal to its full speed, it will do so very gradually, and thereby allows the liquid to pass through the semi-porous mass before the higher centrifugal force causes the material to pack too closely. The liquid passing off at the slower speed tends to leave the solid material in a more permeable condition than it would be if the centrifugal were brought to full speed in a short time.

Description of Centrifugal Machines.—When the machine is started, the water issuing from the jets J_1 J_2 is deflected backwards by the pelton wheel cups C_1, C_2, &c., in the direction shown by the arrow A. As the speed of the wheel increases, the velocity of the spent water in a backward direction correspondingly decreases, until the wheel attains its most efficient speed in relation to the velocity of the water jet; at which time, the water falls clear of the wheel without further backward motion. A further slight increase in the speed of the wheel causes the spent water falling from the wheel to flow in the direction of rotation of the wheel, as shown by arrow B. Then, a portion of this water will enter the scoop H, pass into the float cup E, raise the float D, attached to the lever F, and release the detent G, which in turn releases the rod R, attached to the link L. This action allows the spring S to shut the accelerating valve connected to jet J_2, thus leaving the maintaining jet J_1 to keep up the speed.

The action of this apparatus is remarkably accurate and may be modified by the adjusting valve V, which can be thus made to delay the entrance of the water into the cup E, so that the accelerating jet can be kept open for a longer time.

The trigger T enables the attendant to release the detent G, if for any reason it is desired to close the accelerating valve before the automatic apparatus comes into action.

Calculations of Work Done in Overcoming Inertia, &c.—The following calculations, in connection with the driving of these centrifugals by Watson, Laidlaw & Co.'s water motors, will be interesting and instructive to students:—

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* I am indebted to Watson, Laidlaw & Co., Engineers, Glasgow, the designers and makers of these centrifugals, for having kindly supplied the accompanying four plates to illustrate these interesting and useful machines, as well as for the data from which this was written.

Let Q = Quantity of water from the nozzle jets in lbs. per second.
,, V = The velocity of the water jets in feet per second.
,, F = Force acting on the buckets due to jet, in lbs.
,, v = Velocity of the pelton wheel buckets in feet per second.
,, r = Radius of the buckets in feet from the centre of vertical spindle.
,, k = Radius of gyration of the basket and load = 13 ins. = $\frac{13}{12}$ foot.
,, n = Revolutions of pelton wheel per second = $\frac{1200}{60} = 20$.
,, ω = Angular velocity of basket in radians per second.
,, E = Energy per second (or power), being absorbed by the wheel at any instant, measured in foot-lbs. per second.
,, W = Weight of basket and load in lbs. = 700 lbs.
,, t = Time in seconds from start.
,, E_K = Total kinetic energy in basket and load in foot-lbs.
,, η = Theoretical efficiency of the pelton wheel.

Energy required to overcome inertia :—

$$E_K = \frac{W v^2}{2g} = \frac{W (2\pi r n)^2}{2g} = \frac{2\pi^2 n^2 W k^2}{g}.$$ (See equation (IX.), Lect. XII., Vol. I.)

$$E_K = \frac{2 (3{\cdot}1416)^2 \times 20^2 \times 700 \times (\frac{13}{12})^2}{32{\cdot}2} = 201{,}440 \text{ foot-lbs.}$$

This result takes no account of friction, and is the least total amount of energy which has to be imparted to the revolving parts, whether driven by belt, water, or electric motor, to bring the whole of the revolving parts up to the maximum working speed.

The Efficiency of the Wheel is the ratio of the power actually being taken up by the wheel to the total power passing through the water jet (see Vol. IV., p. 167)—

$$= \frac{4 v (V - v)}{V^2} = \eta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The Force on the Pelton-wheel Buckets due to the change of momentum of the water at any bucket speed v is—

$$= \frac{2 Q (V - v)}{g} = F \text{ lbs.} \quad . \quad . \quad . \quad . \quad (2)$$

The Energy Absorbed by the pelton wheel at bucket velocity v is—

$$F v = \frac{2 Q}{g} (V - v) v = E \text{ foot-lbs. per second.} \quad . \quad (3)$$

Substituting the angular velocity for the linear velocity, we get—

$$E = \frac{2 Q}{g} (V - \omega r) \omega r \text{ foot-lbs. per second.} \quad . \quad (4)$$

In a short time dt, the energy given up to the basket, will be—

$$dE = \frac{2 Q}{g} (V - \omega r) \omega r \, . \, dt \text{ foot-lbs.} \quad . \quad . \quad . \quad (5)$$

Therefore we get, $$dt = \frac{g \times dE}{2 Q (V - \omega r) \omega r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

But, *the Energy Stored up in Basket and Load* is, by equation (IX.), Lecture XII., Vol. I.

$$E_K = \frac{I\omega^2}{2} \text{ foot-lbs.},$$

where I is the moment of inertia of the basket and load.

Or, $$E_K = \frac{W k^2 \cdot \omega^2}{2g} \text{ foot-lbs.} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$\therefore$ $$dE_K = \frac{W k^2 \cdot \omega}{g} d\omega \text{ foot-lbs.} \quad . \quad . \quad . \quad . \quad (8)$$

Now, if the increment of energy absorbed is equal to the increment of energy gained (that is, $dx = dE_K$), then we have, from (6) and (8)—

$$dt = \frac{W k^2}{2Qr}\left(\frac{d\omega}{V - \omega r}\right). \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$\therefore$ $$t = \frac{W k^2}{2Qr}\int_0^{\omega} \frac{1}{V - \omega r} \cdot d\omega.$$

Integrating and simplifying, we get—

$$t = -\frac{W k^2}{2Qr^2} \log_e\left(\frac{V - v}{V}\right) \text{ seconds}, \quad . \quad . \quad (10)$$

where $v = \omega r$. This is the equation of time and bucket velocity. Inverting (10) we get—

$$v = V - Ve^{-\frac{2Qr^2t}{Wk^2}}, \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Now, expressing v in revolutions per minute, we get—

$$\text{Revolutions per minute} = \frac{V - V\left(e^{-\frac{2Qr^2t}{Wk^2}}\right)}{2\pi r} \times 60. \quad . \quad . \quad . \quad . \quad (12)$$

Again, we found the energy per second absorbed by the wheel to be—

$$x = \frac{2Q}{g}(V - v)\,v. \quad \text{(See 3.)}$$

Substituting the value of v from (11) we get—

$$x = \frac{2QV^2}{g}\left\{e^{-\frac{2Qr^2t}{Wk^2}} - e^{-\frac{4Qr^2t}{Wk^2}}\right\}. \quad . \quad . \quad . \quad (13)$$

This is an equation connecting the power absorbed usefully by the pelton wheel with the time from start.

By plotting the above quantities, with time as an abscissa and foot-lbs. per second as one ordinate, whilst revolutions per minute is the other ordinate, we get two curves. One curve shows the energy per second usefully absorbed by the pelton wheel, whilst the other curve gives its speed. It will be evident, that the area under the energy curve (that is, $\int x \cdot dt$) represents the total work usefully absorbed by the wheel; or, in other words, the work stored up in the basket and load as inertia, while the smaller area above this curve represents the work lost on account of the variable speed during the acceleration. The power supplied is assumed to remain constant during that period. The area under the curve can be shown to be 72 per cent. of the whole available area (see Fig., p. 456).

Electrically-Driven Centrifugal Machines (*see Accompanying Plates by Watson, Laidlaw & Co.*). — *Essential Features.* — Here, the centrifugal forms one machine by itself, whilst the electro-motor also constitutes another machine. Each of these machines may, therefore, be constructed in the best manner, quite independently and without necessitating any special adaptation for combining them. The centrifugal may, however, be easily driven by the motor through the medium of an ordinary centrifugal friction clutch.

The motor does not oscillate with the centrifugal, and hence the clearances between the armature and the field poles of the former may be as small as the best electrical and mechanical practice will permit.

Details.—The centrifugal machine is of the "Weston" type, already described. It hangs on ball bearings and rubbers as shown by the accompanying plate. The use of the centrifugal friction clutch between the motor and the centrifugal allows the motor to attain its speed rapidly, whilst the centrifugal follows it up until the motor and the centrifugal are running at the same speed. This action is exactly the same as in the standard "Weston" centrifugal friction pulley, and dispenses with the need for artificial electrical resistances when the motor is first started.*

Oiling of Motor.—The oiling of the motor bearings is effected by a patent device placed at the upper end of the motor spindle. It is so designed that a measured quantity of oil may be automatically admitted into the upper bearing of the motor each time the machine is started. This oil finds its way by gravitation down through suitable channels to each bearing in sequence. After passing through the lower bearing of the motor and the bearings of the spindle, it finally discharges into the brake pulley, which serves as a collecting cup, from which the oil can be withdrawn at intervals of a week or ten days.

Connection between Electric Motor and Framework.—The connection of the motor to the framework is made by two pins or studs, in such a manner, that when one is withdrawn the motor can be raised upon the remaining stud by a worm wheel and nut, so as to disconnect the clutch coupling. The motor can then be swung to one side on this stud or hinge pin, which exposes the top of the spindle for examination or withdrawal as shown by the illustrations. The hinge pin on which the motor turns is made hollow, for the purpose of allowing the electric wire leads to and from the motor to pass upwards through it, so that, when the motor is thus disconnected and turned round, the electrical connections may not be disturbed.

Starting and Stopping the Motor and Centrifugal.—The operations of starting the motor and releasing its brake, as well as that of stopping the motor, and the application of the brake, are all made by one switch lever. Since the act of applying the brake opens the switch of the motor, there is no possibility of the motor being started when the brake is on.

Advantages.—The following are the principal advantages claimed by the designers and makers of this combination of electro-motor and centrifugal on well-established mechanical principles:—(1) The motor is separate from the centrifugal machine. (2) Simplicity of design and construction. (3) The motor, being above the centrifugal, it is out of the way of the attendant, and is not readily splashed by sugar or washing liquid. (4) All working parts are easily and rapidly accessible for examination. (5) Ample bearings and lubrication for the heaviest loads. (6) The system

* See the illustration and description of the "Weston" centrifugal friction pulley in Lecture VIII., Vol. I., of this book.

DISENGAGED
ELECTRIC MOTOR.

INDEX TO PARTS.

I Patent Hollow Post.

J Pin attaching Motor to Stool.

K Driver keyed to Motor Spindle.

L Friction Shoes.

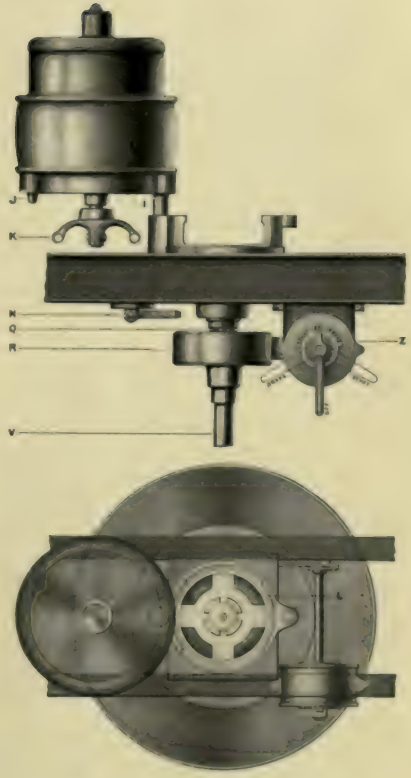
N Worm and Wheel.

Q Nut to adjust Buffers.

R Brake.

V Outer Revolving Spindle.

Z Switch.



Outside View and Plan showing Electric Motor detached from its Weston Centrifugal,
Patented and made by Watson, Laidlaw & Co., Glasgow.

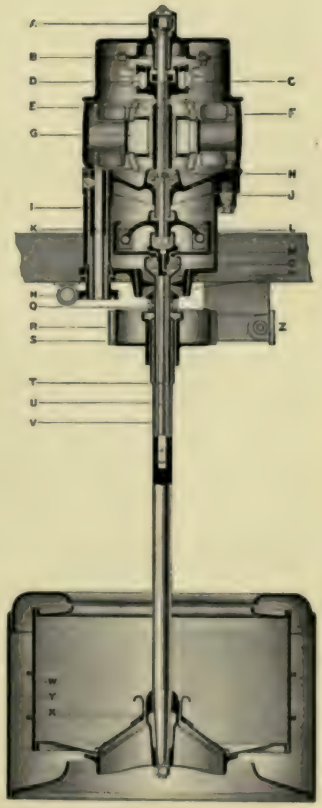
For description, see Professor Jamieson's Text Book of Applied Mechanics.

INDEX TO PARTS.

A Intermittent Oil Cup.
B Brush Holder.
C Brushes.
D Commutator.
E Motor Spindle.
F Field Coils.
G Armature.
H Ball-Bearing of Motor.
I Hollow Post.

J Pin attaching Motor to Stool.
K Driver keyed to Motor Spindle.
L Friction Shoes.
M Coupling fixed to Centrifugal Spindle.
N Worm and Worm Wheel.
O Ball-Bearing of Centrifugal Spindle.
P Indiarubber Buffers.
Q Nut to adjust Buffers.

R Brake.
S Brake Pulley.
T Inner Revolving Spindle.
U Stationary Sleeve.
V Outer Revolving Spindle.
W Basket.
X Discharge Valve.
Y Monitor Case.
Z Switch.



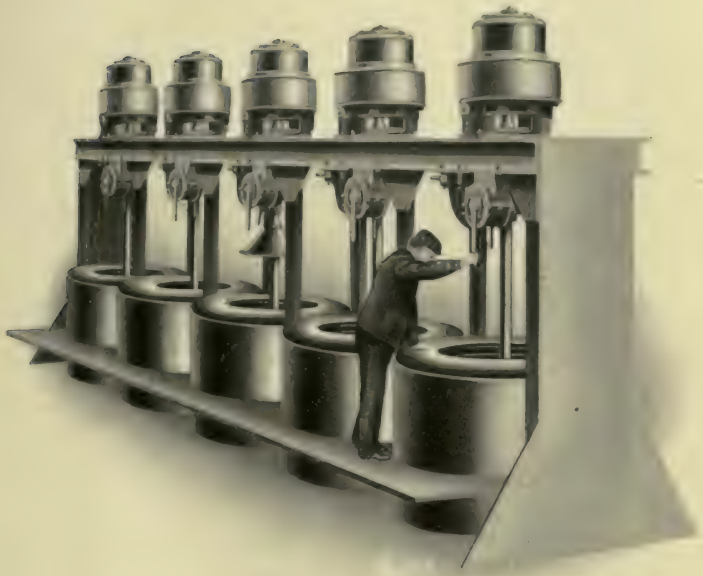
Vertical Section of Electrically-driven Centrifugal Machine, Patented and made by Watson, Laidlaw & Co., Glasgow.

For description, see Professor Jamieson's Text Book of Applied Mechanics.

— BATTERY —

OF

ELECTRICALLY-DRIVEN CENTRIFUGALS.



Electrically-driven Weston Centrifugal Machines, Patented and made by Watson, Laidlaw & Co., Glasgow.

For description, see Professor Jamieson's Text Book of Applied Mechanics.

is equally suitable for direct or for alternating currents. The latter is considered by certain authorities to be the most suitable for driving such machinery which requires to be frequently started and stopped in the manner described above. In any case, it is just as well to be able to suit the different requirements of a continuous or of an alternate current supply.

Considerations of the Different Ways for Driving Centrifugals.—Before examining the relative merits of the three methods of driving centrifugals, it will first be necessary to find out the power actually absorbed by the centrifugal itself, which, for like machines working under like conditions, will be the same, irrespective of the kind of motor employed.

As the fundamental principles which govern the power required to set a centrifugal machine in motion and maintain its speed, are applicable in a like manner to all kinds of machinery, it may be convenient to take a familiar example to illustrate the points which require consideration.

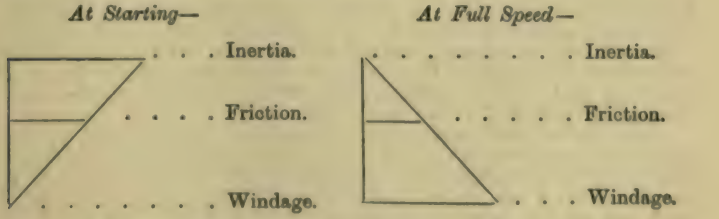
Every one is familiar with a railway train, which gradually increases its speed as it leaves the station, until a certain maximum speed is reached, after which the train's motion is uniform.

Inertia, Friction, and Windage.—It has been observed, that the train does not start at full speed all at once, and that after a time the speed does not increase further, but becomes uniform or constant. The following are the resistances which the force exerted by the engine has to overcome :—

(1) Its *inertia* or the resistance to motion which the weight of the train offers at starting.
(2) At the same time, the *statical* and then *sliding friction* of the moving parts.
(3) The resistance of the atmosphere which may be termed *windage*.

When the train begins to move, *inertia* offers the greatest resistance to a rapid increase of speed or acceleration, then comes friction, and finally windage. As the train gradually increases in speed, these resistances to its motion gradually vary until they assume a different relation at its maximum speed. The inertia of starting will be entirely overcome, whereas windage now offers the greatest resistance to any further increase of speed, and the sliding friction slightly diminishes or remains fairly constant.

The two following diagrams show these changes graphically :—



Here we see, that inertia has decreased from a maximum to a minimum, and windage has risen from a minimum to a maximum, while the friction keeps an intermediate position in both cases.

Work Done in Overcoming Inertia.—The work done in overcoming the *inertia* of the train, from starting till full speed is reached, depends solely upon the weight of the train. Whether the train be made to attain full speed in, say, four minutes from starting or in one minute, the total work done in foot-lbs. will be the same. In the shorter period of one minute, the locomotive will require to exert four times the average horse-power that it does during the four-minute period.

Work Done in Overcoming Friction.—The resistance due to *friction* is modified by the conditions of speed, but is not sufficiently variable for our present purpose to require further investigation.

Work Done in Overcoming Windage.—The rate at which the resistance due to *windage* increases at the beginning of the train's motion is very slow. But, as the train increases in speed, the rate of increase of this resistance gradually increases, until it becomes very great when the train has attained a high velocity.

Application of the Foregoing Principles to Centrifugals.—The conditions under which a centrifugal machine attains its full speed are very similar to those described for a railway train :—

(1) The work done in overcoming the inertia of all the moving parts.
(2) The friction of the bearings.
(3) The windage resistance, chiefly between basket and monitor case.

Take the previous case of a "Weston Centrifugal" having a basket 30 inches diameter and arranged to run at a maximum speed of, say, 1,200 revolutions per minute. Let it be assumed that it is capable of curing 1 ton of dry sugar per hour ; and further, that the average number of charges are 10 per hour, which is equivalent to a discharge of 2 cwts. of dry sugar every six minutes. Then, the complete cycle of operations is as follows :—

2	minutes	accelerating to full speed.
2	,,	running at full speed.
2	,,	stopping and discharging.
Total, 6	minutes	to complete the cycle of operations.

The basket is charged with the material to be dried during the time of acceleration, and all the charge is run into it before full speed is reached.

As a large proportion of the molasses will be discharged during acceleration, consequently a certain amount of work is spent on them during that time, although only 2 cwts. of the load has been brought up to full speed.

It may, therefore, be assumed, that work done on the mass is equivalent to accelerating or raising a load of 300 lbs. from rest to a speed of 1,200 revolutions per minute. In addition to this, there is the weight of the revolving parts of the machine, about 400 lbs., making a gross load of 700 lbs., as illustrated on the facing page.

Work Done on Centrifugal during Acceleration and during Full Speed.—In the previous calculations on the work done in overcoming the inertia of 700 lbs. until the agreed-upon full speed of 1,200 revolutions per minute was obtained, it was found, that an expenditure of 201,440 foot-lbs. of work was required, whatever length of time might have been occupied in the process. It has been found from actual experiment, that the centrifugal, under the conditions now stated, can be run during full speed with full load for an expenditure of about 0·875 H.P., or 28,875 foot-lbs. per

minute. This work is entirely expended in overcoming friction and windage. During acceleration there is also friction and windage to be overcome, but as the average number of revolutions per minute during that period is only half of the number of revolutions at full speed, it is sufficient to assume that the work done on friction and windage, during an acceleration period of 2 minutes, will amount to $\frac{28,875 \times 2 \text{ (minutes)}}{2}$ = 28,875 foot-lbs. Therefore, the total work expended during acceleration will be 201,440 + 28,875 = 230,315 foot-lbs., and the total work expended during a period of 2 minutes at full speed will be 28,875 × 2 = 57,750 foot-lbs. These two quantities may be called the useful work done during the acceleration and full speed periods respectively, and are the least possible under the conditions. *Whatever type of prime mover is used, that amount of work must be done during each cycle.*

Relative Efficiencies of Three Different Ways of Driving Centrifugals.—Having thus determined the least expenditure of work required by the centrifugal under a given set of conditions, we naturally inquire which method of driving is the most efficient. Generally speaking, a few years ago it was customary to drive a battery of centrifugals by intermediate gear from one prime mover; but now, a prime mover may be connected directly to the spindle of each centrifugal.

Here it is assumed, that the prime mover steam engines which drive the batteries of centrifugals, in the first case by shafting and belting, and in the last case through the intervention of a dynamo and electric motors, receive, cut-off, and exhaust their steam at the same rate and same pressures; and, that they are each of 90 per cent. mechanical efficiency. In other words, that the ratio of their B.H.P. to their I.H.P. is ·9.

In many sugar factories all the steam engines exhaust into one or more steam mains, where the back pressure may be maintained at from 5 to 10 lbs. by gauge. This exhaust steam is usefully applied for other purposes, such as heating and boiling. This will, however, not affect the relative or final comparative efficiencies, if we suppose that the steam pumps also exhaust into such mains at the same back pressure as the steam engines, the chief differences being, that the steam pumps receive their steam during their whole stroke instead of by an early cut-off, as in the case of the steam engines.

The following table shows how the power may be transmitted from a steam boiler, of the same efficiency and pressure in each case, to the centrifugals by each of the three methods of driving—viz., by Belting, by Pelton Wheels, and by Electric Motors—when considering a large installation:—

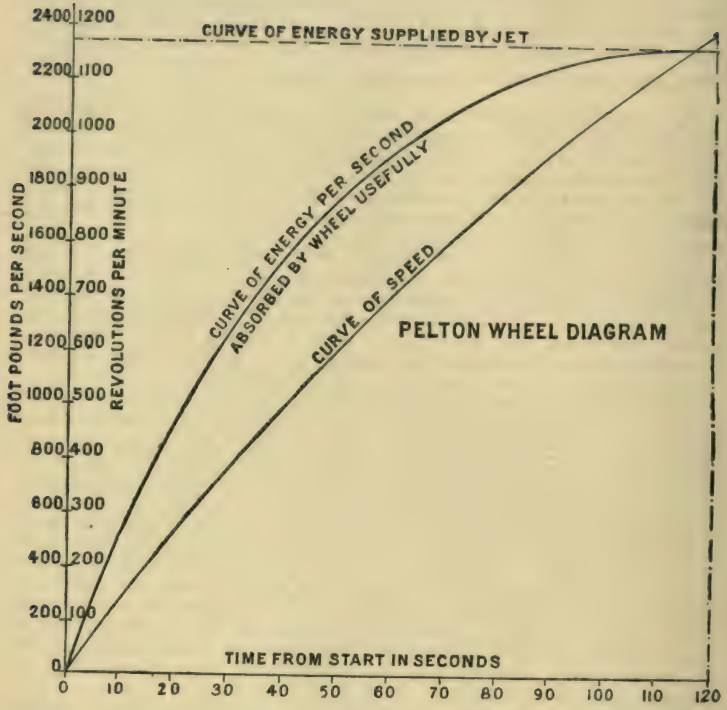


DIAGRAM SHOWING THE ENERGY SUPPLIED BY THE JET TO THE PELTON WHEEL; ALSO, THE ENERGY USEFULLY GOT OUT AND THE ENERGY LOST ON ACCOUNT OF THE VARIABLE SPEED DURING ACCELERATION.

Although the above curves were plotted by the substitution of certain values for the symbol letters in equation (13) at page 451, and therefore represent the probable maxima results obtainable under these conditions from a Pelton-wheel-driven centrifugal; yet, they clearly indicate how the corresponding values derived from actual tests should be treated. Those who have the opportunity of making practical tests of such plant under ordinary working conditions should do so, in order to ascertain the efficiency of the whole as well as of the various elements.

TABLE SHOWING THE DIFFERENT TRANSFORMATIONS BETWEEN THE STEAM BOILER AND THE CENTRIFUGALS, AS WELL AS THE MAXIMUM EFFICIENCIES AT EACH TRANSFORMATION AND THE TOTAL EFFICIENCY OF THE THREE CHIEF METHODS OF DRIVING CENTRIFUGALS.

	Belting.	Water Drive.	Electric Drive.
From steam boiler to prime mover,	Piping and steam engine.	Long piping and steam pump up to water motor during working and standing.	Piping and steam engine.
Max. efficiencies,	90 per cent.	55 per cent.	90 per cent.
1st transformer,	Shafting and belting with friction pulley.	None.	Dynamo.
Max. efficiency,	55 per cent.	...	90 per cent.
2nd transformer,	None.	None.	Cables and Electric motor.
Max. efficiency,	...	...	74 per cent.
Power applied to centrifugal,	Belt pulley applied to centrifugal spindle.	Pelton wheel on centrifugal spindle.	Friction pulley of motor on centrifugal spindle.
Max. efficiency,	60 per cent.	72 per cent.	70 per cent.
Final or total efficiency,	$90 \times 55 \times 60 = 30$ per cent.	$55 \times 72 = 40$ per cent.	$90 \times 90 \times 74 \times 70 = 42$ per cent.

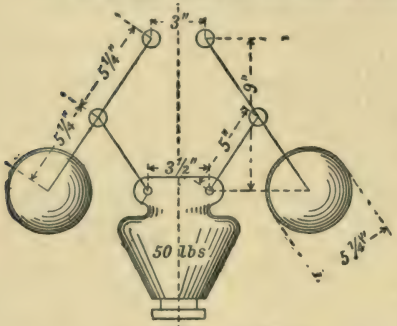
Conclusions.—From the above table it will be seen, that the oldest form of driving centrifugals by shafting, belting, and friction pulleys is the least efficient; whilst the water and electric drives are about equal in efficiency. Where a water accumulator plant, or where steam is abundant and cheap, the water drive may be preferred. But, where a suitable electric plant has been installed, or where electric power can be obtained from a central station, then electric motors may be more convenient.

The following example and answer will be found useful in connection with the designing of Governors :—

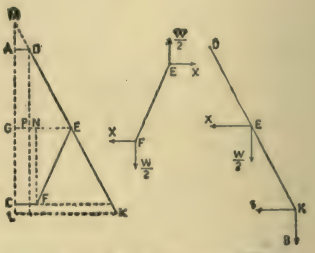
EXAMPLE III.*—Find the weight of counterpoise in a Watt's governor, when you have the speed of the governor, height of cone, centres and diameter of balls fixed. Also, how to get the speed of the governor at mid-position, when you have all the dimensions, as given in the sketch.

ANSWER.—*First Method.*—When the arms D E, E F are unequal, and also A D, C F of the governor are unequal.

Let w = The weight of each ball.
,, N = The number of revolutions per minute.
,, ω = The angular velocity in radians per second.
,, W = The suspended weight, or weight of counterpoise required.
,, H = The height of the cone in inches.



SKETCH OF A LOADED WATT'S GOVERNOR.



ACTION OF THE LOADED WATT GOVERNOR.

Consider the equilibrium of the link E F, in the skeleton diagram; a force whose vertical component is $\frac{W}{2}$ must be acting at the point E, because at the point F this is also the magnitude of the vertical component. If X is the magnitude of the horizontal force acting at E, then, taking moments about F, we get—

$$X = \frac{\frac{1}{2} W \times E N}{N F}$$

Now, if W is known, X is known; or, $X = k W$, where k is equal to $\frac{E N}{2 N F}$, and is obtained from the skeleton diagram.

Consider the equilibrium of the link D E K, and take moments about D. Let F be the centrifugal force of the ball acting at the point K. Then—

$$F \,.\, A L = w \,.\, Q K + \frac{W}{2} \,.\, E P + X \,.\, B P.$$

* This question was asked and answered recently in *The Practical Engineer.*

Now, as all the above values except F are known, if W is known, F can be obtained; and, if the speed is known, N being revolutions per minute—

$$\text{Angular velocity } \omega = \frac{2\pi N}{60}.$$

$$\text{Centrifugal force } F = \frac{w r \omega^2}{g}.$$

Where r = K L in feet, and g = 32·2 feet per second per second; and as X = kW, W can be calculated. To find the weight of each ball, we have—

$$w = \cdot 27 \frac{\pi}{6} d^3.$$

Where d is the diameter of a ball, and the weight of a cubic inch of cast iron is 0·27 lb. When A B = C F and D E = E F, a formula may be used—

$$H = \left(\frac{188 \cdot 2}{N}\right)^2 \frac{w + W\frac{a}{b}}{w}$$

Where a = E M, and b = M K.

Taking the dimensions given by the question, we get the weight of each ball :—

$$w = \cdot 27 \frac{\pi}{6} d^3 = \cdot 27 \times \frac{22}{42} \times (5 \cdot 25)^3 = \mathbf{20 \cdot 4 \text{ lbs.}}$$

$$X = \frac{\frac{1}{2} W \times E N}{N F} = 25 \times \frac{2 \cdot 45}{4 \cdot 36} = \mathbf{14 \cdot 05 \text{ lbs.}}$$

r = 6·91 inches.

Then, substituting numerical values in the equation,

$$F \cdot A L = w \cdot Q K + \frac{W}{2} \cdot E P + X \cdot B P,$$

we get, $F \times 9 = 20 \cdot 4 \times 5 \cdot 41 + 25 \times 2 \cdot 7 + 14 \cdot 05 \times 4 \cdot 5,$

$$9 F = 110 \cdot 2 + 67 \cdot 5 + 63 \cdot 25 = 240 \cdot 95 \text{ lbs.}$$

i.e., $$F = \frac{240 \cdot 95}{9} = \frac{w r \omega^2}{g}.$$

$\therefore$ $$\omega^2 = \frac{g F}{w r} = \frac{32 \cdot 2 \times 240 \cdot 95 \times 12}{9 \times 20 \cdot 4 \times 6 \cdot 91} = 73 \cdot 6.$$

Or, $$\omega = \sqrt{73 \cdot 6} = 8 \cdot 58.$$

Remembering that r must be in feet.

Hence, $$N = \frac{60\,\omega}{2\pi} = \frac{60 \times 8 \cdot 58 \times 7}{2 \times 22} = \mathbf{82 \text{ revolutions per minute.}}$$

The equation, $H = \left(\frac{188 \cdot 2}{N}\right)^2 \frac{w + W\frac{a}{b}}{w}$, will give very nearly the same result.

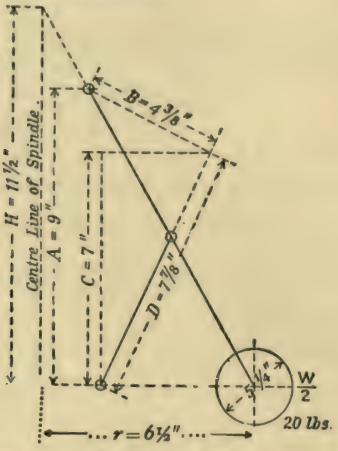
Calculation or measurement gives H = 11·5 inches; $\left\{\begin{matrix} a = 8 \cdot 17 = E M \\ b = 13 \cdot 42 = M K \end{matrix}\right\}$;

$\frac{a}{b} = \frac{8 \cdot 17}{13 \cdot 42} = \cdot 61.$

Therefore, $$N^2 = \frac{(188 \cdot 2)^2}{H} \times \frac{20 \cdot 4 + 50 \times \cdot 61}{20 \cdot 4} = \frac{(188 \cdot 2)^2}{11 \cdot 5} \times \frac{50 \cdot 9}{20 \cdot 4} = 7{,}700.$$

Or, $$N = \sqrt{7{,}700} = \mathbf{87 \cdot 8 \text{ revolutions per minute.}}$$

Second Method.—The centre weight of a governor should bear a definite ratio to the size of engine for which the governor is used. The work of the governor is equal to the weight lifted, multiplied by the height of lift in feet; thus, in the question, it is equal to $50 \times \frac{2{\cdot}25}{12} = 9{\cdot}375$, say, 10 foot-pounds. For an ordinary throttle-valve engine, it is usual to allow ½ foot-pound per I.H.P., so that this governor would be suitable for a 20 I.H.P. engine. Having given the speed, height of cone, or fixed the height of cone, proceed to obtain the weight. Set out the leverages A, B, C, D, as shown by the diagram.



ACTION OF THE LOADED WATT GOVERNOR.

Let N_1 = Speed or number of revolutions per minute (assumed 280).
,, W = Weight in lbs. of counterpoise required.
,, w = Weight of both balls = 40 lbs.
,, r = Radius in feet = ·54.
,, N_2 = Gravity revolutions for an ordinary Watt governor (280 revs. per min. is about the mean speed for these kinds of governors).

$$N_2 = \frac{187{\cdot}5}{\sqrt{H}} = \frac{187{\cdot}5}{\sqrt{11{\cdot}5}} = 55\text{; then, } N_2^2 = 3{,}025.$$

$$N_1 = 280, \text{ and } N_1^2 = 78{,}400.$$

From the formula—

$$W = (N_1^2 - N_2^2) \times \left(w \times r \times {\cdot}00034 \times \frac{A}{B} \times \frac{C}{D}\right)$$

$$W = (78{,}400 - 3{,}025) \times \left(40 \times {\cdot}54 \times {\cdot}00034 \times \frac{9}{4\frac{3}{8}} \times \frac{7}{7\frac{7}{8}}\right)$$

$$\therefore \quad W = 75{\cdot}375 \times {\cdot}013 = 960 \text{ lbs.}$$

The balls, as shown by the sketch, are very heavy. Corresponding with the weight of counterpoise, the calculated ratio is 960 : 40; or, 24 : 1 is much nearer the general practice.

Now, calculating for speed when W = 50 lbs., we have—

$$N_1^2 - N_2^2 = \frac{W}{w \times r \times \cdot00034 \times \frac{A}{B} \times \frac{C}{D}} = \frac{50}{\cdot013}.$$

$$N_1 = \sqrt{\frac{50}{\cdot013} + N_2^2} = \sqrt{\frac{50}{\cdot013} + 3{,}025} = \sqrt{3{,}846 + 3{,}025}$$

$$N_1 = \sqrt{6{,}871} = 82 \text{ revolutions per minute.}$$

The method of obtaining the leverages is set out in dotted lines on the diagram and marked, and can easily be obtained for any other position of the governor balls.

Re "The Balancing of Machinery."

Example IV.—The crank arms and crank pin of a crank shaft are equivalent to a weight of 700 lbs. at 1 foot radius. The shaft is supported in two bearings 5 feet centre to centre, and the central plane of the crank and crank pin is $1\frac{1}{2}$ feet from centre of left-hand bearing. The diameter of shaft at journals is 8 inches. Find the dynamical load on shaft and on each bearing for a speed of 240 revolutions per minute.

If these masses are unbalanced calculate H.P. lost at each bearing due to dynamical load *only*, assuming a coefficient of friction of ·05.

Let w = Equivalent weight of crank pin, &c., = 700 lbs.
,, v = Linear velocity at which the centre of mass of w acts = ωr.
,, ω = Angular velocity in feet per second = $2\pi n$ = $(2\pi r n)$.
,, r = Radius at which w acts = 1 foot.
,, n = Revolutions per second = $\left(\frac{240}{60}\right)$ = 4.
,, d = Diameter of bearing = 8 inches = $\left(\frac{8}{12}\right)$ feet.

See Vol. I., p. 315, Lecture XII., equation XVIII., for centrifugal force F, or called R above and here.

$$R = \frac{w v^2}{g r} \text{ (but } v = 2\pi r n).$$

$$\therefore \quad R = \frac{w(2\pi r n)^2}{g r} = \frac{700\,(22 \times 2 \times 1 \times 4)\,(2 \times 22 \times 1 \times 4)}{32 \times 1 \times 7 \times 7}.$$

$$\therefore \quad R = 700 \times \pi \times 2\pi = 2\pi^2 \times 700 = 2 \times 9{\cdot}87 \times 700.$$

The total dynamical load on the shaft } = R = 19·74 × 700 = **13,828 lbs.**

This dynamical load is distributed between these two bearings in the inverse ratio of the distances of the centres of these bearings from the centre plane of the motion of the centre of the crank pin (see previous and the following figure).

Let A and B be the two bearings 5 feet from centre to centre, and you are given that the centre of bearing A is 1·5 feet from plane of rotation of centre of crank pin.

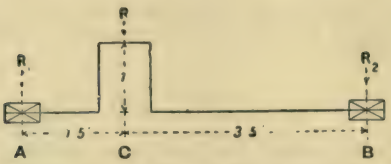
But, *dynamical load* on A and on B together = 13,828 lbs.

Hence—

Pressure on (A + B)	:	Pressure on B alone	::	Distance A to B	:	Distance A to C.
13,828 lbs.	:	R_2	::	5 feet	:	1·5 feet.

$$\therefore \quad R_2 \times 5 = 1{\cdot}5 \times 13{,}828.$$

Or, $$\text{Pressure on B} = R_2 = \frac{{\cdot}3\,(1{\cdot}5) \times 13{,}828}{5} = {\cdot}3 \times 13{,}828.$$



Line Diagram of a Crank Shaft and Bearings.

Dynamical pressure R_2 on bearing B = **4,148·4 lbs.**

$\therefore$ Pressure R_1 on bearing A = (13,828 − 4,148·4).

Dynamical pressure R_1 on bearing A = **9,679·6 lbs., say, 9,680 lbs.**

Turn to Vol. I., Lecture VII., p. 119, for equation.

Work lost in friction in one turn of shaft at both journals = $\pi\, d\, \mu\, R$.

$\therefore$,, ,, ,, at journal A = $\pi\, d\, \mu\, R_1$.

And ,, ,, ,, at journal B = $\pi\, d\, \mu\, R_2$.

From this study p. 120, same Lecture VII., and you see that if $n = 4$ revolutions per second—*i.e.*, 240 per minute.

And taking it as per minute, we have by the formula near foot where $d = 8$ inches = $\left(\frac{8}{12}\right)$ foot, and we are given $\mu = {\cdot}05$, whilst $R_1 = 9{,}680$ lbs.

H.P. lost in friction at $R_1 = \dfrac{\pi\, d\, n\, \mu\, R_1}{33{,}000}$.

,, ,, $= \dfrac{22 \times 8 \times 240 \times {\cdot}05 \times 9{,}680}{7 \times 12 \times 33{,}000}$.

,, ,, $= \dfrac{22 \times 8 \times 240 \times {\cdot}05 \times 9{,}680}{7 \times 12 \times 33{,}000} = 7{\cdot}4$

H.P. lost at R_2 in exactly same way = 3·1

Total H.P. **lost at both bearings, . . 10·5**

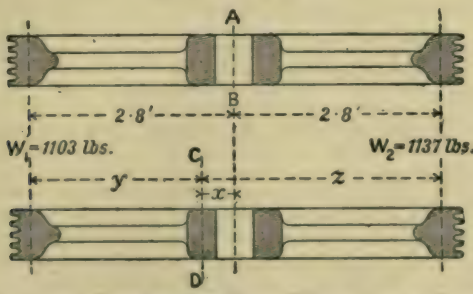
LECTURE XVIII.—Notes and Additions.

Example V.—A rope wheel, weighing 1 ton, and driving the main shaft of a mill at 150 revolutions per minute, caused the bearing nearest to it to heat. The distances of the right-hand and the left-hand bearings from the centre of the wheel are, respectively, 1 foot 9 inches and 5 feet. The shaft was disconnected, and it was found by experiment that the wheel was out of balance (the rim was not turned inside) to the extent of 34 pounds at 2·8 feet radius. Find the distance of the mass centre of the wheel from the centre of the shaft, and the dynamical load on the bearings. Assuming a coefficient of friction of 0·1, find the horse-power required to overcome the friction of the bearings due to the dynamical load alone. The diameter of the shaft is 5 inches. What fraction is the dynamical load of the static load?

Answer.—(*a*) When the sections of the wheel are both equal, the mass centre will be exactly in the middle of the centre line of the shaft—*i.e.*, it coincides with the centre of gravity.

In the accompanying diagrams, the top one represents the mass centre line coinciding with the centre line of shaft when the loads at the extremities of a diameter at the rim are both equal.

$$W_1 = W_2 = 1{,}120 \text{ lbs.}$$



Diagrammatic Sketches of Rope Wheel when Balanced and Unbalanced.

In the bottom diagram, the weights of the respective sections are as shown. Then, the mass centre line C D will be out of truth with the centre line of shaft by a certain distance. Let x feet represent this distance, and taking moments about the line C D, we get—

$$W_1 \times y = W_2 \times z.$$

Or, $1{,}103 \times y = 1{,}137 \times z.$

But, $y + z = 2 \times 2{\cdot}8 \text{ feet} = 5{\cdot}6 \text{ feet.} \qquad \therefore\ y = 5{\cdot}6 - z \text{ feet.}$

Substituting this value, we obtain—

$$z = \frac{1{,}103 \times y}{1{,}137} = \frac{1{,}103\ (5{\cdot}6 - z)}{1{,}137} = \frac{6{,}177}{2{,}240} = 2{\cdot}757 \text{ feet.}$$

$\therefore$ $x = 2{\cdot}8 - 2{\cdot}757 = {\cdot}043$ of a foot.

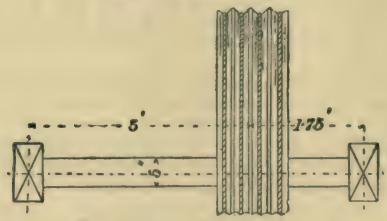
Or, $x = {\cdot}043 \times 12 = {\cdot}516$ of an inch.

Therefore, the mass centre of the unbalanced wheel is nearly $\frac{1}{2}$ inch out of the true centre line of rope pulley.

(*b*) To find the dynamical load, which is the load produced by the unbalanced centrifugal force of the 34 lbs. weight acting at 2·8 feet radius and turning at n revolutions per second $= \frac{150}{60} = 2{\cdot}5$.

$$\therefore \quad \text{Total dynamical load } R = \frac{W v^2}{g r} = \frac{W (2 \pi r n)^2}{g r} = \frac{34 (2 \times 22 \times 2{\cdot}8 \times 2{\cdot}5)^2}{32 \times 7 \times 7 \times 28}.$$

$$\text{,,} \qquad \text{,,} \qquad R = 734{\cdot}5 \text{ lbs.}$$



Sketch of Rope Wheel, Shaft, and Bearings.

Now, in order to get the dynamical load on each bearing, you must divide this total dynamical load in the inverse ratio of the lengths of the respective arms.

$$\therefore \quad \left.\begin{matrix}\text{Dynamical load on the} \\ \text{right-hand bearing}\end{matrix}\right\} = \frac{5}{6{\cdot}75} \times 734{\cdot}5 = \mathbf{544 \text{ lbs., weight.}}$$

$$\text{And,} \quad \left.\begin{matrix}\text{Dynamical load on the} \\ \text{left-hand bearing}\end{matrix}\right\} = \frac{1{\cdot}75}{6{\cdot}75} \times 734{\cdot}5 = \mathbf{190 \text{ lbs., weight.}}$$

(*c*) Now, in order to obtain the H.P. lost by friction on the bearings—

$$\text{H.P} = \frac{\left\{\begin{matrix}\text{coefficient of friction} \times \text{radius of journal in} \\ \text{feet} \times \omega \times \text{dynamical load on bearing}\end{matrix}\right\}}{550}.$$

$$\text{Or,} \quad \text{H.P.} = \frac{\pi d n \mu R}{33{,}000} = \frac{22 \times 5 \times 150 \times {\cdot}1 \times 734{\cdot}5}{7 \times 12 \times 33{,}000} = \frac{73{\cdot}45}{168} = \mathbf{{\cdot}44 \text{ H.P. lost.}}$$

(*d*) The relation of the dynamical load to the statical load is—

$$\frac{\text{Dynamical load}}{\text{Dead load when machine is at rest}} = \frac{734{\cdot}5}{2{,}240} = \mathbf{{\cdot}328, \text{ or } 32{\cdot}8 \text{ per cent.}}$$

LECTURE XVIII.—QUESTIONS.

1. The governor and flywheel of an engine have both the purpose of regulating its speed. Explain how their actions in this respect differ.

2. Explain the principle of *Watt's* pendulum governor, and state its advantages and defects. Various methods have been proposed for improving this form of governor; discuss the action of any such modern apparatus with which you are acquainted.

3. What are the principal essentials of a good steam engine governor? Sketch, in outline, any one form of governor with which you are acquainted, and explain to what extent it is satisfactory according to the conditions which you have laid down, or how it might be improved.

4. Sketch the ordinary pendulum or ball governor of a steam engine. Mark on your drawing some particular line whose length is related to the number of revolutions of the balls. State the relation as nearly as you know it. If the line referred to be shortened in proportion of 2 : 3, how much would the number of revolutions be increased? *Ans.* $\sqrt{3} : \sqrt{2}$.

5. Sketch an ordinary Watt's governor, and explain its action upon the valve with which it is connected. Why is it an improvement to shift the points of suspension so that the arms cross each other?

6. Explain the advantages of the crossed-arm governor for a steam-engine. Find the height of the cone when the engine is making 40 revolutions per minute, and prove the formula on which you rely.

Ans. 1·8 ft.

7. Define the term "isochronous" as applied to governors. How may isochronism be approximately obtained? Prove the formula, connecting the height of the cone of revolution and the number of revolutions per minute, for a simple pendulum governor.

8. Sketch the pendulum governor as Watt made it. From the balls of a common governor, whose collective weight is A, there is hung by a pair of links (of lengths equal to the ball-rods) a load, B, capable of sliding up and down the spindle. Compare the loaded and common governor as regards sensitiveness, the weights of the arms or links being neglected.

9. Find an expression for the height of the cone in a loaded governor when rotating at a given number of revolutions per minute. Show, by a sketch, the connection of the governor with a throttle valve. By what arrangement may the tendency to over-sensitiveness be corrected?

10. Find the height of a simple or "Watt" governor revolving at 80 revolutions per minute. If the same governor had a weight of 40 lbs. attached to the sleeve, the balls weighing 3 lbs. each, what should be its height, supposing the same speed to be maintained, and the link work to be such that the sleeve rises twice as fast as the balls? Neglect the weight of the connecting links. *Ans.* 5·5 inches and 6·5 feet.

11. Find the height of a simple conical pendulum revolving at 80 revolutions per minute. If a loaded governor, making 240 revolutions per minute, had a weight of 20 lbs. attached to the sleeve, the balls weighing 2 lbs. each, what would be its height, the vertical motion of the balls being half that of the sleeve?

12. Sketch and describe any spring-loaded governor, and compare the action of the spring with that of a weight.

13. What objection is there to regulating the speed of an engine by the throttle valve?

14. Explain what is meant by automatic expansion gear, showing wherein lie its special advantages in the economic working of an engine. Sketch such an arrangement and its connections.

15. Explain by the aid of the necessary sketches the construction of either the Armington-Sims or the Westinghouse high-speed flywheel governor and valve gear. Show clearly how in these arrangements the throw and angle of advance of the eccentric are varied, whilst the lead is kept constant. (Robey's and Ransoms, Sims & Jefferies' shaft governors are similar to those asked for.)

16. What special benefit is obtained by adding a relay to a governor? Sketch and describe a relay which automatically follows up the motion of the governor.

17. Sketch Knowles' supplemental governor and describe its action.

18. Describe the pendulum governor of the Otto engine, and point out, by reference to sketches, the manner in which it acts.

19. Explain clearly the arrangement by which the speed of an Otto engine is regulated.

20. Describe any form of inertia governor used for regulating the speed of a gas engine.

21. Describe, with proper sketches, a form of vibrating pendulum regulator as fitted to an Otto gas engine, and explain how it acts, and is made adjustable. Assuming that the pendulum is actuated by the rotation of the gas and air valve, describe the mechanism connecting the end of the valve with the pendulum, showing that it forms a well-known combination in linkwork.

22. Explain why it is necessary to balance high-speed machinery, and describe the most approved method of doing so.

23. What primary law in mechanics asserts itself when some revolving piece of machinery moves at a high velocity, and is unbalanced? A weight of 1 lb. is placed on the rim of a wheel 2 feet in diameter, which revolves upon its axis and is otherwise balanced. The linear velocity of the rim being 30 feet per second, what is the pull on the axis as caused by the weight of 1 lb.? *Ans.* 28·1 lbs.

24. Explain by sketches and description how railway carriage wheels for express trains and their axles are balanced. Give your reasons for and against the common workshop expression that a perfect statical balance is not one when the machine is run at a high speed.

25. Referring to Example III., in this Lecture, find the weight of the counterpoise when N, the revolutions per minute, is 100, and the height of cone, &c., are the same as in the sketch.

26. A machine shaft, 8 feet long between the centres of its bearings, carries two weights of 24 lbs. and 18 lbs. at radii of 9 inches and 18 inches respectively. The planes of rotation of these two weights are 2 feet and 5 feet respectively from the centre of the left-hand bearing, and the angle between their radii is 90°. Find the two weights acting at 12-inch radius, and their relative angular positions which will balance the system when placed in planes 4 feet and 7 feet respectively from the centre of the left-hand bearing. Draw the whole arrangement to scale in plan and end elevation.

27. Explain, by aid of diagrammatic sketches and any analogies you please, how the resistances due to *inertia*, *friction*, and *windage* vary from the time of starting to the stopping of a centrifugal sugar-drying machine.

28. Referring to p. 450, on water-driven centrifugals; let W, the weight of the basket and load, be 600 lbs.; N, the revolutions per minute, be 1,000; and k, the radius of gyration of the basket and load, be 12 inches.

Find (1) the kinetic energy at the above speed; (2) if 90 seconds are occupied in attaining full speed, 2·5 minutes running at full speed, and 100 seconds in bringing the whole to rest, find the foot-lbs. of work done and absorbed per minute during each of these three stages.

29. By aid of good sectional freehand sketches, explain the construction and action of water- and electric-driven centrifugals. Compare the efficiencies of these two methods, and state wherein they are to be preferred to belt-driven machines and why.

30. Explain, by aid of freehand sketches, the difference between the old and the latest methods of suspending the baskets and of oiling the bearings of centrifugal machines.

Lecture XVIII.—A.M.Inst.C.E. Exam. Questions.

1. In a loaded governor, the four equal links of which form a parallelogram with sides 8 inches long, the balls weigh 5 lbs. each, and the load is 20 lbs. Find the speed when the links stand at right angles. Find also what increase of speed is required to raise the load ½ inch higher.
(I.C.E., *Feb.*, 1898.)

2. Describe exactly what is the object of using a load on a watt governor. Prove what you say to be correct. (I.C.E., *Oct.*, 1898.)

3. The rods and links of a Porter's loaded governor are each 1 foot long, the balls each weighing 2 lbs., and the load 12 lbs. The valve is full open when the arms are at 30° to the vertical, and shut when they are at 45°. The velocity ratio between the engine-shaft and the governor-spindle is 2. Find the extreme working speeds of the engine.
(I.C.E., *Oct.*, 1900, and *Feb.*, 1903.)

4. Show that the height of a simple revolving pendulum when making n revolutions per minute is, approximately, $\frac{35{,}000}{n^2}$ inches. The balls of an unloaded governor each weigh 20 lbs., the arms and slide links are all equal, and, when running at 80 revolutions per minute, the actual height of the governor is 7 inches. Calculate the pull on the slides at this speed. (I.C.E., *Oct.*, 1902.)

5. What are the difficulties to be overcome in governing turbines to run at constant speed? Describe two methods of governing. (I.C.E., *Oct.*, 1902.) (See also Lecture V., Vol. IV., on "Turbines.")

6. Explain the use of a governor for a motor, quoting an instance in which it may be dispensed with. State and explain the causes which prevent governors of ordinary design from exactly accomplishing the desired purpose, and mention some devices by which the imperfections can be lessened. State the nature of the special arrangements requisite in the case of a gas engine, a marine engine, *or* a large hydraulic motor. (I.C.E., *Feb.*, 1905.)

7. What is meant by the "stability" of a steam engine governor? Show that with a stable governor the engine will run at lower speed at full load than at light loads. Explain the principle of the Knowles secondary governor, which maintains the speed constant at all loads. (I.C.E., *Feb.*, 1906.)

8. Sketch and describe any one form of spring-loaded governor, and sketch the curve connecting the controlling force with the radius of the path of the mass centre of the balls, assuming that the control is due to the spring alone. (I.C.E., *Oct.*, 1906.)

9. What is the essential difference between a centrifugal and an inertia governor? Give hand-sketches of each kind as applied to the form of governor known as the flywheel type (also known as a shaft governor), and suitable for shifting an eccentric actuating the admission valve.
(I.C.E., *Oct.*, 1907.)

10. A crank-shaft has four cranks spaced at 2 feet intervals, whose phase angles reading from one end are 0°, 90°, 180°, and 270° respectively. The rotating masses are equivalent to masses of 200, 400, 300, and 250 lbs. respectively, all at 1 foot radius. Reduce the centrifugal forces due to rotation of the shaft in its bearings, to a single force and a couple in a plane perpendicular to the single force. (Poinsot's central axis.) Take the angular velocity as unity. (I.C.E., *Oct.*, 1908.)

11. What are the usual assumptions made in calculating the magnitude and position of the balance-weights required on an ordinary uncoupled locomotive engine? Explain how you would calculate these weights. (I.C.E., *Oct.*, 1908.)

12. An engine of 800 I.H.P. runs at 120 revolutions per minute. Within what percentage could its speed be controlled by a governor if its load were suddenly reduced 75 per cent. and as suddenly again increased? Sketch any type of governor with which you are familiar which would control the speed within the limits given by you. (I.C.E., *Oct.*, 1908.)

APPENDICES.

APPENDIX A (p. 472 to p. 477).

(i.) General Instructions by The Board of Education for their Examinations on Applied Mechanics.

(ii.) General Instructions by The City and Guilds of London Institute for their Examination on Mechanical Engineering.

(iii.) Rules and Syllabus of Examinations by The Institution of Civil Engineers for Election of Associate Members.

APPENDIX B (p. 478 to p. 481).

The latest Exam. Papers pertaining to Theory of Machines and set by the governing bodies enumerated under Appendix A.

APPENDIX C (p. 482 to p. 487).

Tables of Constants, Logarithms, Antilogarithms, and Functions of Angles.

APPENDIX A.

MAY EXAMINATION ON SUBJECT VIIA. APPLIED MECHANICS.*

BY THE BOARD OF EDUCATION SECONDARY BRANCH, SOUTH KENSINGTON, LONDON.

GENERAL INSTRUCTIONS.

If the rules are not attended to, your paper will be cancelled.

Immediately before the Examination commences, the following REGULATIONS *are* TO BE READ TO THE CANDIDATES.

Before commencing your work, you are required to fill up the numbered slip which is attached to the blank examination paper.

You may not have with you any books, notes, or scribbling paper.

You are not allowed to write or make any marks upon your paper of questions, or to take it away before the close of the examination.

You must not, under any circumstances whatever, speak to or communicate with one another, and no explanation of the subject of examination may be asked or given.

You must remain seated until your papers have been collected, and then quietly leave the examination room. None of you will be permitted to leave before the expiration of one hour from the commencement of the examination, and no one can be re-admitted after having once left the room.

Your papers, unless previously given up, will all be collected at 10 o'clock.

If any of you break any of these rules, or use any unfair means, you will be expelled, and your paper cancelled.

Before commencing your work, you must carefully read the following instructions:—

Candidates who have applied for examination in the Elementary Stage must confine themselves to that stage. Candidates who have not applied to take the Elementary Stage may take Stage 2, or Stage 3, or, if eligible, Honours, but they must confine themselves to one of them.

Put the number of the question before your answer.

You are to confine your answers *strictly* to the questions proposed.

Such details of your calculations should be given as will show the methods employed in obtaining arithmetical results.

A table of logarithms and functions of angles and useful constants and formulæ is supplied to each candidate. (See end of Appendix to this Book.)

The examination in this subject lasts for three hours.

* *See Appendix B for the latest Exam. Papers.*

City and Guilds of London Institute.

DEPARTMENT OF TECHNOLOGY.

TECHNOLOGICAL EXAMINATIONS.

46.—MECHANICAL ENGINEERING.*

HONOURS GRADE (Written Examination).

INSTRUCTIONS.

The Candidate for Honours must have previously passed in the Ordinary Grade, and is required to pass a Written and Practical Examination. He is requested to state, on the Yellow Form, whether he has elected to be examined in A, Machine Designing, or in B, Workshop Practice (*a*) *Fitting*, (*b*) *Turning*, (*c*) *Pattern-making*. Candidates in Machine Designing must forward their work to London not later than May 6th,† and in Workshop Practice not later than May 13th.†

The number of the question must be placed before the answer in the worked paper.

The Candidate is at liberty to use divided scales, compasses, set squares, calculators, slide rules, and mathematical tables.

Five marks extra will be awarded for every answer worked out with the slide rule, provided the method of working is explained.

The maximum number of marks obtainable is affixed to each question.

Three hours allowed for this paper.

The Candidate is not expected to answer more than *eight* of the following questions, which must be selected from *two* sections *only*.

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* The questions in Sections A and C may be answered from Vols. I. to V. of my *Text-Book on Applied Mechanics*. Section B is printed in my *Text-Book on Steam and Steam Engines*. The Questions for the Ordinary Grade are printed at the end of my *Elementary Manual on Applied Mechanics*.

* *See Appendix B for the latest Exam. Papers.*

† These dates are only approximate, and subject to a slight alteration each year.

The Institution of Civil Engineers.

Extracts from Rules and Syllabus of Examinations for ELECTION OF ASSOCIATE MEMBERS.

Note.—Engineers who desire to enter for the A.M.Inst.C.E. examinations should write *at once* to the Secretary, Great George Street, Westminster, S.W., for the complete Rules, Syllabus, and Application Forms. They will find all the questions relating to the above mentioned subjects which have been set since these examinations commenced in 1897 in my **Text-Books.**

The following extracts are simply printed here to show how far my books upon *Applied Mechanics and Mechanical Engineering; Steam and Steam Engines, including Turbines and Boilers;* as well as *Magnetism and Electricity* (including Munro & Jamieson's *Pocket-Book of Electrical Rules and Tables*), together with my "*Correspondence System of Electrical and Mechanical Engineering Science, as taught by Exercises, Drawings, and Instructions*" cover the Scientific and Practical Knowledge demanded by the Institution, under Part II., **Section A** (1, 2, 3*a*, and 3*b*), as well as **Section B** under Group i., Theory of Heat Engines; Group ii., Hydraulics and Theory of Machines; and Group iii., Applications of Electricity.

Note for Students.—In the regulations for Students one of the subjects which may be selected is that of Elementary Mechanics of Solids and Fluids, for which see my *Elementary Applied Mechanics* when studying this subject. Also, see my *Elementary Magnetism and Electricity* book when reading that part of the Elementary Physics for admission of Students, or correspond with me *re* those two subjects.

Part II.*—*Scientific Knowledge.*

Section A.

1. **Mechanics** (one Paper, *time allowed*, 3 *hours*).
2. **Strength and Elasticity of Materials** (one Paper, *time allowed*, 3 *hours*).

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* Candidates may offer themselves for examination in Sections A and B of Part II. together; or they may enter for Section A alone, and, if successful, may take Section B at a subsequent examination. In the latter case, however, such candidates will not be allowed to present themselves for examination in Section B unless or until they are actually occupied in work as pupils or assistants to practising Engineers. The Council may permit Candidates who have attempted the whole of Part II. at one examination, and have failed in Section B only, to complete their qualification by passing in that section at a subsequent examination, subject to their being then occupied as above stated.

3. *Either* (*a*) **Theory of Structures,**
or (*b*) **Theory of Electricity and Magnetism** (one Paper, *time allowed*, 3 *hours*).

SECTION B.

Two of the following nine subjects—not more than one from any group (one Paper in each subject taken, *time allowed*, 3 *hours for each Paper*):—

Group i.	*Group* ii.	*Group* iii.
Geodesy.	Hydraulics.	Geology and Mineralogy.
Theory of Heat Engines.	Theory of Machines.	Stability and Resistance of Ships.
Thermo- and Electro-Chemistry.	Metallurgy.	Applications of Electricity.

Mathematics.—The standard of Mathematics required for the Papers in Part II. of the examination is that of the mathematical portion of the Examination for the Admission of Students, though questions may be set involving the use of higher Mathematics.

The range of the examinations in the several subjects, in each of which a choice of questions will be allowed, is indicated generally hereunder:—

SECTION A.

1. Mechanics:—

Statics.—Forces acting on a rigid body; moments of forces, composition, and resolution of forces; couples, conditions of equilibrium, with application to loaded structures. The foregoing subjects to be treated both graphically and by aid of algebra and geometry.

Hydrostatics.—Pressure at any point in a gravitating liquid; centre of pressure on immersed plane areas; specific gravity.

Kinematics of Plane Motion.—Velocity and acceleration of a point; instantaneous centre of a moving body.

Kinetics of Plane Motion.—Force, mass, momentum, moment of momentum, work, energy, their relation and their measure; equations of motion of a particle; rectilinear motion under the action of gravity; falling bodies and motion on an inclined plane; motion in a circle; centres of mass and moments of inertia; rotation of a rigid body about a fixed axis; conservation of energy.

2. Strength and Elasticity of Materials:—

Physical properties and elastic constants of cast iron, wrought iron, steel, timber, stone, and cement; relation of stress and strain, limit of elasticity, yield-point, Young's modulus; coefficient of rigidity; extension and lateral contraction; resistance within the elastic limit in tension, compression, shear and torsion; thin shells; strength and deflection in simple cases of bending; beams of uniform resistance; suddenly applied loads.

Ultimate strength with different modes of loading; plasticity, working stress; phenomena in an ordinary tensile test; stress-strain diagram; elongation and contraction of area; effects of hardening, tempering and annealing; fatigue of metals; measurement of hardness.

Forms and arrangements of testing machines for tension, compression, torsion, and bending tests; instruments for measuring extension, compression, and twist; forms of test pieces and arrangements for holding them;

influence of form on strength and elongation; methods of ordinary commercial testing; percentage of elongation and contraction of area; test conditions in specifications for cast iron, mild steel, and cement.

3. (*a*) Theory of Structures:—

Graphic and analytic methods for the calculation of bending moments and of shearing forces, and of the stresses in individual members of framework structures loaded at the joints; plate and box girders; incomplete and redundant frames; stresses suddenly applied, and effects of impact; buckling of struts; effect of different end-fastenings on their resistance; combined strains; calculations connected with statically indeterminate problems, as beams supported at three points, &c.; travelling loads; riveted and pin-joint girders; rigid and hinged arches; strains due to weight of structures; theory of earth-pressure and of foundations; stability of masonry and brickwork structures.

3. (*b*) Theory of Electricity and Magnetism:—

Electrical and magnetic laws, units, standards, and measurements; electrical and magnetic measuring instruments; the theory of the generation, storage, transformation, and distribution of electrical energy; continuous and alternating currents; arc and incandescent lamps; secondary cells.

Section B.

Group i. Theory of Heat Engines:—

Thermodynamic laws; internal and external work; graphical representation of changes in the condition of a fluid; theory of heat engines working with a perfect gas; air- and gas-engine cycles; reversibility, conditions necessary for maximum possible efficiency in any cycle; properties of steam; the Carnot and Clausius cycles; entropy and entropy-temperature diagrams, and their application in the study of heat engines; actual heat engine cycles and their thermodynamic losses; effects of clearance and throttling; initial condensation; testing of heat engines, and the apparatus employed; performances of typical engines of different classes; efficiency.

Group ii. Hydraulics:—

The laws of the flow of water by orifices, notches, and weirs; laws of fluid friction; steady flow in pipes or channels of uniform section; resistance of valves and bends; general phenomena of flow in rivers: methods of determining the discharge of streams; tidal action; generation and effect of waves; impulse and reaction of jets of water; transmission of energy by fluids; principles of machines acting by the weight, pressure, and kinetic energy of water; theory and structure of turbines and pumps.

Theory of Machines:—

Kinematics of machines; inversion of kinematic chains; virtual centres; belt, rope, chain, toothed and screw gearing; velocity, acceleration

and effort diagrams; inertia of reciprocating parts; elementary cases of balancing; governors and flywheels; friction and efficiency; strength and proportions of machine parts in simple cases.

Group iii. Applications of Electricity:—

Theory and design of continuous- and alternating-current generators and motors, synchronous and induction motors and static transformers; design of generating- and sub-stations and the principal plant required in them; the principal systems of distributing electrical energy, including the arrangement of mains and feeders; estimation of losses and of efficiency; principal systems of electric traction; construction and efficiency of the principal types of electric lamps.

☞ Candidates should see, that all their "Forms" are *duly completed* and *passed* by the Council of the Institution of Civil Engineers, Great George Street, Westminster, S.W., *before* 1st January for the February Examination, and *before* the 1st September for the October Examination. Candidates should, therefore, apply to the Secretary for the "Forms," *at least six* months *before* these Examinations, to give them time to make due and proper Application, and to thoroughly *Revise* the subjects upon which they are to be examined with an experienced **Guide** and Tuition by Correspondence.

Examinations Abroad.—The papers of the *October* Examination *only* will be placed before accepted Candidates in India and the Colonies. To enable the Secretary to make arrangements for the Application Forms and Fees, &c., of these Candidates, their Forms, &c., *must* be in the Secretary's hands, *before* the 1st June preceding the October Examinations.

N.B.—*See Appendix B for the latest Exam. Papers in Theory of Machines.*

APPENDIX B.

THE INSTITUTION OF CIVIL ENGINEERS.

ELECTION OF ASSOCIATE MEMBERS.

THEORY OF MACHINES.

Not more than EIGHT *questions to be attempted by any Candidate.*

February, 1909.

1. Sketch the ideal mechanism for a reciprocating steam engine. If the stroke is 3 feet and the length of the connecting-rod five times that of the crank, draw the path of the virtual centre between the crank and cross-head for one complete revolution.

2. Sketch any arrangement of four toothed wheels by which a ratio of $\frac{3}{4}\frac{7}{0}\frac{1}{0}\frac{1}{0}$ could be transmitted in a practicable and satisfactory manner. Give the number of teeth in each wheel.

3. Draw a diagram showing the velocity ratio between the framework and flywheel radius of a donkey pump based on the infinite connecting-rod mechanism. Show the position of all the virtual centres for the mechanism. How could the same velocity ratio be obtained by the use of spur wheels?

4. Design an involute tooth for a pair of spur wheels, of which the smaller wheel is 10 inches in diameter, to transmit 20 H.P. at 400 revolutions per minute of the smaller wheel and an approximate velocity ratio of 3 to 1. How many teeth would you put in each wheel?

5. Briefly explain how a formula may be obtained for the determination of the size of a leather belt to transmit a given horse-power at a given speed. State any assumptions made and give the approximate limits of speed of belt beyond which practical difficulties might be expected to arise and the nature of those difficulties.

6. Explain the terms powerfulness, sensitiveness, isochronism, as applied to governors. Show how each may be attained in an actual governor. Mention any difficulties in the way of making any of these qualities approach perfection.

7. Sketch the arrangement for altering the cut-off in a steam engine with either (1) Joy's valve gear; or (2) Stephenson's gear; or (3) Corliss gear. Show diagrammatically the effect on the steam distribution of such alteration of cut-off.

8. An engine of 100 brake H.P. running at 330 revolutions per minute runs so that its maximum rate is not greater than 335 and its minimum

less than 325 revolutions per minute. Calculate the moment of inertia of the flywheel required. The fluctuation in energy is 5 per cent. each side of the mean per stroke and the load overcome is constant.

9. Sketch and explain the method of adjustment for wear of either (1) a plummer-block for a screw propeller shaft; or (2) a stuffing-box for a locomotive piston-rod; or (3) a differential gear for a motor-car.

10. Why is it not possible to completely balance the moving parts of a reciprocating engine? Explain what compromise is usually adopted.

11. Show how the inertia of the moving parts of a reciprocating engine affects the diagram for the torque on the crank-shaft.

12. Over 200 H.P. has to be transmitted from one shaft running at 200 revolutions per minute to another parallel and in close proximity to it running at 300 revolutions per minute. Sketch any method by which this could be efficiently accomplished, stating the conditions which must be satisfied to obtain a satisfactory result.

October, 1909.

1. The connecting-rod of a steam engine is 52 inches long, and the crank radius is 13 inches. Find the angular velocity of the crank shaft relative to the main bearing, the angular velocity of the crank-pin relative to the connecting-rod bearing, and the angular velocity of the connecting-rod relative to the crosshead pin; when the crank has turned through 30° from the inner dead centre and the revolutions are 200 per minute. The angular velocities are to be expressed in radians per second.

2. The reciprocating parts of a steam engine weigh $8\frac{1}{8}$ tons; the connecting-rod is 17·5 feet long, and the crank radius is 3 feet. Calculate the force required for acceleration of the reciprocating masses at the beginning and end of the stroke respectively, when the speed is 60 revolutions per minute.

3. A train weighing 200 tons is travelling at 40 miles per hour up a bank of 1 in 75, against a tractive resistance of 25 lbs. per ton. Find the horse-power necessary to maintain the speed.

4. The tup of a drop forging plant is sometimes lifted by means of a belt passed over a revolving pulley on a main shaft running above the hammers. The belt over the pulley is arranged so that one end is attached to the tup whilst the other end hangs free, and within reach of the smith operating the hammer. Suppose the arc of contact between the belt and the pulley is 180°, and that the coefficient of friction between the leather and the pulley is 0·5, find what weight of tup could be lifted when the smith exerts a pull of 50 lbs. on the free end of the belt.

5. The cylinders of a four-cylinder vertical marine engine are pitched at equal distances apart. The reciprocating masses belonging to each of the two middle lines of parts weigh 1 ton. The cranks of the middle pair of cylinders are set at an angle of 100°. Find the angle between each outside crank and its neighbour, and the weight of each of the outer reciprocating line of parts, so that the engine shall be in balance for primary forces and couples.

6. Make a sketch of any one form of governor in which the control is effected mainly by a spring. Distinguish between stable and unstable conditions of running, and show, by the aid of a diagram, how the stability is affected by the initial compression (or tension) of the controlling spring.

7. Make a sketch of the application of an epicyclic train of wheels to any purpose which has come under your notice, and show how to find the relative velocity of the first and last piece in the train.

8. The difference between the driving and the back pressures in a steam-engine cylinder is 60 tons when the crank is 30° from the inner dead point. The reciprocating parts weigh 7 tons. The connecting-rod is five times as long as the crank, which is 3 feet radius. Calculate the corresponding value of the turning moment in the crank shaft when the speed is 60 revolutions per minute. (Any graphical methods may be used in the solution of this problem.)

9. Make a sketch of a piston valve in its central position over the ports, and arranged to take steam on the inside of the valve. Assuming the valve to be operated by a simple valve gear with a single eccentric sheave, find the radius of the eccentric sheave, its angular position with regard to the crank, and the steam lap, so that cut-off may take place when the crank is at 120° from the dead point, the lead and the maximum port opening for steam being respectively $\frac{1}{10}$ inch and 1 inch. (Neglect the effect of the obliquity of the connecting-rod.)

19. Make a sketch of a Joy valve motion, and point out clearly how reversal of the engine is determined by the gear, and how the cut-off in the cylinder may be varied.

11. It has been found by experiment that, with the regulator wide open and a given cut-off in a locomotive cylinder, the mean pressure is a function of the piston speed of the form

$$p = a - cv,$$

where a and c are constants, p is the mean pressure in lbs. per square inch, and v is the piston speed in feet per minute. Show from this that there is one piston speed at which the horse-power is a maximum for the given cut-off, and determine what this piston speed is when $c = 0{\cdot}068$ and $a = 99$.

12. Make a sketch of any one type of axle-box suitable for the main journals of the driving axle of a locomotive. Show clearly the arrangements made for lubrication—*or* sketch a bearing for one of the crank-shaft journals of a high-speed engine arranged for forced lubrication.

February, 1910.

1. Draw a diagram showing the velocity of the crosshead in the case of an ordinary steam engine mechanism where the connecting-rod is four times the length of the crank, and the crank, which is 1 foot radius, revolves uniformly at 60 revolutions per minute. Determine from your diagram the maximum crosshead velocity and the corresponding crank angle.

2. Give any graphical construction by means of which the acceleration of the piston of a steam engine can be found for any assigned position of the crank and speed of rotation. Apply the construction to find the acceleration in the case where the crank is 1 foot radius, the connecting-rod 3 feet 6 inches long, and the crank is at 45° from the inner dead centre. Speed 300 revolutions per minute.

3. A 5 horse-power motor, the armature of which runs at 500 revolutions per minute, is connected by a belt to a shaft which is to run at 300 revolutions per minute. Assume that the ratio of the tensions of the tight and

slack sides of the belt is 2 to 1. Find the width of the belt, allowing a maximum tension of 100 lbs. per inch of width. The diameter of the pulley on the armature is 10 inches.

4. Find the speed at which an engine, whose centre of gravity is 5 feet above rail level, would first be unstable on a curve of 10 chains radius without superelevation and 4 feet 8½ inches gauge—*i.e.*, say, 5 feet centre to centre of rails.

5. The velocity ratio between the first motion shaft and the chain barrel of a crane is to be 105 : 1. Design a suitable train of wheels.

6. A flywheel weighs 44·4 tons. Its radius of gyration is 11·5 feet. Find how much energy it stores at 60 revolutions per minute. If the variation of torque on the crank shaft is such that 55 foot-tons of energy are alternately added to and subtracted from the energy stored in the wheel, calculate the corresponding maximum and minimum number of revolutions of the wheel per minute.

7. Sketch any one form of loaded governor, and point out the advantage due to the loading.

8. The reciprocating parts of a two-cylinder locomotive weigh 550 lbs. per cylinder. Two-thirds of this amount is balanced by means of balance weights placed in the driving wheels. The revolving masses are completely balanced. Find the maximum pressure between a driving wheel and the rail at 60 miles per hour from the following data :—Diameter of driving wheel, 7 feet ; static load on driving wheel, 6 tons ; distance centre to centre of cylinders, 2 feet ; distance between the planes in which the mass centres of the balance weights in the driving wheels revolve, 5 feet ; crank radius, 1 foot.

9. Describe a rope dynamometer suitable for measuring the brake horse-power of a small steam engine. An engine provided with two flywheels each 6 feet in diameter has a rope brake fitted to each wheel. When the speed is 200 revolutions per minute the loads hanging on the ropes are respectively 120 and 130 lbs., and the pull on the tail end of each rope is 10 lbs. The girth of the ropes is 2 inches. Find the brake horse-power.

10. Make a sketch of the differential gear used on motor cars, and point out the object of its use.

11. Describe with the aid of sketches a Stephenson link motion. What do you understand by the term equivalent eccentric as applied to a link motion ? Find the equivalent eccentric corresponding to the mid gear position of the motion block for a link motion having the following dimensions :—Angular distance between each eccentric and the crank, 104° ; length of each eccentric-rod, 6 feet 3 inches ; length of the link, 24¾ inches ; radius of each eccentric sheave, 4·3 inches.

12. Make a sketch of the big end of a connecting-rod of any type with which you are familiar, and point out clearly how the wear in the brasses is taken up.

APPENDIX C.

EXAMINATION TABLES.

USEFUL CONSTANTS.

1 Inch = 25·4 millimetres.

1 Gallon = ·1605 cubic foot = 10 lbs. of water at 62° F. ∴ 1 lb. = ·01605 cubic foot.

1 Knot = 6080 feet per hour. 1 Naut = 6080 feet.

Weight of 1 lb. in London = 445,000 dynes.

One pound avoirdupois = 7000 grains = 453·6 grammes.

1 Cubic foot of water weighs 62·3 lbs.

1 Cubic foot of air at 0° C. and 1 atmosphere, weighs ·0807 lb.

1 Cubic foot of Hydrogen at 0° C. and 1 atmosphere, weighs ·00557 lb.

1 Foot-pound = $1{\cdot}3562 \times 10^7$ ergs.

1 Horse-power-hour = 33000 × 60 foot-pounds.

1 Electrical unit = 1000 watt-hours.

Joule's Equivalent to suit Regnault's H, is { 774 ft.-lbs. = 1 Fah. unit. / 1393 ft.-lbs. = 1 Cent. „

1 Horse-power = 33000 foot-pounds per minute = 746 watts.

Volts × amperes = watts.

1 Atmosphere = 14·7 lb. per square inch = 2116 lbs. per square foot = 760 m.m. of mercury = 10^6 dynes per sq. cm. nearly.

A Column of water 2·3 feet high corresponds to a pressure of 1 lb. per square inch.

Absolute temp., $t = \theta° \text{ C.} + 273°{\cdot}7$.

Regnault's $H = 606{\cdot}5 + {\cdot}305\ \theta° \text{ C.} = 1082 + {\cdot}305\ \Theta° \text{ F.}$

$p\, u^{1{\cdot}0646} = 479$

$$\log_{10} p = 6{\cdot}1007 - \frac{B}{t} - \frac{C}{t^2}$$

where $\log_{10} B = 3{\cdot}1812$, $\log_{10} C = 5{\cdot}0871$,

p is in pounds per square inch, t is absolute temperature Centigrade, u is the volume in cubic feet per pound of steam.

$$\pi = 3{\cdot}1416 = \frac{22}{7} = \frac{355}{113} = 10(\sqrt{3} - \sqrt{2}).$$

One radian = 57·3 degrees.

To convert common into Napierian logarithms, multiply by 2·3026.

The base of the Napierian logarithm is $e = 2{\cdot}7183$.

The value of g at London = 32·182 feet per second per second.

TABLE OF LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

TABLE OF LOGARITHMS.—*Continued.*

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56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 6 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6 6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1 1 2	3 3 4	5 6 6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1 2	3 3 4	5 5 6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1 1 2	3 3 4	5 5 6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1 1 2	3 3 4	5 5 6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1 2	3 3 4	5 5 6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1 1 2	3 3 4	5 5 6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1 2	3 3 4	4 5 6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1 2	2 3 4	4 5 6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1 1 2	2 3 4	4 5 6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1 1 2	2 3 4	4 5 5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1 1 2	2 3 4	4 5 5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1 1 2	2 3 4	4 5 5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1 1 2	2 3 4	4 5 5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1 1 2	2 3 3	4 5 5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1 1 2	2 3 3	4 5 5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1 1 2	2 3 3	4 4 5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1 1 2	2 3 3	4 4 5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1 1 2	2 3 3	4 4 5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1 1 2	2 3 3	4 4 5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1 1 2	2 3 3	4 4 5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1 1 2	2 3 3	4 4 5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1 2	2 3 3	4 4 5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1 1 2	2 3 3	4 4 5
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87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0 1 1	2 2 3	3 4 4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0 1 1	2 2 3	3 4 4
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90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0 1 1	2 2 3	3 4 4
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94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0 1 1	2 2 3	3 4 4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0 1 1	2 2 3	3 4 4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0 1 1	2 2 3	3 4 4
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98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1 1	2 2 3	3 4 4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0 1 1	2 2 3	3 3 4

TABLE OF ANTILOGARITHMS.

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·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0 0 1	1 1 1	2 2 2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0 0 1	1 1 1	2 2 2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0 1 1	1 1 2	2 2 2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0 1 1	1 1 2	2 2 2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0 1 1	1 1 2	2 2 3
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0 1 1	1 1 2	2 2 2
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·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0 1 1	1 1 2	2 2 3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0 1 1	1 1 2	2 2 3
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·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0 1 1	1 2 2	2 3 3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0 1 1	1 2 2	2 3 3
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·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0 1 1	2 2 2	3 3 3
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·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0 1 1	2 2 2	3 3 4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0 1 1	2 2 2	3 3 4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0 1 1	2 2 2	3 3 4
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·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1 1 2	3 3 4	5 5 6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1 1 2	3 4 4	5 6 6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1 1 2	3 4 4	5 6 6

TABLE OF ANTILOGARITHMS.—*Continued.*

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·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
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·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
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·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
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·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8891	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

TABLE OF FUNCTIONS OF ANGLES.

Angle.		Chord.	Sine.	Tangent.	Co-tangent.	Cosine.			
Degrees	Radians.								
0°	0	000	0	0	∞	1	1·414	1·5708	90°
1	·0175	·017	·0175	·0175	57·2900	·9998	1·402	1·5533	89
2	·0349	·035	·0349	·0349	28·6363	·9994	1·389	1·5359	88
3	·0524	·052	·0523	·0524	19·0811	·9986	1·377	1·5184	87
4	·0698	·070	·0698	·0699	14·3007	·9976	1·364	1·5010	86
5	·0873	·087	·0872	·0875	11·4301	·9962	1·351	1·4835	85
6	·1047	·105	·1045	·1051	9·5144	·9945	1·338	1·4661	84
7	·1222	·122	·1219	·1228	8·1443	·9925	1·325	1·4486	83
8	·1396	·140	·1392	·1405	7·1154	·9903	1·312	1·4312	82
9	·1571	·157	·1564	·1584	6·3138	·9877	1·299	1·4137	81
10	·1745	·174	·1736	·1768	5·6713	·9848	1·286	1·3963	80
11	·1929	·192	·1908	·1944	5·1446	·9816	1·272	1·3788	79
12	·2094	·209	·2079	·2126	4·7046	·9781	1·259	1·3614	78
13	·2269	·226	·2250	·2309	4·3315	·9744	1·245	1·3439	77
14	·2443	·244	·2419	·2493	4·0108	·9703	1·231	1·3265	76
15	·2618	·261	·2588	·2679	3·7321	·9659	1·218	1·3090	75
16	·2793	·278	·2756	·2867	3·4874	·9613	1·204	1·2915	74
17	·2967	·296	·2924	·3057	3·2709	·9563	1·190	1·2741	73
18	·3142	·313	·3090	·3249	3·0777	·9511	1·176	1·2566	72
19	·3316	·330	·3256	·3443	2·9042	·9455	1·161	1·2392	71
20	·3491	·347	·3420	·3640	2·7475	·9397	1·147	1·2217	70
21	·3665	·364	·3584	·3839	2·6051	·9336	1·133	1·2043	69
22	·3840	·382	·3746	·4040	2·4751	·9272	1·118	1·1868	68
23	·4014	·399	·3907	·4245	2·3559	·9205	1·104	1·1694	67
24	·4189	·416	·4067	·4452	2·2460	·9135	1·089	1·1519	66
25	·4363	·433	·4226	·4663	2·1445	·9063	1·075	1·1345	65
26	·4538	·450	·4384	·4877	2·0503	·8988	1·060	1·1170	64
27	·4712	·467	·4540	·5095	1·9626	·8910	1·045	1·0996	63
28	·4887	·484	·4695	·5317	1·8807	·8829	1·030	1·0821	62
29	·5061	·501	·4848	·5543	1·8040	·8746	1·015	1·0647	61
30	·5236	·518	·5000	·5774	1·7321	·8660	1·000	1·0472	60
31	·5411	·534	·5150	·6009	1·6643	·8572	·985	1·0297	59
32	·5585	·551	·5299	·6249	1·6003	·8480	·970	1·0123	58
33	·5760	·568	·5446	·6494	1·5399	·8387	·954	·9948	57
34	·5934	·585	·5592	·6745	1·4826	·8290	·939	·9774	56
35	·6109	·601	·5736	·7002	1·4281	·8192	·923	·9599	55
36	·6283	·618	·5878	·7265	1·3764	·8090	·908	·9425	54
37	·6458	·635	·6018	·7536	1·3270	·7986	·892	·9250	53
38	·6632	·651	·6157	·7813	1·2799	·7880	·877	·9076	52
39	·6807	·668	·6293	·8098	1·2349	·7771	·861	·8901	51
40	·6981	·684	·6428	·8391	1·1918	·7660	·845	·8727	50
41	·7156	·700	·6561	·8693	1·1504	·7547	·829	·8552	49
42	·7330	·717	·6691	·9004	1·1106	·7431	·813	·8378	48
43	·7505	·733	·6820	·9325	1·0724	·7314	·797	·8203	47
44	·7679	·749	·6947	·9657	1·0355	·7193	·781	·8029	46
45°	·7854	·765	·7071	1·0000	1·0000	·7071	·765	·7854	45°
			Cosine.	Co-tangent.	Tangent.	Sine.	Chord.	Radians.	Degrees.
								Angle.	

1910 MAY EXAMINATION ON SUBJECT VII. APPLIED MECHANICS.*

(*b*) MACHINES AND HYDRAULICS.

BY THE BOARD OF EDUCATION SECONDARY BRANCH, SOUTH KENSINGTON, LONDON.

FOR GENERAL INSTRUCTIONS, SEE APPENDIX A.

STAGE 2.†

INSTRUCTIONS.

You must not attempt more than EIGHT *questions in all, including Nos.* 21 *and* 22—*that is to say, although* 21 *and* 22 *are not compulsory, you are not allowed to take more than six questions in addition to Nos.* 21 *and* 22.

3. Describe a differential pulley block. The diameters of the two grooves are 12 and 11½ inches, what is the velocity ratio? Experiments are made on this pulley block when a load W is lifted by an effort E. When W was 600 lbs. E was 26 lbs., and when W was 300 lbs. E was 18 lbs.; what is E probably when W is 800 lbs.? What is the efficiency when W is 800 lbs.? (B. of E., S. 2, Div. *b*, 1910.)

5. State the following in horse-power hours:—(*a*) Five million foot-pounds, (*b*) three Board of Trade electrical units, (*c*) 20 Fahrenheit heat units, (*d*) 20 Centigrade heat units, (*e*) the calorific energy of 1 lb. of coal or 8,500 Centigrade heat units, (*f*) the Calorific energy of 1 lb. of petrol, 12,400 Centigrade heat units, (*g*) the Calorific energy of 1 cubic foot of coal gas, 380 Centigrade heat units. (B. of E., S. 2, Div. *b*, 1910.)

9. An electromotor lifts 80 tons of grain 100 feet high; the electric energy costs 40 pence at the rate of 2½ pence per unit. How much electric energy is used? What is the efficiency of the lifting arrangements? (B. of E., S. 2, Div. *b*, 1910.)

13. A Hydraulic Company charges 15 pence per thousand gallons of water at 700 lbs. per square inch; what is the cost per horse-power-hour? (B. of E., S. 2, Div. *b*, 1910.)

14. A body has a simple harmonic motion, the total length of one swing being 2 feet; it makes one swing in half a second, that is, its periodic time is one second. Make a diagram showing its velocity and another showing its acceleration at every point of its path. What are its maximum velocity and acceleration? (B. of E., S. 2, Div. *b*, 1910.)

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* Students are referred to Vols. II. and III. for Questions under Div. (*a*) *Materials and Structures.*

† See Vol. I.—*Applied Mechanics*—for Questions 7, 10, 12, and 15.

STAGE 3.*

Instructions.

You must not attempt more than EIGHT *questions in all, including Nos.* 21 *and* 22*—that is to say, although* 21 *and* 22 *are not compulsory, you are not allowed to take more than six questions in addition to Nos.* 21 *and* 22.

21.† Describe, with the help of neatly drawn sketches, which should be roughly to scale, only *one* of the following, (*a*), (*b*), (*c*), or (*d*) :—(*a*) An accumulator ; show in detail how the ram is packed, and the valves for admitting the water to and drawing the water from the accumulator. (*b*) A swivel bearing for use with line shafting provided with means for raising and lowering the centre point of the bearing. What advantages has this type of bearing over the old form of pedestal ? (*c*) A countershaft fitted with fast and loose pulleys. Show in detail the striking gear, and the arrangements for lubricating the brass bushing of the loose pulley. (*d*) Any form of quick-return motion suitable for use on a shaping machine. Show how the stroke of the tool may be varied ?

(B. of E., S. 2 and 3, Div. *b*, 1910.)

22. Answer only *one* of the following, (*a*), (*b*), or (*c*) :—(*a*) Describe, with sketches of the apparatus, any simple method of determining the frictional losses in a straight length of pipe, when water is flowing through it at a steady velocity. (*b*) Describe, with sketches of the apparatus, how you would determine the velocity ratio and the mechanical efficiency under different loads of a screw-jack. Will a screw-jack overhaul ? (*c*) Describe any experimental method of determining the moment of inertia of a small flywheel. You must state clearly all the observations you would require to make, and how you would obtain the desired result from these observations. (B. of E., S. 2 and 3, Div. *b*, 1910.)

28. An electric motor running at 1,500 revolutions per minute has a pulley 10 inches in diameter. The belt is 8 inches wide, and $\frac{1}{4}$ inch thick, and its arc of contact is 160°. The allowable stress in the belt is 400 lbs. per square inch, and the coefficient of friction between belt and pulley is 0·2. What is the maximum horse-power which can be transmitted ?

(B. of E., S. 3, Div. *b*, 1910.)

30. A body weighing 161 lbs. has a simple harmonic motion, the total length of one swing being 2 feet ; the periodic time is 1 second. Make a diagram showing its velocity and another showing its acceleration at every point of its path. What force is giving to the body this motion ? What is its greatest value ? (B. of E., S. 3, Div. *b*, 1910.)

33. Explain, with sketches, only one, (*a*) or (*b*) :—(*a*) How Mr. Schlick uses gyrostats to destroy the rolling of ships. (*b*) How Mr. Brennan uses gyrostats in his mono-rail waggons. (B. of E., S. 3, Div. *b*, 1910.)

34. A weight of 16·1 lbs. hangs from a spiral spring whose stiffness is such that an axial force of 1 lb. elongates it 0·01 foot. The motion is resisted by fluid friction which in pounds is numerically equal to ten times the velocity. The point of support gets a downward displacement $y = 0{\cdot}1 \sin 9t$, find the motion of the weight when its natural vibration has been destroyed by friction. (B. of E., S. 3, Div. *b*, 1910.)

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* See Vol. I.—*Applied Mechanics*—for Questions 24, 27, 29, 31, and 32.
† See Vol. I.—*Applied Mechanics*—for helps to Parts (*b*) and (*c*).

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